.

LECTURE NOTES

ON

MATHEMATICS-II

ACADEMIC YEAR 2021-22

I B.TECH –II SEMISTER(R20)

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DEPARTMENT OF HUMANITIES AND BASIC SCIENCES

VSM COLLEGE OF ENGINEERING

RAMACHANDRAPURAM

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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY: KAKINADA
KAKINADA – 533 003, Andhra Pradesh, India
DEPARTMENT OF ELECTRONICS AND COMMUNICATION ENGINEERING

I Year - II Semester		L	T	P	C
		3	0	0	3
MATHEMATICS-II					

Course Objectives:
- To instruct the concept of Matrices in solving linear algebraic equations
- To elucidate the different numerical methods to solve nonlinear algebraic equations
- To disseminate the use of different numerical techniques for carrying out numerical integration.
- To equip the students with standard concepts and tools at an intermediate to advanced level mathematics to develop the confidence and ability among the students to handle various real world problems and their applications.

Course Outcomes: At the end of the course, the student will be able to
- develop the use of matrix algebra techniques that is needed by engineers for practical applications (L6)
- solve system of linear algebraic equations using Gauss elimination, Gauss Jordan, Gauss Seidel (L3)
- evaluate the approximate roots of polynomial and transcendental equations by different algorithms (L5)
- apply Newton's forward & backward interpolation and Lagrange's formulae for equal and unequal intervals (L3)
- apply numerical integral techniques to different Engineering problems (L3)
- apply different algorithms for approximating the solutions of ordinary differential equations with initial conditions to its analytical computations (L3)

UNIT – I: Solving systems of linear equations, Eigen values and Eigen vectors: (10hrs)
Rank of a matrix by echelon form and normal form – Solving system of homogeneous and non-homogeneous linear equations – Gauss Eliminationmethod – Eigen values and Eigen vectors and properties (article-2.14 in text book-1).

Unit – II: Cayley–Hamilton theorem and Quadratic forms: (10hrs)
Cayley-Hamilton theorem (without proof) – Applications – Finding the inverse and power of a matrix by Cayley-Hamilton theorem – Reduction to Diagonal form – Quadratic forms and nature of the quadratic forms – Reduction of quadratic form to canonical forms by orthogonal transformation. Singular values of a matrix, singular value decomposition (text book-3).

UNIT – III: Iterative methods: (8 hrs)
Introduction– Bisection method–Secant method – Method of false position– Iteration method – Newton-Raphson method (One variable and simultaneous Equations) – Jacobi and Gauss-Seidel methods for solving system of equations numerically.

UNIT – IV: Interpolation: (10 hrs)
Introduction– Errors in polynomial interpolation – Finite differences– Forward differences– Backward differences –Central differences – Relations between operators – Newton's forward and backward formulae for interpolation – Interpolation with unequal intervals – Lagrange's interpolation formula– Newton's divide difference formula.

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UNIT – V: Numerical differentiation and integration, Solution of ordinary differential equations with initial conditions: (10 hrs)

Numerical differentiation using interpolating polynomial – Trapezoidal rule– Simpson's 1/3rd and 3/8th rule– Solution of initial value problems by Taylor's series– Picard's method of successive approximations– Euler's method – Runge-Kutta method (second and fourth order).

Text Books:

1. **B. S. Grewal,** Higher Engineering Mathematics, 44th Edition, Khanna Publishers.
2. **B. V. Ramana,**Higher Engineering Mathematics, 2007 Edition, Tata Mc. Graw Hill Education.
3. **David Poole,** Linear Algebra- A modern introduction, 4th Edition, Cengage.

Reference Books:

1. **Steven C. Chapra,** Applied Numerical Methods with MATLAB for Engineering and Science,Tata Mc. Graw Hill Education.
2. **M. K. Jain, S.R.K. Iyengar and R.K. Jain,** Numerical Methods for Scientific and Engineering Computation, New Age International Publications.
3. **Lawrence Turyn,** Advanced Engineering Mathematics, CRC Press.

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DEPARTMENT OF HUMANITIES AND BASIC SCIENCES

Course Title	Year-Sem	Branch	Contact Periods/Week	Sections
Mathematics-II	1-2	Electrical & electronics Engineering	6	-

COURSE OUTCOMES: At the end of the course, the student will be able to

1. Develop the use of matrix algebra techniques that is needed by engineers for practical applications(K2)
2. Solve system of linear algebraic equations using Gauss elimination, Gauss Jordan, Gauss Seidel(K1)
3. Evaluate the approximate roots of polynomial and transcendental equations by differentalgorithms (K3)
4. Apply Newton's forward & backward interpolation and Lagrange's formulae for equal andunequal intervals (K2)
5. Apply numerical integral techniques to different Engineering problems (K3)
6. Apply different algorithms for approximating the solutions of ordinary differential equations withinitial conditions to its analytical computations (K4)

Unit/item No.	Outcomes		Topic	Number of periods	Total periods	Book Reference	Delivery Method
1	**CO1:** Solving systems of linear equations, Eigen values and Eigen vectors		**UNIT-1**		10	T1,T3, R2	Chalk & Talk, & Tutorial
		1.1	Rank of a matrix by echelon form and normal form	2			
		1.2	Solving system of homogeneous and	2			
		1.3	non- homogeneous linear equations	2			
		1.4	Gauss Eliminationmethod.	2			
		1.5	Eigen values and Eigen vectors and properties	2			
2	**CO2:** Cayley–Hamilton theorem and Quadratic forms		**UNIT-2**		10	T1,T3, R2	Chalk & Talk, & Tutorial
		2.1	Cayley-Hamilton theorem (without proof) – – Applications	2			
		2.2	Finding the inverse and power of a matrix by Cayley-Hamilton theorem	2			
		2.3	Reduction to Diagonal form – Quadratic forms and nature of the quadratic forms	2			
		2.4	Reduction of quadratic form to canonical forms by orthogonal transformation.	2			
		2.5	Singular values of a matrix, singular value decomposition	2			

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3	CO3: Iterative methods	3.1	Introduction– Bisection method	2	15	T1,T3, R2	Chalk & Talk, & Tutorial
		3.2	Secant method	2			
		3.3	Method of false position	2			
		3.4	Iteration method	2			
		3.5	Newton- Raphson method (One variable and simultaneous Equations)	3			
		3.6	Jacobi and GaussSeidel methods for solving system of equations numerically	4			
4	**<u>CO4</u>:** : Interpolation	**UNIT-4**			15	T1,T3, R2	Chalk & Talk, & Tutorial
		4.1	Introduction– Errors in polynomial interpolation	2			
		4.2	Finitedifferences–Forward differences–Backward differences	4			
		4.3	Central differences – Relations between operators	2			
		4.4	Newton's forward and backward formulae for interpolation	3			
		4.5	Interpolation with unequal intervals – Lagrange's interpolation formula	3			
		4.6	Newton's divide difference formula	1			
5	**<u>CO5</u>**: Numerical differentiation and integration, Solution of ordinary differential equations with initial conditions	**UNIT-5**					
		5.1	Numerical differentiation using interpolating polynomial	1	10	T1,T3, R2	Chalk & Talk, Tutorial
		5.2	Trapezoidal rule	2			
		5.3	Simpson's 1/3rd and 3/8th rule	2			
		5.4	Solution of initial value problems by Taylor's series	2			
		5.5	Picard's method of successive approximations	1			
		5.6	Problems on Filters	1			
		5.7	Euler's method –Runge-Kutta method (second and fourth order)	1			

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LIST OF TEXT BOOKS AND AUTHORS

Text Books:

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2. B.V.Ramana,Higher Engineering Mathematics, 2007 Edition, Tata Mc. Graw HillEducation.
3. David Poole, Linear Algebra- A modern introduction, 4 thEdition, Cengage.

Reference Books:

R1. Steven C. Chapra, Applied Numerical Methods with MATLAB for Engineering andScience,Tata Mc. Graw Hill Education.
R2. M. K. Jain, S.R.K. Iyengar and R.K. Jain, Numerical Methods for Scientific and EngineeringComputation, New Age International Publications.
R3. Lawrence Turyn, Advanced Engineering Mathematics, CRC Press.

Faculty Member Head of the Department PRINCIPAL

Unit-1

Linear System of Equations

Real and complex matrices and linear system of Equation

Matrix Definition:-

A system of mn numbers (real and complex) arranged in the form of an ordered set of m rows, each row consisting of an ordered set of n numbers between [ ] or ( ) or || || is called a matrix of order (or) type $m\times n$

Each of mn numbers constituting the $m\times n$ matrix is called an element of the matrix. Thus we write a matrix

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m\times n} = [a_{ij}]_{m\times n}$$

where $1 \le i \le m$, $1 \le j \le n$

In relation to a matrix, we call the numbers as a scalars.

Type of Matrices

Definition:-

1. If $A = [a_{ij}]_{m\times n}$ and $m=n$, then A is called a square matrix. A square matrix A of order

$n \times n$ is something called as a n-rowed matrix

A (or) simply a square matrix of order n.

Eg:- $\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$ is 2nd order matrix

2. A matrix which is not a square matrix is called a rectangular matrix

Eg:- $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 4 \end{bmatrix}$ is a 2×3 matrix

3. A matrix of order $1 \times m$ is called a row matrix

Eg:- $[1 \ 2 \ 3]_{1 \times 3}$

4. A matrix of order $n \times 1$ is called a column matrix

Eg:- $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}_{3 \times 1}$

* Row and column matrices are also called as row and column vectors respectively

5. If $A = [a_{ij}]_{n \times n}$ such that $a_{ij} = 1$ for $i = j$ and $a_{ij} = 0$ for $i \neq j$, then A is called a unit matrix

It is denoted by I_n.

Eg: $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

6. If $A = [a_{ij}]_{m \times n}$ such that $a_{ij} = 0 \ \forall \ i$ and j then A is called zero matrix (or) a NULL matrix.

It is denoted by '0' (or) more clearly $O_{m \times n}$

Eg: $O_{2 \times 3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2 \times 3}$

⇒ Diagonal element of a square matrix and Principal diagonal

Definition:-

1. In a matrix $A = [a_{ij}]_{n \times n}$, the elements a_{ij} of A for which $i = j$ (i.e., $a_{11}, a_{22}, \ldots a_{nn}$) are called the diagonal element of A. The line along which the diagonal elements lie is called the Principal diagonal of A.
2. A square matrix of all whose element except those in leading diagonal are zero is called diagonal matrix. If $d_1, d_2, \ldots d_n$ are diagonal element of a diagonal matrix A, then A is written as

$A = diag(d_1, d_2, \ldots d_n)$

Ex:- $A = diag(3, 1, -2) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

3. A diagonal matrix whose leading diagonal elements are equal is called a scalar matrix.

Ex:- $B = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Equal Matrix:

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are said to be equal if and only if

⇒ A & B are of the same type (or order)

⇒ $a_{ij} = b_{ij}$ for every i and j

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Algebra of Matrices:

Let $A = [a_{ij}]_{m \times n}$, $B = [b_{ij}]_{m \times n}$ between be two matrix $C = [c_{ij}]_{m \times n}$ where $c_{ij} = a_{ij} + b_{ij}$ is called the sum of the matrices A and B the sum of A and B is called and denoted by A+B

Thus $[a_{ij}]_{m \times n} + [b_{ij}]_{m \times n} = [a_{ij} + b_{ij}]_{m \times n}$ and

$[a_{ij}]_{m \times n} + [b_{ij}]_{m \times n}$

Difference of two Matrices

If A, B are two matrices of the same type (order) then $A + (-B)$ is taken as $A - B$

Multiplication of a Matrix by a Scalar

Let A be a matrix. The Matrix obtained by matrix multiplying every element of A by k. a scalar is called the product of A by k and is denoted by KA (or) AK

Thus if $A = [a_{ij}]_{m \times n}$, then

$KA = [Ka_{ij}]_{m \times n}$ and $[Ka_{ij}]_{m \times n} = k[a_{ij}]_{m \times n} = KA$

Properties:

$\Rightarrow$ $OA = O$ (null matrix), $(-1)A = -A$, called the negative of A.

$\Rightarrow$ $K_1(K_2A) = (K_1K_2)A = K_2(K_1A)$ where K_1, K_2 are scalars

$\Rightarrow$ $KA = 0 \Rightarrow A = 0$ if $k \neq 0$

iv) $k_1 A = k_2 A$ and A is not a null matrix $\Rightarrow k_1 = k_2$

Matrix Multiplication

Let $A = [a_{ik}]_{m\times n}$ and $B = [b_{kj}]_{n\times p}$, then the matrix $C = [c_{ij}]_{m\times p}$ where $c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}$ is called the product of the matrices A and B in that order and we write $C = AB$

In the product AB, the matrix A is called the pre-factor and B the post-factor

If the number of coloumns of A is equal to the number of rows in B then the matrices are said to be comfortable for multiplication in that order

Positive Integral powers of square Matrices.

Let A be a square matrix then A^2 is defined as $A \cdot A$. Now, by the Associative law

$A^2 A = (A \cdot A) A = A(AA) = A \cdot A^2$ so that we can write

$$A^2 A = AA^2 = A \cdot A \cdot A = A^3$$

similarly we have $AA^{m-1} = A^{m-1} A = A^m$

where m is a positive integer

Furthur we have $A^m A^n = A^{m+n}$ and $(A^m)^n = A^{mn}$

where m, n are positive integer

Note:

$$I^n = I \; ; \; O^n = O$$

Trace of a Square Matrix

Let $A = [a_{ij}]_{n \times m}$ then trace of the square matrix A is define as $\sum_{i=1}^{n} a_{ii}$ and is denoted by $tr(A)$

Thus $tr(A) = \sum_{i=1}^{n} a_{ij} = a_{11} + a_{22} + \cdots + a_{nn}$

* Properties:

If A and B are square matrices of other n and l is any scalar, then

$\Rightarrow$ $tr(lA) = l\,trA$

$\Rightarrow$ $tr(A+B) = trA + trB$

$\Rightarrow$ $tr(AB) = tr(BA)$

Triangulan Matrix

A square matrix all of whose elements below the leading diagonal are zero is called an upper triangulan matrix. A square matrix all of whose elements above the leading diagonal are zero is called a lower triangular matrix

Ex:. $\begin{bmatrix} 1 & 2 & -3 & 0 \\ 0 & 4 & 2 & 1 \\ 0 & 0 & -6 & 2 \\ 0 & 0 & 0 & 8 \end{bmatrix}$ is an upper triangular matrix

and $\begin{bmatrix} 7 & 0 & 0 & 0 & 0 \\ 5 & 3 & 0 & 0 & 0 \\ -4 & 6 & 0 & 0 & 0 \\ 2 & +1 & -8 & 5 & 0 \\ 2 & 0 & 4 & 1 & 6 \end{bmatrix}$ is an lower triangular matrix

⇒ If A is a square matrix such that $A^2 = A$ then A is called idempotent

⇒ If A is a square matrix such that $A^m = 0$ where m is a positive integer, then A is called Nilpotent. If m is least positive integer such that $A^m = 0$, then A is called Nilpotent of index m.

⇒ If A is a square matrix such that $A^2 = I$ then A is called involuntery

The transpose of a Matrix

Definition:

The matrix obtained from any given matrix A by inter changing its rows and columns is called the transpose of A. It is denoted by A' or A^T

the transpose of A is

If $A = [a_{ij}]_{m \times n}$, then the transpose

$$A' = [b_{ji}]_{n \times m}, \text{ where } b_{ji} = a_{ij}$$

Also $(A')' = A$

Note:

If A' and B' be the transpose of A and B, respectively, then

⇒ $(A')' = A$

⇒ $(A+B)' = A' + B'$, A and B being of the

⇒ $(kA)' = kA'$, k is a scalar

⇒ $(AB)' = B'A'$, A & B being comformable for multiplication.

Determinants:

Minors and co-factors of a square Matrix

let $A = [a_{ij}]_{n \times m}$ be a square matrix. when from A the elements of i^{th} row and j^{th} column are deleted the determinant of $(n-1)$ rows matrix M_{ij} is called the minor of a_{ij} of A and is denoted by $|M_{ij}|$. The signed minor $(-1)^{i+j} |M_{ij}|$ is called the co-factor of a_{ij} and is denoted by A_{ij}

Thus if $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then

$$|A| = a_{11} |M_{11}| - a_{12} |M_{12}| + a_{13} |M_{13}|$$

$$= a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13}$$

Notes:

1. Determinant of the square matrix A can be defined as.

$$|A| = a_{21} A_{21} + a_{22} A_{22} + a_{23} A_{23} = a_{31} A_{31} + a_{32} A_{32} + a_{33} A_{33}$$

(or)

$$|A| = a_{11} A_{11} + a_{21} A_{21} + a_{31} A_{31} = a_{12} A_{12} + a_{22} A_{22} + a_{32} A_{32}$$

$$= a_{13} A_{13} + a_{23} A_{23} + a_{33} A_{33}$$

Therefore in a determinant the sum of the products of the elements of any row or column with their are corresponding co-factors is called to the value of the determinant

2. If A is a square matrix of order n then. $|kA| = k^n |A|$, where k is a scalar

3. If A is a square matrix of order n, then $|A| = |A^T|$

4. If A and B be two square matrices of the same order then $|AB| = |A| \cdot |B|$

* Adjoint of a Square Matrix

Let A be a square matrix of order n the transpose of the matrix got from A by replacing the elements of A by the corresponding co-factors is called the adjoint of A and is denoted by adj A

Note: For any scalar k, $adj(kA) = k^{n-1}\, adj A$

* Singular and Non-singular Matrices:

Definations:

A square matrix A is is said to be singular if $|A| = 0$ if $|A| \neq 0$, then A is said to be non-singular. Thus only non-singular matrix possess inverses.

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Note :-

If A, B are non - singular then AB, the product is also non-singular matrixes is also non singular

Inverse of a Matrix :-

let A be any square matrix B, it exsists such that $AB = BA = I$, then B is called inverse of A and is denoted by A^{-1}

Note:-

For AB, BA to be both defined and equal it is necessary that A and B are both square matrices of same order Thus a non - square matrix cannot have inverse.

Inventible

A Matrix is said to be inverting, if it possess inverse.

Crammer's Rule (Determinant) Method

The solution of the system of linear equation

$a_1 x + b_1 y + c_1 z = d_1;$

$a_2 x + b_2 y + c_2 z = d_2;$

$a_3 x + b_3 y + c_3 z = d_3;$ is given by

$x = \frac{\Delta_1}{\Delta} \Rightarrow y = \frac{\Delta_2}{\Delta}; z = \frac{\Delta_3}{\Delta} \; (\Delta \neq 0)$, where

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} ; \quad \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix} ; \quad \Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

we notice that $\Delta_1, \Delta_2 \cdot \Delta_3$ are the detenminant obtained from Δ on replacing the $1^{st}, 2^{nd}$ and 3^{rd} Columns by d' values respectively

Symmetric Matrix :-

A square Matrix 'A' = $[a_{ij}]$ is said to be symmetric if $a_{ij} = a_{ji}$ for every i and j

Thus, A is a symmetric matrix $\Leftrightarrow A = A'$ or $A' = A$

Skew - Symmetric Matrix

A squore matrix $A = [a_{ij}]$ is said to be skew symmetric if $a_{ij} = a_{ji}$, for every i and j

Thus A is a skew symmetric matrix $\Leftrightarrow A = A$
$A' = A$

Note:

Every diagonal element of a skew-symmetric matrix is necessarily zero since

$$a_{ii} = -a_{ii} \Rightarrow a_{ii} = 0$$

Ex :. $\begin{bmatrix} a, & h, & g \\ h, & b, & f \\ g, & f, & c \end{bmatrix}$ is a symmetric matrix

$$\begin{bmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{bmatrix}$$ is a skew - symmetric matrix

Properties :

1) * A is symmetric

* KA is symmetric

2) A is skew - symmetric

KA is skew- symmetric

Orthogonal Matrix

A square matrix 'A' is said to be orthogonal $AA' = A'A = I$, that is $A^T = A^{-1}$

Solved Examples

1. proved that $\begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix}$ is orthogonal

Soln: $A = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix}$

$$A^T = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix}$$

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$$A \cdot A^T = \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & \frac{-2}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{-2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{9}+\frac{4}{9}+\frac{4}{9} & \frac{2}{9}+\frac{2}{9}-\frac{4}{4} & \frac{2}{9}-\frac{4}{9}+\frac{2}{9} \\ \frac{2}{9}+\frac{2}{9}-\frac{4}{9} & \frac{4}{9}+\frac{1}{9}+\frac{4}{9} & \frac{4}{9}-\frac{2}{9}-\frac{2}{9} \\ \frac{2}{9}+\frac{4}{9}+\frac{2}{9} & \frac{4}{9}-\frac{2}{9}-\frac{2}{9} & \frac{4}{9}+\frac{4}{9}+\frac{1}{4} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

$$A \cdot A^T = I_3$$

∴ Given matrix is an orthogonal

2. $\begin{bmatrix} 2 & -3 & 1 \\ 4 & 3 & 1 \\ 3 & 1 & 9 \end{bmatrix}$ is orthogonal

<u>Solu</u> let $A = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 3 & 1 \\ 3 & 1 & 9 \end{bmatrix}$, $A^T = \begin{bmatrix} 2 & 4 & -3 \\ -3 & 3 & 1 \\ 1 & 1 & 9 \end{bmatrix}$

$$A \cdot A^T = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 3 & 1 \\ 3 & 1 & 9 \end{bmatrix} \begin{bmatrix} 2 & 4 & -3 \\ -3 & 3 & 1 \\ 1 & 1 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 4+9+1 & 8-9+1 & -6-3+9 \\ 8-9+1 & 16+9+1 & -12+3+9 \\ -6-3+9 & -12+3+9 & 9+1+81 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 0 \\ 0 & 26 & 0 \\ 0 & 0 & 91 \end{bmatrix} \neq I_3$$

Given matrix is not an orthogonal

3. Find the values of A, B and c when

$$\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$ is orthogonal

Solu) let $A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$, $A^T = \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$

$$A \cdot A^T = \begin{bmatrix} 0 & 2b & c \\ a & b & c \\ a & -b & c \end{bmatrix} \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix}$$

$$= \begin{bmatrix} 0+4b^2+c^2 & 0+2b^2-c^2 & 0-2b^2+c^2 \\ 0+2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ 0-2b^2+c^2 & a^2-b^2-c^2 & a^2-b^2+c^2 \end{bmatrix}$$

Given that $A \cdot A^T = I_3$

$$= \begin{bmatrix} 4b^2+c & 2b^2-c^2 & -2b^2+c^2 \\ 2b^2-c^2 & a^2+b^2+c^2 & a^2-b^2-c^2 \\ -2b^2+c^2 & a^2-b^2-c^2 & a^2+b^2+c^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{aligned} 2b^2-c^2 &= 0 \to (1) \\ a^2-b^2-c^2 &= 0 \to (2) \\ 4b^2+c^2 &= 1 \to (3) \\ a^2+b^2+c^2 &= 1 \to (4) \end{aligned}$$

from (1) $2b^2-c^2=0$

$c^2 = 2b^2$

from (2) $a^2-b^2-c^2=0$ | $a^2 = 3b^2$

$a^2-b^2-2b^2=0$ | $a = \sqrt{3}b$

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Rank of a Matrix

* If A is a null matrix we define its rank will be "zero". If A
* If A is a non zero matrix we say that R is the rank of A if the following conditions are satisfied

1. Every $(r+1)^{th}$ order minor of A is zero
2. There exsist atleast one rth orden minor of A which is not zero
3. Rank of A is denoted by "$\rho(A)$"

Note:
* Every matrix will have a rank
* Rank of A matrix is unique
* Rank of A is ≥ 1 when A is a non-zero matrix
* If A is a matrix of orden $m \times n$ then rank of A is $\leq \min(m, n)$.
* If rank of $A = r$ then every minor of A of orden $(r+1)$ or more is zero.
* rank of the Identity matrix I_n is 'n'
* If A is matrix of order 'n' and A is non-singular ($|A| \neq 0$) then rank of $A = n$.

1. Find the rank of the matrix

i) $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$ ii) $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$

Solu: i) Given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 4 \\ 7 & 10 & 12 \end{bmatrix}$$

$$|A| = 1(48-40) - 2(36-28) + 3(30-28)$$

$$= 8 - 16 + 6$$

$$= -2 \neq 0$$

$$\therefore \rho(A) = 3$$

ii) $|A| = \begin{vmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{vmatrix}$

$$|A| = 3(4-4) + 1(-12+12) + 2(-6+6)$$

$$|A| = 0$$

$$\rho(A) < 3$$

A minor of order 2×2 of A is $\begin{vmatrix} 3 & -1 \\ -6 & 2 \end{vmatrix} = 6-6=0$

" " " $\begin{vmatrix} -1 & 2 \\ 2 & 4 \end{vmatrix} = -4-4 = -8 \neq 0$

$$\rho(A) = 2$$

iii) H.W $\begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$ iv) $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$ v) $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix}$

Solu) iii) Given matrix

$$A = \begin{bmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} -1 & 0 & 6 \\ 3 & 6 & 1 \\ -5 & 1 & 3 \end{vmatrix}$$

$$= -1(18+5) - 0(9+5) + 6(3+30)$$

$$= -1(23) - 0 + 6(33)$$

$$= -23 + 198$$

$$|A| = 185 \neq 0$$

$$\therefore \rho(A) = 3$$

iv) Given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= 1(24-25) - 2(18-20) + 3(15-16)$$

$$= 1(-1) - 2(-2) + 3(-1)$$

$$= -1 + 4 - 3$$

$$|A| = 0$$

$$\rho(A) < 3$$

A minor of order 2x2 of A is $\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$

$$= 4 - 6 = -2 \neq 0$$

$$\rho(A) = 2$$

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v) Given matrix

$$\begin{bmatrix} 2 & -1 & 3 & 1 \\ 1 & 4 & -2 & 1 \\ 5 & 2 & 4 & 3 \end{bmatrix} \Rightarrow |A| = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 4 & -2 \\ 5 & 2 & 4 \end{vmatrix}$$

Here A minor of 3x3 of A is

$\rho(A) \le \min(3,4)$

$\rho(A) \le 3$

$$|A| = 2(16+4) + 1(4+10) + 3(2-20)$$
$$= 2(20) + 1(14) + 3(-18)$$
$$= 40 + 14 - 54$$
$$|A| = 0$$

A minor of order 3x3 of A is $\begin{vmatrix} -1 & 3 & 1 \\ 4 & -2 & 1 \\ 2 & 4 & 3 \end{vmatrix}$

$$|A| = -1(-6-4) - 3(12-2) + 1(16+4)$$
$$= -1(-10) - 3(10) + 1(20)$$
$$= 10 - 30 + 20$$
$$|A| = 0$$

A minor of order 3x3 of A is $\begin{vmatrix} 2 & 3 & 1 \\ 1 & -2 & 1 \\ 5 & 4 & 3 \end{vmatrix}$

$$|A| = 2(-6-4) - 3(3-5) + 1(4+10)$$
$$= 2(-10) - 3(-2) + 1(14)$$
$$= -20 + 6 + 14$$
$$= -20 + 20$$
$$|A| = 12 \ne 0 = 0$$

26
14
12

$\rho(A) = 3$

$$|A| = \begin{vmatrix} 2 & -1 & 1 \\ 1 & 4 & 1 \\ 5 & 2 & 3 \end{vmatrix} = 2(12-2) + 1(3-5) + 1(2-20)$$
$$= 2(10) + 3(-2) + 1(-18)$$
$$= 20 - 6 - 18$$
$$= 0$$

Now A minor of order 2x2 of A is

$$\begin{vmatrix} 2 & -1 \\ 1 & 4 \end{vmatrix} = 8+1 = 9 \neq 0$$

$$\rho(A) = 2$$

13) From ③

$$4b^2 + c^2 = 1$$

$$4b^2 + 2b^2 = 1$$

$$6b^2 = 1$$

$$b^2 = \frac{1}{6}$$

$$b = \frac{1}{\sqrt{6}}$$

$$c^2 = 2b^2 = 2 \cdot \frac{1}{\cancel{6}\,3} = \frac{1}{3} \Rightarrow c = \frac{1}{\sqrt{3}}$$

$$a = \sqrt{3},\ b = \frac{1}{\sqrt{6}},\ c = \frac{1}{\sqrt{3}}$$

Date
26|11|2018

Conjugate of the Matrix

The matrix obtained from any given matrix A are replacing its elements by the co-rresponding Conjugate Complex Numbers is called the Conjugate of A. It is denoted by '$\bar{A}$'.

Ex: $A = \begin{bmatrix} 2+3i & 0 & i \\ i+2 & 2i-3 & 7 \end{bmatrix}$

$$\bar{A} = \begin{bmatrix} 2-3i & 0 & -i \\ -i+2 & -2i-3 & 7 \end{bmatrix}$$

Note:

1. If $\bar{A}$ and $\bar{B}$ be the conjugates of A and B respectively then

* $\overline{(\bar{A})} = A$
* $\overline{(A \pm B)} = \bar{A} \pm \bar{B}$
* $\overline{(KA)} = \bar{K}\bar{A}$
* $\overline{(AB)} = \bar{A} \cdot \bar{B}$

The transpose of the conjugate of a square matrix

* If A is a square matrix and its conjugate is $\bar{A}$ then the transpose of $\bar{A}$ is $(\bar{A})^T$
* The transposed conjugate of A is denoted by (transposed) 'A^θ'
* Therefore $(\bar{A})^T = \overline{(A^T)} = A^\theta$

Ex:. $A = \begin{bmatrix} 5 & 3-i & -2i \\ 6 & 1+i & 4-i \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 5 & 3+i & 2i \\ 6 & 1-i & 4+i \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 5 & 6 \\ 3+i & 1-i \\ 2i & 4+i \end{bmatrix} = A^\theta$

Note

1. If A^θ and B^θ be the transposed conjugates of A and B respectively

* $(A^\theta)^\theta = A$
* $(A \pm B)^\theta = A^\theta \pm B^\theta$
* $(KA)^\theta = \bar{K}A^\theta$ where K is a complex number.
* $(AB)^\theta = B^\theta . A^\theta$

Hermitian Matrix

A square matrix A such that $(\bar{A})^T = A$ is called a Hermitian Matrix

Ex: $A = \begin{bmatrix} 4 & 1+3i \\ 1-3i & 7 \end{bmatrix} \Rightarrow \bar{A} = \begin{bmatrix} 4 & 1-3i \\ 1+3i & 7 \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 4 & 1+3i \\ 1-3i & 7 \end{bmatrix} = A$

$\therefore$ A is a Hermitian Matrix

Skew Hermitian Matrix

A square matrix A such that $(\bar{A})^T = -A$ is called a skew Hermitian Matrix

Ex:

$A = \begin{bmatrix} -3i & 2+i \\ -2+i & -i \end{bmatrix}, \bar{A} = \begin{bmatrix} 3i & 2-i \\ -2-i & i \end{bmatrix}$

$(\bar{A})^T = \begin{bmatrix} 3i & -2-i \\ 2-i & i \end{bmatrix} = -A$

$\therefore$ A is a skew-Hermitian Matrix

Note:

1. It should be noted that elements are the leading diagonals must be all zero or all are purely imaginary

Unitary Matrix

A square matrix A such that $(\bar{A})^T = A^{-1}$ is called a unitary Matrix

$\therefore A^{\theta}.A = AA^{\theta} = I$

Date 28/11/18

1. If A and B are Hermitian matrices prove that $AB-BA$ is a skew Hermitian Matrices

Solu| Given that A, B are Hermitian matrices

$$(\bar{A})^T = A \quad , (\bar{B})^T = B$$

$$(\overline{AB-BA})^T = (\overline{AB} - \overline{BA})^T$$

$$= (\overline{AB})^T - (\overline{BA})^T$$

$$= (\bar{A}\bar{B})^T - (\bar{B}\bar{A})^T$$

$$= (\bar{B})^T(\bar{A})^T - (\bar{A})^T(\bar{B})^T$$

$$= BA - AB$$

$$(\overline{AB-BA})^T = -(AB-BA)$$

$\therefore AB-BA$ is a skew-Hermitian Matrix

2. If A is a Hermitian matrix prove that iA is a skew Hermition matrix

solu| Since A is a Hermitian matrix

$$(\bar{A})^T = A \Rightarrow A^\theta = A$$

$$(iA)^\theta = \bar{i} A^\theta$$

$$= -iA$$

$$(iA)^\theta = -(iA)$$

$\therefore iA$ is a skew-Hermitian matrix

3. If A is a skew Hermitian prove that iA is a Hermitian matrix

Solu| Since A is a skew Hermitian matrix

$$A^\theta = -A$$

$$(iA)^\theta = \bar{i} A^\theta$$

$$= -i - A$$

$$(iA)^\theta = iA$$

∴ iA is Hermitian matrix

4. show that every square matrix is uniquely expressible as the sum of a Hermitian matrix and a skew Hermitian matrix

soln since A is a square matrix

$$(A+A^\theta)^\theta = A^\theta + (A^\theta)^\theta = A^\theta + A$$

$$(A+A^\theta)^\theta = A+A^\theta$$

∴ $A+A^\theta$ is a Hermitian matrix

$\frac{1}{2}(A+A^\theta) = P$ is also a Hermitian matrix

Now $(A-A^\theta)^\theta = A^\theta - (A^\theta)^\theta$

$= A^\theta - A$

$= -(A-A^\theta)$

∴ $(A-A^\theta)$ is a skew - Hermitian matrix

∴ $\frac{1}{2}(A-A^\theta) = Q$ is also a skew Hermatian matrix

$$P+Q = \frac{1}{2}(A+A^\theta) + \frac{1}{2}(A-A^\theta)$$

$$= A$$

∴ A square matrix A is uniquely expressible. a sum of Hermitian and skew Hermitian matrix.

5. If $A = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$ then show that A is a Hermitian matrix and iA is a skew Hermition matrix

soln $[A] = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ 2-5i & 3-i & 4 \end{bmatrix}$

$A^\theta = \begin{bmatrix} 3 & 7+4i & -2-5i \\ 7-4i & -2 & 3-i \\ 2+5i & 3+i & 4 \end{bmatrix}$

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$$(A+A^\theta)^\theta = A^\theta + (A^\theta)^\theta$$
$$= A^\theta + A$$
$$= A + A^\theta$$

solu

$$\therefore A + \bar{A}^T = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ 2-5i & 3-i & 4 \end{bmatrix} + \begin{bmatrix} 3 & 7+4i & -2-5i \\ 7-4i & -2 & 3-i \\ 2+5i & 3+i & 4 \end{bmatrix}$$

$$A + A^\theta = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3+i \\ 2-5i & 3-i & 4 \end{bmatrix} + \begin{bmatrix} 3 & 7-4i & 2+5i \\ 7+4i & -2 & 3+i \\ -2-5i & 3-i & 4 \end{bmatrix}$$

Given matrix //

$$A = \begin{bmatrix} 3 & 7-4i & -2+3i \\ 7+4i & -2 & 3+i \\ 2-5i & 3+i & 4 \end{bmatrix}$$

$$\bar{A}^T = \begin{bmatrix} 3 & 7+4i & -2-3i \\ 7-4i & -2 & 3-i \\ 2+5i & 3+i & 4 \end{bmatrix} \Rightarrow A^\theta = \begin{bmatrix} 3 & 7-4i & 2+5i \\ 7+4i & -2 & 3+i \\ -2-3i & 3-i & 4 \end{bmatrix}$$

$$(\bar{A})^T = A$$

$$iA = i\begin{bmatrix} 3 & 7-4i & -2+3i \\ 7+4i & -2 & 3+i \\ 2-5i & 3-i & 4 \end{bmatrix}$$

wrong

$$= \begin{bmatrix} 3i & 7i+4 & -2i-3 \\ 7i-4 & -2i & 3i-1 \\ 2i+5 & 3i+1 & 4i \end{bmatrix}$$

$$(\overline{iA}) = \begin{bmatrix} -3i & -7i+4 & +2i-3 \\ -7i-4 & 2i & -3i-1 \\ -2i+5 & -3i+1 & -4i \end{bmatrix}$$

$$(\overline{iA})^T = \begin{bmatrix} -3i & -7i-4 & -2i+5 \\ -7i+4 & 2i & -3i+1 \\ 2i-3 & -3i-1 & -4i \end{bmatrix} = -i\begin{bmatrix} 3 & 7+4 \\ & A \end{bmatrix}$$

solu] Given $\bar{A} = \begin{bmatrix} 3 & 7+4i & -2-5i \\ 7-4i & -2 & -3i \\ -2+5i & 3i & 4 \end{bmatrix}$

B

$(\bar{A})^T = \begin{bmatrix} 3 & 7-4i & -2+5i \\ 7+4i & -2 & 3i \\ -2-5i & 3-i & 4 \end{bmatrix} = A$

$\therefore A$ is a Hermition matrix

$iA = \begin{bmatrix} 3i & 7i+4 & -2i-5 \\ 7i-4 & -2i & 3i-1 \\ -2i+5 & +3i+1 & 4i \end{bmatrix}$

$\bar{iA} = \begin{bmatrix} -3i & -7i+4 & 2i-5 \\ -7i-4 & 2i & -3i-1 \\ +2i+5 & -3i+1 & -4i \end{bmatrix}$

$(\bar{iA})^T = \begin{bmatrix} -3i & -7i-4 & 2i+5 \\ -7i+4 & 2i & -3i+1 \\ 2i-5 & -3i-1 & -4i \end{bmatrix}$

$= -i\begin{bmatrix} 3i & 7+4i & 2+5i \\ 7-4i & -2 & 3+i \\ -2+5i & 3+i & 4 \end{bmatrix}$

$(\bar{iA})^T = -iA$

Date 29/11/18 6. Express the matrix $\begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$ as the sum of Hermition matrix and skew Hermition matrix

solu] Given matrix $A = \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$

$\bar{A} = \begin{bmatrix} 1-i & 2 & 5+5i \\ -2i & 2-i & 4-2i \\ -1-i & -4 & 7 \end{bmatrix}$

$(\bar{A})^T = A^\theta = \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$

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$$A + A^{\theta} = \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix} + \begin{bmatrix} 1-i & -2i & -1-i \\ 2 & 2-i & -4 \\ 5+5i & 4-2i & 7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2-2i & 4-6i \\ 2i+2 & 4 & 2i \\ 4+6i & -2i & 14 \end{bmatrix}$$

$$P = \frac{1}{2}(A + A^{\theta}) = \begin{bmatrix} 1 & 1-i & 2-3i \\ i+1 & 2 & i \\ 2+3i & -i & 7 \end{bmatrix}$$

P is a Hermitian matrix

$$A - A^{\theta} = \begin{bmatrix} 2i & 2+2i & 6-4i \\ 2i-2 & 2i & 8+2i \\ -6-4i & -8+2i & 0 \end{bmatrix}$$

$$Q = \frac{1}{2}(A - A^{\theta}) = \begin{bmatrix} i & 1+i & 3-2i \\ i-1 & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$$

Q is a skew Hermitian matrix

$$P + Q = \frac{1}{2}(A + A^{\theta}) + \frac{1}{2}(A - A^{\theta})$$

$$= \begin{bmatrix} 1 & 1-i & 2-3i \\ i+1 & 2 & i \\ 2+3i & -i & 7 \end{bmatrix} + \begin{bmatrix} i & 1+i & 3-2i \\ i-1 & i & 4+i \\ -3-2i & -4+i & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+i & 2 & 5-5i \\ 2i & 2+i & 4+2i \\ -1+i & -4 & 7 \end{bmatrix}$$

$\therefore$ A square matrix can be expressed in sum of Hermitian and skew Hermitian matrix

$$= A$$

7. Express the matrix $\begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$ as the sum of a Hermitian and skew Hermitian matrix

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solu) Given $A = \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$

$$\bar{A} = \begin{bmatrix} -i & 2+3i & 4-5i \\ 6-i & 0 & 4+5i \\ i & 2+i & 2-i \end{bmatrix}$$

$$(\bar{A})^T = A^\theta = \begin{bmatrix} -i & 6-i & i \\ 2+3i & 0 & 2+i \\ 4-5i & 4+5i & 2-i \end{bmatrix}$$

$$(A+A^\theta) = \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix} + \begin{bmatrix} -i & 6-i & i \\ 2+3i & 0 & 2+i \\ 4-5i & 4+5i & 2-i \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 8-4i & 4+6i \\ 8+4i & 0 & 6-4i \\ 4-6i & 6+4i & 4 \end{bmatrix}$$

$$P = \frac{1}{2}(A+A^\theta) = \begin{bmatrix} 0 & 4-2i & 2+3i \\ 4+2i & 0 & 3-2i \\ 2-3i & 3+2i & 2 \end{bmatrix}$$

P is a Hermitian matrix.

$$A - A^\theta = \begin{bmatrix} 2i & -4-2i & 4+4i \\ 4-2i & 0 & 2-6i \\ -4+4i & -2-6i & 2i \end{bmatrix}$$

$$Q = \frac{1}{2}(A-A^\theta) = \begin{bmatrix} i & -2-i & 2+2i \\ 2-i & 0 & 1-3i \\ -2+2i & -1-3i & i \end{bmatrix}$$

$\therefore$ Q is a skew Hermitian matrix

$$P+Q = \frac{1}{2}(A+A^\theta) + \frac{1}{2}(A-A^\theta)$$

$$= \begin{bmatrix} 0 & 4-2i & 2+3i \\ 4+2i & 0 & 3-2i \\ 2-3i & 3+2i & 2 \end{bmatrix} + \begin{bmatrix} i & -2-i & 2+2i \\ 2-i & 0 & 1-3i \\ -2+2i & -1-3i & i \end{bmatrix}$$

$$= \begin{bmatrix} i & 2-3i & 4+5i \\ 6+i & 0 & 4-5i \\ -i & 2-i & 2+i \end{bmatrix}$$

$$= A$$

A square matrix can be expressed in the sum of the Hermitian and skew Hermitian matrix

Date 30/11/2018

Echelon form of a Matrix

A matrix is said to be in Echelon form if it has the following properties

1. zero rows if any are below any non-zero row
2. The first non-zero entry in each non-zero row is equal to one.
3. The no.of zeroes before the first non-zero elements in a row is less than the no.of such zeroes in the next row.

Note: The condition 2 is optional

Important Note

The no.of non-zero rows in the row Echelon form of A is the rank of A.

Ex:. $A = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\rho(A) = 2$ $\quad \rho(B) = 3$ $\quad \rho(C) = 2$

1. Reduce the Matrix $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ into Echelon form and hence find its ranck

solu) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - 6R_1$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$ $R_4 \to R_4 - R_3$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_4 \to R_4 - R_2$

$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_2 \leftrightarrow R_3$

$\therefore \rho(A) = 3$

2. Reduce the Matrix $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ into Echelon form and hence find its rank

solu) $A = \begin{bmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$ $R_2 \to R_2 + R_1$

$= \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & -11 & 8 & -5 \\ 0 & 4 & -3 & 2 \end{bmatrix}$ $R_2 \to R_2 + R_1$, $R_3 \to R_3 + 2R_1$, $R_4 \to R_4 - R_1$

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$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 2 & -1 \\ 0 & 0 & -6 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} R_3 \to 2R_3 - 11R_2 \\ R_4 \to R_4 + 2R_2 \end{array}$$

$$\sim \begin{bmatrix} -1 & -3 & 3 & -1 \\ 0 & -2 & 0 & -1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_4 \\ R_3 \to R_3 + 6R_4 \end{array}$$

$$\sim \begin{bmatrix} -1 & -3 & 3 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} R_1 \to R_1 + R_3 \\ R_2 \to R_2 + R_3 \end{array}$$

$$\sim \begin{bmatrix} -1 & -3 & 3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \frac{R_2}{-2}; \; R_3 \leftrightarrow R_4$$

H.W

3. $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ 4. $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$

5. $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$ 6. $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$ N7. $\begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$

Solu) 3.

$$\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix} \; R$$

$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad \begin{array}{l} R_{[illegible]} \to R_{[illegible]} - R_{[illegible]} \\ R_3 \to R_3 - R_4 \end{array}$$

$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & -1 & 0 & 2 \\ -2 & 0 & -2 & 0 \end{bmatrix} \quad R_4 \to R_4 + R_1$$

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$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 2 & 2 & -2 \\ 0 & -2 & 2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad \begin{aligned} R_2 &\to R_2 - R_3 \\ R_3 &\to R_3 - R_2 \end{aligned}$$

$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix} \quad \begin{aligned} R_2 &\to R_2 + R_3 \\ R_3 &\to R_3 + R_2 \end{aligned}$$

$$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} R_2 &\to R_2 - R_3 \\ R_3 &\to R_3 - R_2 \end{aligned}$$

$1 \quad 1 \quad 1 \quad -1$

$\rho(A) = 2$

4. $$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix} \qquad \sim \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -10 & -14 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_2 \leftrightarrow R_4$$

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -5 & -7 \\ 0 & 0 & -10 & -14 \end{bmatrix} \quad \begin{aligned} R_2 &\to R_2 - 2R_1 \\ R_3 &\to R_3 - 4R_1 \\ R_4 &\to R_4 - R_3 \end{aligned}$$

$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & -14 \end{bmatrix} \quad \begin{aligned} R_2 &\to R_2 - R_3 \\ R_3 &\to 2R_3 - R_4 \end{aligned}$$

$\rho(A) = 2$

5. $$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ -24 & 0 & -7 & -16 \\ 0 & 0 & 10 & 10 \end{bmatrix} \quad \begin{aligned} R_2 &\to 3R_2 - 3R_1 \\ R_3 &\to R_3 + R_1 \end{aligned} \qquad \rho(A) = 3$$

6. $\begin{bmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 5 & 3 & 14 & 4 \\ -5 & 0 & -8 & -1 \\ 8 & 0 & 20 & 4 \end{bmatrix}$ $R_2 \to 3R_2 - R_1$, $R_3 \to 3R_3 + R_1$

$\sim \begin{bmatrix} 5 & 3 & 14 & 4 \\ -5 & 0 & -8 & -1 \\ 2 & 0 & 5 & 1 \end{bmatrix}$ $R_3 \to \frac{R_3}{4}$ $\quad \therefore \rho(A) = 3$

① $\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -1 & -1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ $R_2 \to 2R_2 + R_1$, $R_3 \to 2R_3 + R_1$

$\sim \begin{bmatrix} -2 & -1 & -3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R.$ $R_3 \to 3R_3 + R_2$, $R_4 \to 3R_4 \cancel{+} - R_2$

$\rho(A) = 2$

7. $= \begin{bmatrix} 1 & -2 & 0 & 1 \\ 2 & -1 & 1 & 0 \\ 3 & -3 & 1 & 1 \\ -1 & -1 & -1 & 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & +3 & 1 & -2 \\ 0 & 3 & 1 & -2 \\ 0 & -3 & -1 & 2 \end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 + R_1$

$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & -2 \\ 0 & -3 & -1 & 2 \end{bmatrix}$ $R_2 \to R_2 - R_3$

$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -3 & -1 & 2 \end{bmatrix}$ $R_3 \to R_3 + R_4$

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$$\sim \begin{bmatrix} 1 & -2 & 0 & 1 \\ 0 & -3 & -1 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad R_2 \leftrightarrow R_4 \qquad \rho(A) = 2$$

Date 1/12/2018

Reduction to Normal Form

Every $m \times n$ - matrix of rank "r" can be reduced to the form is I_R (or) $\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}$ by a finite change of elementory row or coloumn operations this form is called normal form or first Cananical form of a matrix

1. Reduce the matrix $A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$

solu) Given matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 0 \\ -2 & 4 & 3 & 0 \\ 1 & 0 & 2 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 8 & 5 & 0 \\ 0 & -2 & 1 & -8 \end{bmatrix} \quad R_2 \to R_2 + 2R_1, \; R_3 \to R_3 - R_1$$

$$\simeq \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 8 & 5 & 0 \\ 0 & -2 & 1 & 1 \end{bmatrix} \quad C_2 \to C_2 - 2C_1, \; C_3 \to C_3 - C_1, \; C_4/8$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 8 & 5 & 0 \\ 0 & 0 & 9 & 4 \end{bmatrix} \quad R_3 \to 4R_3 + R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 9 & 1 \end{bmatrix} \quad C_2/8, \; C_4/4.$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 9 & 1 \end{bmatrix} \quad C_3 \to C_3 - 5C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 9 \end{bmatrix} \quad C_3/9$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad C_4 \to C_4 - C_3$$

$$\sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix} \quad \therefore \rho(A) = 3$$

2. $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$ by Canonical form

$$\sim \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 2 & 4 & 1 \\ 0 & 4 & 8 & 2 \end{bmatrix} \quad \begin{matrix} R_3 \to R_3 - R_1 \\ R_4 \to R_4 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \to R_4 - 2R_3$$

$$\sim \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 0 & 0 & 4 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_3 \to 3R_3 - 2R_2$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 6 & 8 & 1 \\ 0 & 0 & 8 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} C_2 \to 2C_2 - C_1 \\ C_3 \to 2C_3 - 3C_1 \\ C_4 \to C_4 - 2C_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} C_1/2 \\ C_2/6 \\ C_3/8 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_4 \to C_4 - C_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad C_3 \to C_3 - C_2$$

$$\sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix} \quad \therefore \ \rho(A) = 3$$

H.W

3. $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

Solu| Given matrix

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -5 & -3 & -7 \\ 0 & -7 & 9 & -1 \\ 0 & -6 & 3 & -4 \end{bmatrix} \quad \begin{array}{l} R_2 \to 2R_2 - R_1 \\ R_3 \to 2R_3 - 3R_1 \\ R_4 \to R_4 - 3R_1 \end{array}$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & -10 & -3 & -7 \\ 0 & -14 & 9 & -1 \\ 0 & -12 & 3 & -4 \end{bmatrix} \quad \begin{array}{l} C_2 \to 2C_2 - 3C_1 \\ C_3 \to 2C_3 + C_1 \\ C_4 \to 2C_4 + C_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -5 & -1 & -7 \\ 0 & -7 & 3 & -1 \\ 0 & -6 & 1 & -4 \end{bmatrix} \quad \begin{array}{l} C_1/2 \\ C_2/2 \\ C_3/3 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -7 \\ 0 & -22 & 3 & -1 \\ 0 & -11 & 1 & 4 \end{bmatrix} \quad \begin{array}{l} C_2 \to C_2 - 5C_3 \\ C_3 \to \end{array} \qquad -6-5$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -7 \\ 0 & -22 & 0 & -22 \\ 0 & -11 & 1 & 4 \end{bmatrix} \quad R_3 \to R_3 + 3R_2 \quad R_4 \to 2R_4 - 1$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -7 \\ 0 & -22 & 0 & -22 \\ 0 & 0 & 2 & 18 \end{bmatrix} \quad R_4 \to 2R_4 - R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -7 \\ 0 & -2 & 0 & 22 \\ 0 & 0 & 2 & 18 \end{bmatrix} \quad C_2/11, \ C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -7 \\ 0 & 1 & -22 & 22 \\ 0 & 0 & -4 & 18 \end{bmatrix} \quad \begin{matrix} C_2/2 \\ C_3 \to 7C_3 \pm C_4 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -7 & 0 \\ 0 & 1 & 22 & -22 \\ 0 & 0 & 18 & -4 \end{bmatrix} \quad C_3 \leftrightarrow C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & -22 \\ 0 & 0 & 8 & -4 \end{bmatrix} \quad C_3 \to C_3 + C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -7 & 0 \\ 0 & 1 & 0 & +11 \\ 0 & 0 & 8 & 2 \end{bmatrix} \quad C_4 \to C_4/-2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & -7 & 0 \\ 0 & 0 & 8 & 2 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 1 \end{bmatrix} \quad \begin{matrix} \frac{R_3}{-7} \\ \frac{R_4}{2} \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_4 \to R_4 - 4R_3$$

$\frac{14}{\frac{15}{4}}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad C_4 \to C_4 - 11C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_4 \leftrightarrow R_3 \sim \begin{bmatrix} I_4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\rho(A) = 4$$

4. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ 5. $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 3 \end{bmatrix}$ 6. $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -3 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$

7. $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$ 8. $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$ 9. $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$ 11. $\begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$

Solu) 11.

$$A = \begin{bmatrix} 2 & 3 & 7 \\ 3 & -2 & 4 \\ 1 & -3 & -1 \end{bmatrix}$$

$$A \sim \begin{bmatrix} 2 & 3 & 7 \\ 0 & -13 & -13 \\ 0 & -9 & -9 \end{bmatrix} \quad \begin{array}{l} R_2 \to 2R_2 - 3R_1 \\ R_3 \to 2R_3 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 2 & 3 & 7 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} R_2 \to \frac{R_2}{-13} \\ R_3 \to \frac{R_3}{-9} \end{array}$$

$$\sim \begin{bmatrix} 2 & 3 & 7 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad R_2 \to R_2 - 3R_3$$

$$\sim \begin{bmatrix} 2 & 3 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad C_3 \to C_3 - C_2$$

$$\sim \begin{bmatrix} 2 & 0 & 4 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad R_1 \to R_1 - 3R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad R_1 \to R_1/2$$

$$\sim \begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C_2 \leftrightarrow C_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad C_2 \to C_2 - 2C_1$$

4 $$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -3 & -2 & -5 \\ 0 & -6 & -4 & -22 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 2 \\ 0 & -6 & -4 & -22 \end{bmatrix} \quad \begin{matrix} R_2 \to 2R_2 + 3R_1 \\ R_3 \to R_3 + 3P \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 5 & 2 \\ 0 & 3 & 2 & 11 \end{bmatrix} \quad R_3 \to \frac{R_3}{-2}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 10 & 2 \\ 0 & 3 & -5 & 5 \end{bmatrix} \quad \begin{matrix} C_3 \to 2C_3 - 3C_2 \\ C_4 \to C_4 - 2C_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & -1 & 5 \end{bmatrix} \quad C_2 \to \frac{C_3}{5}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 2 \\ 0 & 3 & -6 & 5 \end{bmatrix} \quad \begin{array}{l} C_2 \to C_2 - 2C_1 \\ C_3 \to C_3 - C_4 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 1 & 1 & 5 \end{bmatrix} \quad \frac{C_2}{3}, \frac{C_3}{-2}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \begin{array}{l} C_2 \to C_2 - C_3 \\ C_4 \to C_4 - 5C_3 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad C_2 \leftrightarrow C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad \frac{C_2}{2} \quad \sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\rho(A) = 3$$

5 $$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 2 \\ 0 & -4 & -11 & 3 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 6R_1 \end{array}$$

$$\sim \begin{bmatrix} 2 & 0 & -2 & 2 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 2 \\ 0 & 0 & -3 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 \to 2R_1 + R_3 \\ R_4 \to R_4 - R_3 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 0 & -3 & 2 \\ 0 & -2 & -4 & 1 \\ 0 & 0 & -3 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 \to \frac{R_1}{2} \\ R_3 \to \frac{R_3}{2} \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & -3 & 1 \\ 0 & 0 & -2 & 1 \end{bmatrix} \quad \begin{array}{l} C_2 \to \frac{C_2}{-2} \\ C_3 \to C_3 + C_4 \end{array}$$

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$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 3 \\ 0 & 2 & 0 & -1 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad \begin{array}{l} R_2 \to 2R_2 - R_4 \\ R_3 \to 2R_3 - 3R_4 \\ R_4 \to R_4 - 2R_2 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad \begin{array}{l} R_4 \to 3R_1 \cdot \; C_2 \to \frac{C_2}{2} \\ C_4 \to C_4 - C_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad C_4 \to C_4 + C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad C_4 \to \frac{C_4}{3}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad C_2 \leftrightarrow C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \end{bmatrix} \quad C_3 \leftrightarrow C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad R_4 \to R_4 + R_2$$

$$\sim \begin{bmatrix} I_3 & 0 \\ 0 & 0 \end{bmatrix} \quad \rho(A) = 3$$

6. $$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -3 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 3 & -1 & -1 \\ 0 & -5 & -3 & -5 \\ 0 & -7 & 9 & -1 \\ 0 & -6 & 3 & -4 \end{bmatrix} \quad \begin{array}{l} R_2 \to 2R_2 - R_1 \\ R_3 \to 2R_3 - 3R_1 \\ R_4 \to R_4 - 3R_1 \end{array}$$

$$\sim \begin{bmatrix} 2 & 0 & -1 & +1 \\ 0 & -14 & -3 & +5 \\ 0 & 20 & 9 & +1 \\ 0 & 3 & 3 & +4 \end{bmatrix} \quad \begin{array}{l} C_2 \to C_2 + 3C_3 \\ C_4 \to \frac{C_4}{-1} \end{array}$$

$$\sim \begin{bmatrix} 2 & 0 & -1 & 1 \\ 0 & -14 & -3 & 5 \\ 0 & 20 & 9 & 1 \\ 0 & 3 & 3 & 4 \end{bmatrix} \quad \begin{array}{l} \frac{C_1}{2} \\ R_4 \to 14R_4 + 3R_2 \end{array}$$

$$\begin{array}{l} +14\,(0 \;\; 3 \;\; 3 \;\; 4) \\ \;\;\;\; 0 \;\; +42 \;\; +42 \;\; +56 \\ 3\,(0 \;\; -14 \;\; -3 \;\; 5) \\ \;\;\;\; 0 \;\; -42 \;\; -9 \;\; 15 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -14 & -3 & 5 \\ 0 & 20 & 9 & 1 \\ 0 & 3 & 3 & 4 \end{bmatrix} \quad \begin{array}{l} C_3 \to C_3 + C_1 \\ C_4 \to C_4 - C_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -14 & -1 & 5 \\ 0 & 20 & 3 & 1 \\ 0 & 3 & 1 & 4 \end{bmatrix} \quad C_3 \to \frac{C_3}{3}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 5 \\ 0 & -22 & 3 & 1 \\ 0 & -11 & 1 & 4 \end{bmatrix} \quad C_2 \to C_2 - 14C_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 5 \\ 0 & 2 & 3 & 1 \\ 0 & 1 & 1 & 4 \end{bmatrix} \quad C_2 \to \frac{C_2}{-11}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 5 \\ 0 & 0 & 1 & -7 \\ 0 & 1 & 1 & 4 \end{bmatrix} \quad R_3 \to R_3 - 2R_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 1 & -7 \\ 0 & 1 & 0 & 11 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 + R_3 \\ R_4 \to R_4 - R_3 \end{array}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & -7 & 1 & 0 \\ 0 & 11 & 0 & 1 \end{bmatrix} \quad C_2 \leftrightarrow C_4$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 11 & 0 & 1 \end{bmatrix} \quad R_3 \to 2R_3 - 7R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 11 & 0 & 1 \end{bmatrix} \quad R_2 \to \frac{R_2}{-2}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_4 \to R_4 - 11R_2$$

$$\sim \begin{bmatrix} I_4 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\rho(A) = 4$$

7. $$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 3 & -4 \\ 2 & 3 & 5 & -5 \\ 3 & -4 & -5 & 8 \end{bmatrix}$$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 2 & -5 \\ 0 & 1 & 3 & -7 \\ 0 & -7 & -8 & 5 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 6 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 6 & -30 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 - R_2 \\ R_3 \to R_3 - R_2 \\ R_4 \to R_4 + 7R_2 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 6 \\ 0 & 1 & 2 & -5 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 1 & -5 \end{bmatrix} \quad R_4 \to \frac{R_4}{6}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & 6 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 2R_3 \\ R_4 \to R_4 - R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad \begin{matrix} C_3 \to C_3 + C_1 \\ C_4 \to C_4 - 6C_1 \end{matrix}$$

$$C_4 \to C_4 + 9C_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & -3 \end{bmatrix} \quad C_4 \to C_4 + 2C_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R_4 \to \frac{R_4}{-3}$$

$\therefore \rho(A) = 4$

8 $$A = \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 5 & 5 & 0 & 5 \\ 0 & 10 & 10 & 0 & 10 \\ -1 & 1 & 2 & 2 & 4 \end{bmatrix} \quad \begin{aligned} R_2 &\to R_2 + 2R_1 \\ R_3 &\to R_3 + R_1 \\ R_4 &\to \frac{R_4}{3} \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 5 & 5 & 0 & 5 \end{bmatrix} \quad \begin{aligned} R_2 &\to \frac{R_2}{5} \\ R_3 &\to \frac{R_3}{10} \\ R_4 &\to R_4 + R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 & -2 & -3 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} R_1 &\to R_1 - 4R_2 \\ R_3 &\to R_3 - R_2 \\ R_4 &\to R_4 - 5R_2 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} C_3 &\to C_3 + C_1 \\ C_4 &\to C_4 - 2C_1 \\ C_5 &\to C_5 + 3C_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad \begin{aligned} C_3 &\to C_3 - C_2 \\ C_5 &\to C_5 - C_2 \end{aligned}$$

when $\begin{bmatrix} I_2 & 0 \\ 0 & 0 \end{bmatrix}$ $\therefore \rho(A) = 2$

9 $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$

$\sim \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $R_3 \to R_3 - 3R.$

$\sim \begin{bmatrix} 1 & 2 & -5 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$ $R_1 \to R_1 + R_4$

$\sim \begin{bmatrix} 1 & 2 & -5 & -1 \\ 0 & -2 & 6 & 2 \\ 0 & -5 & 15 & 5 \\ 0 & -1 & 3 & 1 \end{bmatrix}$ $R_2 \to R_2 - R_1$, $R_3 \to R_3 - 3R_1$, $R_4 \to R_4 - R_1$

$\sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & -2 & 6 & 2 \\ 0 & -1 & 3 & 1 \\ 0 & -1 & 3 & 1 \end{bmatrix}$ $R_1 \to R_1 + R_2$, $R_3 \to \frac{R_3}{5}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & +2 & 6 & 2 \\ 0 & +1 & 3 & 1 \\ 0 & +1 & 3 & 1 \end{bmatrix}$ $C_2 \to \frac{C_2}{-1}$, $C_3 \to C_3 - C_1$, $C_4 \to C_4 - C_1$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ $C_3 \to \frac{C_3}{3}.$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$ $R_2 \to \frac{R_2}{2}$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $C_3 \to C_3 - C_2$, $C_4 \to C_4 - C_2$

$\sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to R_3 - R_2$, $R_4 \to R_4 - R_2$

Date
3/12/2018 System of Linear Simultaneous equations

1. Write the following equations in matrix form $Ax=B$ and solve for x by finding A^{-1} where $Ax=B$ and solve for x by finding A^{-1}

$x+y-2z=3$; $2x-y+z=0$; $3x+y-z=8$

Solu) Given Equations

$$x+y-2z=3$$
$$2x-y+z=0 \qquad AX=B \Rightarrow X=A^{-1}B$$
$$3x+y-z=8$$

when $A=\begin{bmatrix}1 & 1 & -2\\ 2 & -1 & 1\\ 3 & 1 & -1\end{bmatrix}$; $X=\begin{bmatrix}x\\ y\\ z\end{bmatrix}$; $B=\begin{bmatrix}3\\ 0\\ 8\end{bmatrix}$

consider $A=I_3A$.

$$\begin{bmatrix}1 & 1 & -2\\ 2 & -1 & 1\\ 3 & 1 & -1\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}A$$

$$\begin{bmatrix}1 & 1 & -2\\ 0 & -3 & 5\\ 0 & -2 & 5\end{bmatrix} \begin{matrix}R_2\to R_2-2R_1\\ R_3\to R_3-3R_1\end{matrix} = \begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ -3 & 0 & 1\end{bmatrix}A$$

$$\begin{bmatrix}1 & 1 & -2\\ 0 & -3 & 5\\ 0 & 0 & 5\end{bmatrix} R_3\to 3R_3-2R_2 = \begin{bmatrix}1 & 0 & 0\\ -2 & 1 & 0\\ -5 & -2 & 3\end{bmatrix}A$$

$$\begin{bmatrix}3 & 0 & -1\\ 0 & -3 & 0\\ 0 & 0 & 1\end{bmatrix} \begin{matrix}R_1\to 3R_1+R_2\\ R_2\to R_2-R_3\\ R_3/5\end{matrix} = \begin{bmatrix}1 & 1 & 0\\ 3 & 3 & -3\\ -1 & -2/5 & 3/5\end{bmatrix}A$$

$$\begin{bmatrix}3 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \begin{matrix}R_1\to R_1+R_3\\ R_2/-3\end{matrix} = \begin{bmatrix}0 & 3/5 & 3/5\\ -1 & -1 & 1\\ -1 & -\frac{2}{5} & 3/5\end{bmatrix}A$$

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$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \frac{R_1}{3} = \begin{bmatrix} 0 & \frac{1}{5} & 1/5 \\ -1 & -1 & 1 \\ -1 & -2/5 & 3/5 \end{bmatrix} A$$

$$I_3 = CA. \Rightarrow C = \begin{bmatrix} 0 & \frac{1}{5} & 1/5 \\ -1 & -1 & 1 \\ -1 & -2/5 & 3/5 \end{bmatrix}.$$

$$\boxed{\therefore AA^{-1} = A^{-1}A = I}$$

$$\therefore A^{-1} = C$$

$$X = A^{-1}B$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1/5 & 1/5 \\ -1 & -1 & 1 \\ -1 & -2/5 & 3/5 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 8 \end{bmatrix}$$

$$= \begin{bmatrix} 0 + 0 + 8/5 \\ -3 - 0 + 8 \\ -3 - 0 + 24/5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8/5 \\ 5 \\ 9/5 \end{bmatrix}$$

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$\therefore x = 8/5,\ y = 5,\ z = 9/5$

2. For Non-Homogeneous System

* Consistent

The system $AX = B$ is consistent if and only if it has a solution

rank of A = rank of AB and

1. The $\rho(A) = \rho(AB) = n$ then the system has unique solution.

where n = unknown variables

2. If $\rho(A) = \rho(AB) < n$ then the system is consistent but therse exsist infinite number of solutions.

3. If the $\rho(A) \neq \rho(AB)$ then the system is <u>inconsistent</u> and it has no solution.

1. Show that the equations $x+y+z=4$; $2x+5y-2z=3$ and $x+7y-7z=5$ are not consistent (only row operations)

<u>Solu</u> Given Equations

$$x+y+z=4$$
$$2x+5y-2z=3$$
$$x+7y-7z=5$$

can be expressed as $AX=B$

$$A=\begin{bmatrix}1 & 1 & 1\\ 2 & 5 & -2\\ 1 & 7 & -7\end{bmatrix};\ B=\begin{bmatrix}4\\3\\5\end{bmatrix};\ X=\begin{bmatrix}x\\y\\z\end{bmatrix}$$

Consider Argumented matrix

$$[AB]=\begin{bmatrix}1 & 1 & 1 & 4\\ 2 & 5 & -2 & 3\\ 1 & 7 & -7 & 5\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 1 & 1 & 4\\ 0 & 3 & -4 & -5\\ 0 & 6 & -8 & 1\end{bmatrix} \quad \begin{matrix}R_2 \to R_2 - 2R_1\\ R_3 \to R_3 - R_1\end{matrix}$$

$$\sim\begin{bmatrix}1 & 1 & 1 & 4\\ 0 & 3 & -4 & -5\\ 0 & 0 & 0 & 11\end{bmatrix} \quad R_3 \to R_3 - 2R_2$$

$$\rho(A)=2;\quad \rho(AB)=3$$

$$\therefore \rho(A) \neq \rho(AB)$$

Hence given equation are inconsistent and it has no solution

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2. Solve the Equations $x+y+z=9$, $2x+5y+7z=52$ and $2x+y-z=0$

Solu) Given equations

$$x+y+z=9$$
$$2x+5y+7z=52$$
$$2x+y-z=0$$

$52 - 18 = 34$, $34 - 36 = -2$

Given equations can be expressed as

$$A=\begin{bmatrix}1&1&1\\2&5&7\\2&1&-1\end{bmatrix};\ X=\begin{bmatrix}x\\y\\z\end{bmatrix};\ B=\begin{bmatrix}9\\52\\0\end{bmatrix}$$

Argumented matrix

$$[AB]=\begin{bmatrix}1&1&1&9\\2&5&7&52\\2&1&-1&0\end{bmatrix}$$

$$\sim\begin{bmatrix}1&1&1&9\\0&3&5&34\\0&-1&-3&-18\end{bmatrix}\begin{matrix}R_2\to R_2-2R_1\\R_3\to R_3-2R_1\end{matrix}$$

$$\sim\begin{bmatrix}1&1&1&9\\0&3&5&34\\0&0&-4&-20\end{bmatrix}R_3\to 3R_3+R_2$$

$$\rho(A)=3,\ \rho(AB)=3,\ n=3$$

$$\therefore \rho(A)=\rho(AB)=n$$

Given system is consistent and it has unique no.of so solutions

$$x+y+z=9$$
$$3y+5z=34 \to ①$$
$$-4z=-20$$
$$z=5$$

$$① \to 3y+25=34$$
$$3y=34-25$$
$$=9$$

$$y = 3$$
$$x + 3 + 5 = 9$$
$$x = 1$$

$\therefore x = 1, y = 3, z = 5$

3. Solve the system of linear equations by matrix method $x + y + z = 6$; $2x + 3y - 2z = 2$; $5x + y + 2z = 13$

Solu) Given equations

$$x + y + z = 6$$
$$2x + 3y - 2z = 2$$
$$5x + y + 2z = 13$$

~~Argum~~

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$$

Now $AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & -2 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -4 \\ 0 & -4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 5R_1 \end{matrix} = \begin{bmatrix} 6 \\ -10 \\ -17 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -4 \\ 0 & 0 & -19 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 + 4R_2 = \begin{bmatrix} 6 \\ -10 \\ -57 \end{bmatrix} \quad \begin{matrix} -17 \\ -40 \end{matrix}$$

$$x + y + z = 6$$
$$y - 4z = -10$$
$$-19z = -57$$
$$z = 3$$
$$y - 12 = -10$$
$$y = 2$$
$$x + 2 + 3 = 6$$
$$x = 1$$

4. Examine the following equations are consistent or inconsistent

1) $x - 4y + 7z = 8$
$3x + 8y - 2z = 6$
$7x - 8y + 26z = 31$

2) $x + 2y - z = 3$
$3x - y + 2z = 1$
$2x - 2y + 3z = 2$
$x - y + z = -1$

solu/ 1) Given equations

$x - 4y + 7z = 8$
$3x + 8y - 2z = 6$
$7x - 8y + 26z = 31$

Can be expressed as

$$A = \begin{bmatrix} 1 & -4 & 7 \\ 3 & 8 & -2 \\ 7 & -8 & 26 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 8 \\ 6 \\ 31 \end{bmatrix}$$

Consider Argumented matrix $[AB]$

$$[AB] = \begin{bmatrix} 1 & -4 & 7 & 8 \\ 3 & 8 & -2 & 6 \\ 7 & -8 & 26 & 31 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -4 & 7 & 8 \\ 0 & 20 & -23 & -18 \\ 0 & 20 & -23 & -25 \end{bmatrix} \quad \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 7R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -4 & 7 & 8 \\ 0 & 20 & -23 & -18 \\ 0 & 0 & 0 & -7 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$\rho(A) = 2$; $\rho(AB) = 3$; $n = 3$

$\rho(A) \neq \rho(AB)$

Hence given equations are inconsistent and it has no solution

2) Given equations

$x + 2y - z = 3$
$3x - y + 2z = 1$
$2x - 2y + 3z = 2$
$x - y + z = -1$

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Given equations can be expressed as

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \end{bmatrix}$$

consider argumented matrix

$$[A\,B] = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 3 & -1 & 2 & 1 \\ 2 & -2 & 3 & 2 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & -6 & 5 & -4 \\ 0 & -3 & 2 & -4 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 - 2R_1 \\ R_4 \to R_4 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & 5 & 20 \\ 0 & 0 & -1 & -4 \end{bmatrix} \quad \begin{array}{l} R_3 \to 7R_3 - 6R_2 \\ R_4 \to 7R_4 - 3R_2 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & -1 & 3 \\ 0 & -7 & 5 & -8 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 5 & 20 \end{bmatrix} \quad R_3 \leftrightarrow R_4$$

$\rho(A) = \cancel{3}\,4$; $\rho(AB) = 4$; $n = 4$

$$\rho(A) \neq \rho(AB) = n$$

$\therefore$ The given system is ~~in~~consistent and it has unique solution

$$x + 2y - z = 3$$
$$-7y + 5z = -8$$
$$-z = -4$$
$$5z = 20$$

$-7y + 5(4) = -8$

$-7y + 20 = -8$

$-7y = -28$

$y = 4$

$x + 2(4) - 4 = 3$

$x + 4 = 3$

$x = -1$

$x + 2y - z = 3$
$-1 + 2(4) - 4 = 3$
$-5 + 8 = 3$
$3 = 3$

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5. For what values of λ the equations $x+y+z=1$; $x+2y+4z=\lambda$; $x+4y+10z=\lambda^2$ have a solution and solve them completely in each case.

Solu) Given equation

$$\left.\begin{aligned} x+y+z &= 1 \\ x+2y+4z &= \lambda \\ x+4y+10z &= \lambda^2 \end{aligned}\right\} \rightarrow (1)$$

can be expressed as a matrix form of system (1)

$AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

Argumented matrix

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 3 & 9 & \lambda^2-1 \end{bmatrix} \begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda-1 \\ 0 & 0 & 0 & \lambda^2-3\lambda+2 \end{bmatrix} R_3 \rightarrow R_3 - 3R_2$$

$\rho(A) = \rho(AB) = 3$

But given that the system has a solution. It must be consistent. So that

$$\lambda^2 - 3\lambda + 2 = 0$$

$$(\lambda - 2)(\lambda - 1) = 0$$

$$\lambda = 1, 2$$

case (i)

if $\lambda = 1$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$; $\rho(AB) = 2$, $n = 3$

$\therefore \rho(A) = \rho(AB) = r < n$

Given equation are consistent and will have infinite no. of solutions.

$$x + y + z = 1$$

$$y + 3z = 0$$

let $n - r = 3 - 2 = 1$ L.I.S

let $z = k$

$$y + 3k = 0$$

$$y = -3k$$

$$x - 3k + k = 1$$

$$x = 1 + 2k$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1+2k \\ -3k \\ k \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + k \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

case (ii)

if $\lambda = 2$

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$; $\rho(AB) = 2$, $n = 3$

$\therefore \rho(A) = \rho(AB) = r < n$

Given equation consistent and will have infinite no. of solutions

$x + y + z = 1$

$y + 3z = 1$

let $z = k$

$y + 3k = 1$

$y = 1 - 3k$

$x + \cancel{1} - 3k + k = \cancel{1}$

$x = 2k$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2k \\ 1-3k \\ k \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + k \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$$

2 If $a+b+c \neq 0$, show that the system of equation $-2x + y + z = a$, $x - 2y + z = b$, $x + y - 2z = c$ has no solution. If $a+b+c = 0$. show that it has infinitely many solutions. show that it.

Solu) Given Equations

$$\left.\begin{aligned} -2x + y + z &= a \\ x - 2y + z &= b \\ x + y - 2z &= c \end{aligned}\right\} \rightarrow ①$$

system ① can be expressed as the matrix form of $AX = B$

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} ; B = \begin{bmatrix} a \\ b \\ c \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Argumented matrix

$$[AB] = \begin{bmatrix} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & -2 & c \\ 1 & -2 & 1 & b \\ -2 & 1 & 1 & a \end{bmatrix} R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & c \\ 0 & -3 & 3 & b-c \\ 0 & 3 & -3 & a+2c \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 + 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & c \\ 0 & -3 & 3 & b-c \\ 0 & 0 & 0 & a+b+c \end{bmatrix} R_3 \to R_3 + R_2$$

if $a+b+c \neq 0$

$\rho(A) = 2 ; \rho(AB) = 3$

$\rho(A) \neq \rho(AB)$

Given system are inconsistent and will have no solution.

if $a+b+c = 0$

$\rho(A) = 2 ; \rho(AB) = 2 ; n = 3$

$\rho(A) = \rho(AB) = r < n$

Given equations are consistent and will have infinite no. of solutions.

$x + y - 2z = c$

$-3y + 3z = b - c$

$n-r = 3-2 = 1$ L.I.S

let $z = k$

$-3y + 3k = b - c$

$3y = 3k - b + c$

$y = k - \frac{b}{3} + \frac{c}{3}$

$x + k - \frac{b}{3} + \frac{c}{3} - 2k = c$

$x = k + \frac{b}{3} + \frac{2c}{3}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k + \frac{b}{3} + \frac{2c}{3} \\ k - b/3 + c/3 \\ k \end{bmatrix}$$

$$= k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} b/3 + 2c/3 \\ -b/3 + c/3 \\ 0 \end{bmatrix}$$

H.W

3. Solve the system of linear equations by matrix method

i) (over) $x+y+z = 6$
$2x+3y-2z = 2$
$5x+y+2z = 13$

ii) $x+y+4z = 6$
$x+2y-2z = 6$
$x+y+z = 6$

iii) $x+y+2z = 4$
$2x-y+3z = 9$
$3x-y-z = 2$

iv) $x+y+z = 6$
$x+2y+3z = 14$
$x+4y+7z = 30$

iv) Solu) Given Equations <u>Matrix</u> <u>Method</u>

$x+y+z = 6$
$x+2y+3z = 14$
$x+4y+7z = 30$

These equations can be expressed

$AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}; \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad B = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 3 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 6 \\ 8 \\ 24 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 - 3R_2 = \begin{bmatrix} 6 \\ 8 \\ 0 \end{bmatrix}$$

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24

$x + y + z = 6$

$y + 2z = 8$ let $z = 0$

$y + 2(0) = 8$

$y = 8$

$x + 8 + 0 = 6$

$x = 6 - 8$

$x = -2$

$\therefore x = -2 ; y = 8 ; z = 0$

(or)

Consistent Method

let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 7 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 14 \\ 30 \end{bmatrix}$

Argumented matrix

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 1 & 4 & 7 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 3 & 6 & 24 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - 3R_2$$

$\therefore \rho(A) = 2 \; ; \; \rho(B) = 2 \; ; \; n = 3$

$\rho(A) = \rho(AB) < n$

The given system of equations is consistent and has infinite no. of solutions

$x + y + z = 6$;

$y + 2z = 8$;

let $z = k$

$y + 2k = 8$

$y = 8 - 2k$;

$\cancel{8} - \cancel{2k} + \cancel{2k} = \cancel{8}$

$x + 8 - 2k + k = 6$

$x - k = 6 - 8$

$x - k = -2$

$x = -2 + k$.

$x = k - 2$

iii) Matrix Method

Given equations

$x + y + 2z = 4$

$2x - y + 3z = 9$.

$3x - y - z = 2$

these equations can be expressed as $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix} ; \; B = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix} ; \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & -4 & -7 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix} = \begin{bmatrix} 4 \\ 1 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & -4 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ -10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -3 & -1 \\ 0 & 0 & -17 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to 3R_3 - 4R_2 = \begin{bmatrix} 4 \\ 1 \\ -34 \end{bmatrix}$$

$x+y+2z = 4$

$-3y - z = 1$

$-17z = -34$

$z = 2$

$-3y - 2 = 1$

$-3y = 3$

$y = -1$

$x - 1 + \cancel{4} = \cancel{4}$

$x = 1$

$[\because x+y+2z = 4$

$1-1+4 = 4$

$0=0]$

$\therefore x = 1, \; y = -1, \; z = 2$

(or)

<u>Consistent</u> <u>Method</u>

$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & -1 & 3 \\ 3 & -1 & -1 \end{bmatrix}; \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \; B = \begin{bmatrix} 4 \\ 9 \\ 2 \end{bmatrix}$

Argumented matrix

$$[A\,B] = \begin{bmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & 3 & 9 \\ 3 & -1 & -1 & 2 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & 1 & 1 \\ 0 & -4 & -7 & -10 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -1 & 1 \\ 0 & 0 & -17 & -34 \end{bmatrix} \quad R_3 \to 3R_3 - 4R_2$$

$\rho(A) = 3$; $\rho(AB) = 3$; $n = 3$

$\rho(A) \neq \rho(AB) = n$

The given system of equation is ~~in~~consistent it has ~~a~~ a solution.

$$x + y + 2z = 4$$
$$-3y - z = 1$$
$$-17z = -34$$
$$z = 2$$

; $-3y - 2 = 1$; $-3y = 3$; $y = -1$

; $x - 1 + 4 = 4$; $x = 1$

$\therefore x = 1$; $y = -1$; $z = 2$

ii) Given Equations

$$x + y + 4z = 6$$
$$x + 2y - 2z = 6$$
$$x + y + z = 6$$

$A = \begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & -2 \\ 1 & 1 & 1 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$

<u>Matrix method</u>

The given system of equations can be expressed as $AX = B$

$$\begin{bmatrix} 1 & 1 & 4 \\ 1 & 2 & -2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 6 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 4 \\ 0 & 1 & -6 \\ 0 & 0 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 6 \\ 0 \\ 0 \end{bmatrix}$$

$$x + y + 4z = 6$$
$$y - 6z = 0$$
$$-3z = 0$$
$$z = 0$$

; $y = 0$; $x = 6$; $z = 0$

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Consistent method

Argumented matrix,

$$[A\,B] = \begin{bmatrix} 1 & 1 & 4 & 6 \\ 1 & 2 & -2 & 6 \\ 1 & 1 & 1 & 6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 4 & 6 \\ 0 & 1 & -6 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix} R_2 \to R_2 - R_1$$

$$\rho(A) = 3 \; ; \; \rho(AB) = 3 \; ; \; n = 3$$

$$\therefore \; \rho(A) = \rho(AB) = n$$

The given system of equations is consistent and has unique solution

$$x + y + 4z = 6$$
$$y - 6z = 0 \quad ; \; y = 0 ; \; x + 0 + 0 = 6$$
$$-3z = 0 \qquad x = 6$$
$$z = 0$$

Date 6/12/18

4. Find the values of λ for which the system of equations $3x - y + 4z = 3$; $x + 2y - 3z = -2$; $6x + 5y + \lambda z = -3$ will have infinite no of solutions solve them with the λ values

Solu) Given equations

$$\left.\begin{aligned} 3x - y + 4z &= 3 \\ x + 2y - 3z &= -2 \\ 6x + 5y + \lambda z &= -3 \end{aligned}\right\} \to (1)$$

system (1) can be expressed in a matrix form

$$AX = B$$

where $A = \begin{bmatrix} 3 & -1 & 4 \\ 1 & 2 & -3 \\ 6 & 5 & \lambda \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 3 \\ -2 \\ -3 \end{bmatrix}$

$$[AB] = \begin{bmatrix} 3 & -1 & 4 & 3 \\ 1 & 2 & -3 & -2 \\ 6 & 5 & \lambda & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 7 & \lambda-8 & -9 \end{bmatrix} \begin{matrix} \\ R_2 \to 3R_2 - R_1 \\ R_3 \to R_3 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 0 & \lambda+5 & 0 \end{bmatrix} R_3 \to R_3 - R_2$$

If $\lambda+5=0$

$\lambda = -5$

$\rho(A) = 2$; $\rho(AB) = 2$; $n=3$

$\rho(A) = \rho(AB) = r < n$

Given equations have infinite no. of solutions

$\lambda = -5$ then

$$[AB] = \begin{bmatrix} 3 & -1 & 4 & 3 \\ 0 & 7 & -13 & -9 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$n - r = 3 - 2 = 1$ L.I.S

$3x - y + 4z = 3$

$7y - 13z = -9$

let $z = k$

$7y - 13k = -9$

$7y = -9 + 13k$

$y = \frac{-9}{7} + \frac{13}{7}k$

$3x + \frac{9}{7} - \frac{13}{7}k + 4k = 3$

$3x = -\frac{15}{7}k + \frac{12}{7}$

$x = \frac{4}{7} - \frac{5}{7}k$

$\therefore x = \frac{4}{7} - \frac{5}{7}k$; $y = \frac{-9}{7} + \frac{13}{7}k$, $z = k$

5. Find whether the following set of equations are consistent

$x_1 + x_2 + x_3 + x_4 = 0$

$x_1 + x_2 + x_3 - x_4 = 4$

$x_1 + x_2 - x_3 + x_4 = -4$

$x_1 - x_2 + x_3 + x_4 = 2$

Solu) Given equations

$$\left.\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 0 \\ x_1 + x_2 + x_3 - x_4 &= 4 \\ x_1 + x_2 - x_3 + x_4 &= -4 \\ x_1 - x_2 + x_3 + x_4 &= 2 \end{aligned}\right\} \rightarrow (1)$$

set (1) can be expressend in a matrix form

$AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 0 \\ 4 \\ -4 \\ 2 \end{bmatrix}$$

Argumented matrix

$$[AB] \sim \begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & -1 & 4 \\ 1 & 1 & -1 & 1 & -4 \\ 1 & -1 & 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 4 \\ 0 & 0 & -2 & 0 & -4 \\ 0 & -2 & 0 & 0 & 2 \end{bmatrix} \begin{matrix} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \\ R_4 \rightarrow R_4 - R_1 \end{matrix}$$

$\rho(A) = 4$, $\rho(AB) = 4$; $n = 4$

$\rho(A) = \rho(AB) = r = n$

Given equations are consistent and will have a unique solution

Consistency of system of Homogeneous linear Equations

1. Consider a system of m- homogeneous linear equations in n- unknowns.

$$a_{11}x_1+a_{12}x_2+a_{13}x_3+\dots+a_{1n}x_n=0$$
2. $a_{21}x_1+a_{22}x_2+a_{23}x_3+\dots+a_{2n}x_n=0$
3. $a_{31}x_1+a_{32}x_2+a_{33}x_3+\dots+a_{3n}x_n=0$
4. $a_{m1}x_1+a_{m2}x_2+\dots+a_{mn}x_n=0$ $\rightarrow$ (i)

system one can be return as in a matrix form

$$AX=0.$$

* If $\rho(A)=n$ then the system of equations have only trivial solution i.e., zero solution
* If $\rho(A)<n$ then the system of equations have an infinite no of non-trivial solutions, in this case $n-r$ linearly independent solution

1. Solve $x+y-2z+3w=0$; $x-2y+z-w=0$; $4x+y-5z+8w=0$; $5x-7y+2z-w=0$; Given equation

solu) Given equation

$$\left.\begin{aligned} x+y-2z+3w&=0\\ x-2y+z-w&=0\\ 4x+y-5z+8w&=0\\ 5x-7y+2z-w&=0\end{aligned}\right\}\rightarrow (i)$$

system (i) can be expressed in the form of a matrix

$$AX=0$$

$$A=\begin{bmatrix}1&1&-2&3\\1&-2&1&-1\\4&1&-5&8\\5&-7&2&-1\end{bmatrix};\ X=\begin{bmatrix}x\\y\\z\\w\end{bmatrix};\ O=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

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$$\sim \begin{bmatrix} 1 & 1 & -2 & 3 & 0 \\ 0 & -3 & 3 & -4 & 0 \\ 0 & -3 & 3 & -4 & 0 \\ 0 & -12 & 12 & -16 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - 4R_1 \\ R_4 \to R_4 - 5R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & -2 & 3 & 0 \\ 0 & -3 & 3 & -4 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_3 \to R_3 - R_2 \\ R_4 \to R_4 - 4R_2 \end{matrix}$$

$\rho(A) = 2$, $n = 4$

$r < n$

Given equation have infinite no. of solutions of non-trivial solution.

$n - r = 4 - 2 = 2$ L.I.S

$x + y - 2z + 3w = 0$

$-3y + 3z - 4w = 0$

let $w = k_1$, $z = k_2$

$3y = 3k_2 - 4k_1$

$y = k_2 - \frac{4}{3}k_1$

$x + k_2 - \frac{4}{3}k_1 - 2k_2 + 3k_1 = 0$

$x - k_2 + \frac{5}{3}k_1 = 0$

$x = k_2 - \frac{5}{3}k_1$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} k_2 - \frac{5}{3}k_1 \\ k_2 - \frac{4}{3}k_1 \\ k_2 \\ k_1 \end{bmatrix}$$

2. Solve $x+y-3z+2w=0$, $2x-y+2z-3w=0$
$3x-2y+z-4w=0$, $-4x+y-3z+w=0$

Solu) Given Equations

$$\left.\begin{array}{l} x+y-3z+2w=0 \\ 2x-y+2z-3w=0 \\ 3x-2y+z-4w=0 \\ -4x+y-3z+w=0 \end{array}\right\} \rightarrow (1)$$

Given system of equations (1) can be expressed

as $AX=0$

$$A = \begin{bmatrix} 1 & 1 & -3 & 2 \\ 2 & -1 & 2 & -3 \\ 3 & -2 & 1 & -4 \\ -4 & 1 & -3 & 1 \end{bmatrix};\quad X = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix};$$

$$\sim = \begin{bmatrix} 1 & 1 & -3 & 2 \\ 0 & -3 & 8 & -7 \\ 0 & -5 & 10 & -10 \\ 0 & 5 & -15 & 9 \end{bmatrix} \begin{array}{l} \\ R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \\ R_4 \rightarrow R_4 + 4R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & -3 & 2 \\ 0 & -3 & 8 & -7 \\ 0 & 0 & -10 & 5 \\ 0 & 0 & -5 & -8 \end{bmatrix} \begin{array}{l} \\ \\ R_3 \rightarrow 3R_3 - 5R_2 \\ R_4 \rightarrow 3R_4 + 5R_2 \end{array}$$

$$\begin{array}{r} -35 \\ 27 \\ \hline -8 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & -3 & 2 \\ 0 & -3 & 8 & -7 \\ 0 & 0 & -10 & 5 \\ 0 & 0 & 0 & -21 \end{bmatrix} R_4 \rightarrow 2R_4 - R_3$$

$\therefore \rho(A) = 4, \; n = 4$

$r = n$

Given Equation have trivial Solution

$x = 0 ; y = 0 ; z = 0 ; w = 0$

3 Solve $x + y + w = 0 ; \; y + z = 0 ; \; x + y + z + w = 0 ;$
$x + y + 2z = 0$

Soln Given equations

$$\left.\begin{array}{l} x + y + w = 0 \\ y + z = 0 \\ x + y + z + w = 0 \\ x + y + 2z = 0 \end{array}\right\} \rightarrow ①$$

system of Equation ① can have be expressed in the form

$Ax = 0$

$$A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 2 & 0 \end{bmatrix} ; X \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & -1 \end{bmatrix} \quad \begin{array}{l} R_3 \rightarrow R_3 - R_1 \\ \\ R_4 \rightarrow R_4 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \quad R_4 \rightarrow R_4 - 2R_3$$

$\sim \rho(A) = 4 \quad n = 4$

$\therefore$ The given system of equations can have trivial solution.

$x = 0 ; y = 0 ; z = 0 ; w = 0$

4. solve the system of equations $x+2y+(2+k)z=0$
Date $2x+(2+k)y+4z=0$, $7x+13y+(18+k)z=0$ for all valu
8/12/18 of k

Solu) Given Equations

$$\left.\begin{aligned} x+2y+(2+k)z &= 0 \\ 2x+(2+k)y+4z &= 0 \\ 7x+13y+(18+k)z &= 0 \end{aligned}\right\} \to (1)$$

system (1) can be expressed as a matrix form of $AX=B$ where

$$A = \begin{bmatrix} 1 & 2 & 2+k \\ 2 & 2+k & 4 \\ 7 & 13 & 18+k \end{bmatrix};\ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The given system has a solution for all values of k if the system has a non-trivial solution. i.e.,

$$\rho(A) < n \quad ; n=3$$

$$\rho(A) < 3$$

Given matrix A is 3x3 matrix so that

$$|A| = 0$$

$$|A| = \begin{vmatrix} 1 & 2 & 2+k \\ 2 & 2+k & 4 \\ 7 & 13 & 18+k \end{vmatrix} = 0$$

$$1[(18+k)(2+k)-52] - 2[2(18+k)-28] + (2+k)[26-7(2+k)] = 0$$

$$36+18k+2k+k^2-52-2(36+2k-28)+(2+k)(26-14-7k)=0$$

$$k^2+20k-16-16-4k+24-14k+12k-7k^2=0$$

$$-6k^2+14k-8=0$$

$$3k^2-7k+4=0$$

$$3k^2-3k-4k+4=0$$

$$3k(k-1)-4(k-1)=0$$

$$(k-1)(3k-4)=0$$

12
-4 -3

$(k-1)(3k-4)=0$

$k=1 ; k=4/3$

case (i)

If $k=1$

$$A=\begin{bmatrix}1 & 2 & 3\\ 2 & 3 & 4\\ 7 & 13 & 19\end{bmatrix}$$

$$\sim\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & -2\\ 0 & -1 & -2\end{bmatrix}\quad \begin{array}{l}R_2\to R_2-2R_1\\ R_3\to R_3-7R_1\end{array}$$

$$\sim\begin{bmatrix}1 & 2 & 3\\ 0 & -1 & -2\\ 0 & 0 & 0\end{bmatrix}\quad R_3\to R_3-R_2$$

$\rho(A)=2,\ n=3,\ \rho(A)<n$

When $k=1$ the system has a non-trivial solution

$n-r=3-2=1$ L.I.S

$x+2y+3z=0$

$-y-2z=0$

Let $z=k$

$y=-2k$

$x-4k+3k=0$

$x=k$

$$\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}k\\ -2k\\ k\end{bmatrix}=k\begin{bmatrix}1\\ -2\\ 1\end{bmatrix}$$

case (ii)

If $k=4/3$

$$A=\begin{bmatrix}1 & 2 & 10/3\\ 2 & 10/3 & 4\\ 7 & 13 & 58/3\end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 10/3 \\ 0 & -2/3 & -8/3 \\ 0 & -1 & -12/3 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 7R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 10/3 \\ 0 & -2/3 & -8/3 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to \frac{2}{3} R_3 - R_2$$

$\rho(A) = 2$; $n = 3$

$\therefore \rho(A) < n$

$n - r = 3 - 2 = 1$ L.I.S

$x + 2y + 10/3 z = 0$, $\quad -\frac{2}{3} y - \frac{8}{3} z = 0$

let $z = k$

$-\frac{2}{3} y - \frac{8}{3} k = 0$; $\quad y = \cancel{\frac{8}{3}} k \times -\cancel{\frac{3}{2}}$

$-\frac{2}{3} y = \frac{8}{3} k$; $\quad y = -4k$

$x - 8k + \frac{10}{3} k = 0$

$x = \frac{14}{3} k$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{14}{3}k \\ -4k \\ k \end{bmatrix} = k \begin{bmatrix} 14/3 \\ -4 \\ 1 \end{bmatrix}$$

-14
10
$-\frac{4}{3}$

5. Solve the system $\lambda x + y + z = 0$; $x + \lambda y + z = 0$; $x + y + \lambda z = 0$; if it has non-zero solutions only

<u>solu</u> Given Equations

$$\left.\begin{matrix} \lambda x + y + z = 0 \\ x + \lambda y + z = 0 \\ x + y + \lambda z = 0 \end{matrix}\right\} \to ①$$

Then system ① can be expressed in the matrix form $Ax = 0$

$$A = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Given that given system has a non trivial solution

$\rho(A) < n \quad ; n=3$

$\rho(A) < 3$

Given matrix is a 3X3 matrix so that

$|A| = 0$

$$|A| = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{bmatrix} = 0$$

$\lambda(\lambda^2 - 1) - 1(\lambda - 1) + 1(1-\lambda) = 0$

$\lambda^3 \lambda - \lambda + 1 + 1 - \lambda = 0$

$\lambda^3 - 3\lambda + 2 = 0$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$

$(\lambda-1)(\lambda^2+\lambda-2) = 0$

$(\lambda-1)(\lambda^2+2\lambda-\lambda-2) = 0$

$(\lambda-1)(\lambda(\lambda+2)-1(\lambda+2)) = 0$

$(\lambda-1)(\lambda-1)(\lambda+2) = 0$

$\lambda = 1, 1, -2$

case (i)

$\lambda = 1$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{array}$$

$\therefore \rho(A) = 1, \; n = 3$

$\rho(A) < n$

$n - r = 3 - 1 = 2 \quad L \cdot I \cdot S$

$x + y + z = 0$

$y = k_1; \; z = k_2$

$x + k_1 + k_2 = 0$

$x = -(k_1 + k_2)$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -(k_1 + k_2) \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore$ For $\lambda = 1$ the system has a non trivial solution

case (ii)

$\lambda = -2$

$$A = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix}$$

$$\sim \begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 3 & -3 \end{bmatrix} \begin{matrix} R_2 \to 2R_2 + R_1 \\ R_3 \to 2R_3 + R_1 \end{matrix}$$

$$\sim \begin{bmatrix} -2 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 + R_2$$

$\rho(A) = 2; \; n = 3$

$\rho(A) < n$

$\therefore$ For $\lambda = -2$ the system has a non trivial solution

$n - r = 3 - 2 = 1 \quad L \cdot I \cdot S$

$z = k$

$-2x + y + z = 0$

$-3y + 3z = 0$

$-3y + 3k = 0$

$-\cancel{3}y = -\cancel{3}k$

$y = k$

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$$-2x + k + k = 0$$
$$-2x = -2k$$
$$x = k$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ k \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

H.W

6 show that the only real number λ for which the system $x+2y+3z=\lambda x$; $3x+y+2z=\lambda y$; $2x+3y+z=\lambda z$. has non-zero solution is 6 and solve them when $\lambda=6$

Solu:- Given system can be expressed as $AX=0$ where

$$A = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } 0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Here number of variables $n=3$

The given system of equations possess a non-zero solution, if

Rank of $A < n$

$$\rho(A) < 3$$

For this $|A| = 0$

$$\begin{bmatrix} 1-\lambda & 2 & 3 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{bmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\begin{vmatrix} 6-\lambda & 6-\lambda & 6-\lambda \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)\begin{vmatrix} 1 & 1 & 1 \\ 3 & 1-\lambda & 2 \\ 2 & 3 & 1-\lambda \end{vmatrix} = 0$$ 7.

8.

$$(6-\lambda)\begin{vmatrix} 1 & 0 & 0 \\ 3 & -(\lambda+2) & -1 \\ 2 & 1 & -(\lambda+1) \end{vmatrix} \begin{matrix} C_2 \to C_2 - C_1 \\ C_3 \to C_3 - C_1 \end{matrix} = 0$$

$$(6-\lambda)\left[1\left((\lambda+2)(\lambda+1)+1\right) - 0\left(-3(\lambda+1)-2\right) + 0\left(3+2(\lambda+1)\right)\right] = 0$$

$$(6-\lambda)\left[\lambda^2+2\lambda+\lambda+2+1-0+0\right]=0 \Rightarrow (6-\lambda)[\lambda^2+3\lambda+3] =$$

$$(6-\lambda)[\lambda^2+3\lambda+3] =$$

$$\therefore \lambda = 6$$

$$\cancel{(6-\lambda)\left[4\lambda+3\right]=0}$$

$$\cancel{24\lambda - 4\lambda^2 + 18 - 3\lambda = 0}$$

$$\cancel{21\lambda + 18 = 4\lambda^2}$$

$$\cancel{4\lambda^2 - 21\lambda - 18 = 0}$$

7 solu

$\therefore$ Here $\lambda = 6$ is the only real value and other value are complex. when $\lambda = 6$, the given system becomes

$$A = \begin{bmatrix} -5 & 2 & 3 \\ 3 & -5 & 2 \\ 2 & 3 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore \rho(A) = 2;\ n = 3$

$n - r = 3 - 2 = 1$ L.I.S

$$\sim \begin{bmatrix} -5 & 2 & 3 \\ 0 & -19 & 19 \\ 0 & 19 & -19 \end{bmatrix} \begin{matrix} R_2 \to 5R_2 + 3R_1 \\ R_3 \to 5R_3 + 2R_1 \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -5 & 2 & 3 \\ 0 & -19 & 19 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 + R_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-5x + 2y + 3z = 0$; $-5x + 2k + 3k = 0$

$-19y + 19z = 0$

$\cancel{-}5x = \cancel{-}5k$

$z = k$

$x = k$

$-19y + 19k = 0$

$-19y = -19k$

$y = k$;

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ k \\ k \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} k$$

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Gauss -

Solutions of Linear systems Direct Methods

1) Gaussian Elimination Method

This method of solving system of n linear Equations in n unknowns consists of eliminating the co-efficients in such a way that the system reduces to upper triangular system which may be solved by backward substitution.

1. solve the Equations $2x+y+z=10; 3x+2y+3z=18; x+4y+9z=16$; by using Gauss elimination method.

solu/ Given Equations

$$\left.\begin{array}{l} 2x+y+z=10 \\ 3x+2y+3z=18 \\ x+4y+9z=16 \end{array}\right\} \rightarrow (1)$$

system (1) can be expressed in the form $AX=B$

where $A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$; $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B=\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$

Argumented matrix

$$[AB] = \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix} \begin{array}{l} R_2 \rightarrow 2R_2 - 3R_1 \\ R_3 \rightarrow 2R_3 - R_1 \end{array}$$

$$\begin{array}{r} 70 \\ 21 \\ \hline 59 \end{array} \qquad \begin{array}{r} 22 \\ 22 \\ \hline 20 \end{array}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix} R_3 \rightarrow R_3 - 7R_2$$

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which is a upper triangular matrix

$$2x+y+z=10; \quad y+3z=6$$

$$-4z=-20$$

$$z=5$$

$y+3(5)=6$; $2x-9+5=10$

$y=6-15$ $2x=14$

$y=-9$ $x=7$;

$x=7$; $y=-9$; $z=5$

2. Solve $3x+y-z=3$; $2x-8y+z=-5$; $x-2y+9z=8$ by Gaussian elimination method

Given Equations

$$\left.\begin{aligned} 3x+y-z&=3 \\ 2x-8y+z&=-5 \\ x-2y+9z&=8 \end{aligned}\right\} \rightarrow ①$$

system ① can be expressed in the form $AX=B$

$$A=\begin{bmatrix} 3 & 1 & -1 \\ 2 & -8 & 1 \\ 1 & -2 & 9 \end{bmatrix}; \quad X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad B=\begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$$

Argumented matrix

$$[AB]=\begin{bmatrix} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{bmatrix}$$

$$\sim\begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & -7 & 28 & 21 \end{bmatrix} \begin{matrix} \\ R_2 \rightarrow 3R_2-2R_1 \\ R_3 \rightarrow 3R_3-R_1 \end{matrix}$$

-24
-2

$$\sim\begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & -1 & 4 & 3 \end{bmatrix} R_3 \rightarrow \frac{R_3}{7}$$

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$$\sim \begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & 0 & 99 & 99 \end{bmatrix} R_3 \to 26R_3 + R_2$$

$\begin{array}{r} 1 \\ 26 \\ 3 \\ \hline 78 \\ 21 \\ \hline 99 \end{array}$ $\begin{array}{r} 2 \\ 26 \\ 4 \\ \hline 104 \\ 5 \\ \hline 99 \end{array}$

which is a upper triangular matrix

$$3x + y - z = 3$$

$$-26y + 5z = -21$$

$$\cancel{99}z = \cancel{99}$$

$$z = 1$$

$$-26y + 5 = -21$$

$$-26y = -21 - 5$$

$$\cancel{-26}y = \cancel{-26}$$

$$y = 1$$

$$3x + \cancel{1} - \cancel{1} = 3$$

$$x = 1$$

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$\therefore x = 1,\ y = 1,\ z = 1$

3. solve $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$ by using Gauss-Jordan-Method (only row operations)

solu) Given Equations

$$\left.\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 3z &= 18 \\ x + 4y + 9z &= 16 \end{aligned}\right\} \to (1)$$

system (1) can be expressed in the form $AX = B$ where

$$[A\ B] = \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix} \begin{array}{l} R_2 \to 2R_2 - 3R_1 \\ R_3 \to 2R_3 - R_1 \end{array}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix} R_3 \to R_3 - 7R_2$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_3 \to R_3/-4$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 5 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - 3R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 14 \\ 0 & 1 & 0 & -9 \\ 0 & 1 & 1 & 5 \end{bmatrix} \quad R_1 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_1 \to R_1/2$$

$x = 7$; $y = -9$; $z = 5$

H.W.

4. Solve the equations $x+y+z=6$; $3x+3y+4z=20$; $2x+y+3z=13$; using partial pivoting Goussion eliminotion method.

Solu) Given Equations

$$\left.\begin{aligned} x+y+z &= 6 \\ 3x+3y+4z &= 20 \\ 2x+y+3z &= 13 \end{aligned}\right\} \to (1)$$

system (1) can be expressed in the form

$AX = B$ where

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 6 \\ 20 \\ 13 \end{bmatrix}.$$

Argumented matrix

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - 3R_1 \\ R_3 \to R_2 - 2R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

which is a upper triangular matrix

$$x+y+z=6 \qquad ; \quad x+1+2=6$$
$$x=3$$
$$-y+z=1 \quad ; \quad -y+2=1$$
$$z=2 \qquad -y=-1$$
$$y=1 \;;$$

$$\therefore x=3;\; y=1;\; z=2$$

5. Solve the Equations $3x+y+2z=3$; $2x-3y-z=-3$; $x+2y+z=4$ by using Gauss Elimination method

solu) Given Equations

$$\left.\begin{array}{l} 3x+y+2z=3 \\ 2x-3y-z=-3 \\ x+2y+z=4 \end{array}\right\} \to (1)$$

system (1) can be expressed in the form $AX=B$

where $A=\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$; $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B=\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$

Argumented matrix

$$[AB] = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & -5 & -1 & -9 \end{bmatrix} \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & 0 & 8 & -8 \end{bmatrix} \quad R_3 \to 7R_3 - 5R_2.$$

which is an upper triangular matrix

$x+2(2)-1=4$; $x+2y+z=4$

$x+4-1=4$ $-7y-3z=-11$; $-7y-3(-1)=-11$

$x=1$ $8z=-8$ $-7y+3=-11$

$z=-1$ $-7y=-14$

$y=2$

$\therefore x=1$; $y=2$; $z=-1$

6. Solve the equations $10x+y+z=12$; $2x+10y+z=13$ and $x+y+5z=7$ by Gauss - Jordon Method

<u>Solu</u> Given Equations

$$\left.\begin{aligned} 10x+y+z&=12 \\ 2x+10y+z&=13 \\ x+y+5z&=7 \end{aligned}\right\} \to (1)$$

system (1) can be expressed in the form

$$AX = B$$

$$A = \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

Argumented matrix

$$[AB] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 9 & 49 & 58 \end{bmatrix} \quad \begin{aligned} R_2 &\to 5R_2 - R_1 \\ R_3 &\to 10R_3 - R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & 1 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 2365 & 2365 \end{bmatrix} \quad R_3 \to 49R_3 - 9R_2$$

$$\sim \begin{bmatrix} 20 & 1 & 1 & 1 \\ & & & \end{bmatrix} \quad R_2 \leftrightarrow$$

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$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix} \quad \begin{matrix} R_2 \longleftrightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 10R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & -8 & -44 & -51 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix} \quad R_1 \rightarrow R_1 + R_3$$

$$\sim \begin{bmatrix} -1 & +8 & +44 & +51 \\ 0 & 8 & -9 & -1 \\ 0 & 9 & 49 & 58 \end{bmatrix} \quad \begin{matrix} R_1 \rightarrow \frac{R_1}{-1} \\ R_3 \rightarrow \frac{R_3}{-1} \end{matrix}$$

392
464

$$\sim \begin{bmatrix} -1 & 8 & 44 & 51 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 473 & 473 \end{bmatrix} \quad R_3 \rightarrow 8R_3 - 9R_2$$

$$\sim \begin{bmatrix} -1 & 8 & 44 & 51 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 \rightarrow \frac{R_3}{473}$$

$$\sim \begin{bmatrix} -1 & 0 & 53 & 52 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{matrix} R_1 \rightarrow R_1 - R_2 \\ R_2 \rightarrow R_2 + 9R_3 \end{matrix}$$

$$\sim \begin{bmatrix} -1 & 0 & 53 & 52 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_2 \rightarrow \frac{R_2}{8}$$

$$\sim \begin{bmatrix} +1 & 0 & 0 & +1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_1 \rightarrow \frac{R_1 - 53R_3}{-1}$$

$\therefore x = 1 \; ; \; y = 1 \; ; \; z = 1$

7. Solve the Equations
$10x_1 + x_2 + x_3 = 12$; $x_1 + 10x_2 - x_3 = 10$ and $x_1 - 2x_2 + 10x_3 = 9$ by Gauss - Jordan method

Date
15/12/18

2. Eigen values Eigen vectors & Quadratic

Let $A = [a_{ij}]_{m\times n}$ matrix a non-zero vector x is said to be characteristic vector of A if there exist a scalar λ such that $Ax = \lambda x$. If $Ax = \lambda X$, $(X \neq 0)$ we say that X is Eigen vector or characteristic vector of A corresponding to the Eigen values or characteristic vectors or values $\lambda(A)$

Note: $A - \lambda I$ is called characteristic matrix of A. also determinant $A - \lambda I$ is a polynomial in λ are degree 'n'

* $|A - \lambda I| = 0$ is called the characteristic equation of A. This will be polynomial equation in λ of degree 'n'. Here 'A' is $n\times n$ matrix (square matrix) & I is the $n\times n$ unit matrix i.e., should be satisfied

1. Find the Eigen values and Eigen vectors of the following matrix

i) $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$

Solu) Given matrix

$$A = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5-\lambda & -2 & 0 \\ -2 & 6-\lambda & 2 \\ 0 & 2 & 7-\lambda \end{bmatrix}$$

The characteristic equation of A is

$|A - \lambda I| = 0$

$$\begin{vmatrix} 5-\lambda & -2 & 0 \\ -2 & 6-\lambda & 2 \\ 0 & 2 & 7-\lambda \end{vmatrix} = 0$$

$(5-\lambda)[(6-\lambda)(7-\lambda)-4] + 2[-2(7-\lambda)-0] + 0 = 0$

$(5-\lambda)(42 - 13\lambda + \lambda^2 - 4) - 28 + 4\lambda = 0$

$(5-\lambda)(\lambda^2 - 13\lambda + 38) - 28 + 4\lambda = 0$

$5\lambda^2 - 65\lambda + 190 - \lambda^3 + 13\lambda^2 - 38\lambda - 28 + 4\lambda = 0$

$-\lambda^3 + 18\lambda^2 - 99\lambda + 162 = 0$

$\lambda^3 - 18\lambda^2 + 99\lambda - 162 = 0$

$\lambda = 3 \Rightarrow 27 - 162 + 297 - 162 = 0.$

$$\begin{array}{c|cccc} 3 & 1 & -18 & 99 & -162 \\ & 0 & 3 & -45 & 162 \\ \hline & 1 & -15 & 54 & \boxed{0} \end{array}$$

$(\lambda^2 - 15\lambda + 54)(\lambda - 3) = 0$

$\lambda - 3 = 0$ | $\lambda^2 - 15\lambda + 54 = 0$

$\lambda = 3$ | $(\lambda-6)(\lambda-9) = 0$

$\lambda = 6, 9$

$\therefore \lambda = 6, 9, 3$, are the characteristics of A or Eigen values or roots of A

<u>case (I)</u>

If $\lambda = 3$ then $(A - \lambda I)X = 0$

$$\begin{bmatrix} 2 & -2 & 0 \\ -2 & 3 & 2 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\sim \begin{bmatrix} 2 & -2 & 0 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 / 2 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & -2 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

$2x - 2y = 0$; $\quad \rho(A) = 2$; $n = 3$

$y + 2z = 0$ $\quad$ let $z = k$

$n - r = 3 - 2 = 1$ L.I.S $\quad y + 2k = 0$

$y = -2k$

$2x - 2(-2k) = 0$

$2x = -4k$

$x = -2k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2k \\ -2k \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ -2 \\ 1 \end{bmatrix}$$

<u>case-II</u>

If $\lambda = 6$ then $(A - \lambda I)x = 0$

$$\begin{bmatrix} -1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -1 & -2 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 \cancel{+R_1} \end{matrix}$$

$$\sim \begin{bmatrix} -1 & -2 & 0 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} R_2 \to \frac{R_2}{2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -1 & -2 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 - R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n = 3$

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$n - r = 3 - 2 = 1$ L.I.S

$-x - 2y = 0$; $2y + z = 0$; $z = K$

$-x + 2\left(\frac{K}{2}\right) = 0$ $\quad$ $2y + K = 0$

$2y = -K$

$\cancel{-}x = \cancel{-}2K$ $\quad$ $y = \frac{-K}{2}$

$x = K$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} K \\ -K/2 \\ K \end{bmatrix} = K \begin{bmatrix} 1 \\ -1/2 \\ 1 \end{bmatrix}$$

Case - III

If $\lambda = 9$ then $(A - \lambda I)x = 0$

$$\begin{bmatrix} -4 & -2 & 0 \\ -2 & -3 & 2 \\ 0 & 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 0 \\ -2 & -3 & 2 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad R_1 \to \frac{R_1}{-2}, \; R_3 \to \frac{R_3}{2} \quad = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 0 \\ 0 & -2 & 2 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad R_2 \to R_2 + R_1$$

R

$$\sim \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad R_2 \to \frac{R_2}{2} \quad = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad R_3 \to R_3 + R_2 \quad = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n = 3$

$2x + y = 0$; $z = K$

$-y + z = 0$ $\quad$ $2x + K = 0$

$-y + K = 0$ $\quad$ $2x = -K$

$-y = -K$ $\quad$ $x = \frac{-K}{2}$

$y = K$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k/2 \\ k \\ k \end{bmatrix} = k\begin{bmatrix} -1/2 \\ 1 \\ 1 \end{bmatrix}$$

ii) $A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$ iii) $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

ii)

solu) Given matrix

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & -2 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 2 & 2 \\ 0 & 0 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 & -1 \\ 0 & 2-\lambda & 2 \\ 0 & 0 & -2-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & -1 \\ 0 & 2-\lambda & 2 \\ 0 & 0 & -(2+\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)\left[-(2-\lambda)(2+\lambda) - 0\right] - 2[0] - 1(0-0) = 0$$

$$(1-\lambda)\left[-(4 - \cancel{2\lambda} + \cancel{2\lambda} - \lambda^2)\right] = 0$$

$$(1-\lambda)\left[\cancel{2\lambda} - 4 + \lambda^2\right] = 0$$

$$(\lambda^2 - 4)(1-\lambda) = 0$$

$$\lambda^2 = 4 \quad 1 = \lambda$$

$$\lambda = \pm 2$$

$$\lambda = 1, 2, -2$$

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$\therefore \lambda = 1, 2, -2$ are the Eigen roots of A

case (I)

If $\lambda = 1$ then $(A-\lambda I)X = 0$

$$\begin{bmatrix} 0 & 2 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 5 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_2 \rightarrow 2R_2 - R_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 & -1 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \rightarrow 5R_3 + 3R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n = 3$

$n-r = 3-2 = 1$ L.I.S

$2y - z = 0$; $5z = 0$; $x = k$

$2y - 0 = 0$; $z = 0$

$2y = 0$

$y = 0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ 0 \\ 0 \end{bmatrix} = k\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

case (ii) If $\lambda = 2$ then $(A-\lambda I)X = 0$

$$\begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 2 \\ 0 & 0 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -1 & 2 & -1 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \rightarrow 2R_3 + 4R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n = 3$

$n-r = 3-2 = 1$ L.I.S

$$-x+2y-z=0$$
$$2z=0 \quad ; x=k$$
$$z=0$$
$$-k+2y-0=0$$
$$2y=k$$
$$y=\frac{k}{2}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ k/2 \\ 0 \end{bmatrix} = k\begin{bmatrix} 1 \\ 1/2 \\ 0 \end{bmatrix}$$

<u>Case-III</u>

If $\lambda = -2$ then $(A-\lambda I)X = 0$

$$= \begin{bmatrix} 3 & 2 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 3 & 2 & -1 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_2 \to \frac{R_2}{2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2; n=3$ $\quad 3x+2y-z=0$ $\quad z=k \; ; \; 3x+2\left(\frac{-k}{2}\right)-k=0$

$n-r = 3-2$ $\quad 2y+z=0$ $\quad 3x-2k=0$

$= 1$ $\quad 2y+k=0$ $\quad 3x=2k$

L.I.S $\quad 2y=-k$ $\quad x=\frac{2}{3}k$

$$y=-\frac{k}{2}$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{2}{3}k \\ -k/2 \\ k \end{bmatrix} = k\begin{bmatrix} 2/3 \\ -1/2 \\ 1 \end{bmatrix}$$

3. Given matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

The characteristics matrix of A is

$$A-\lambda I = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2-\lambda & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{vmatrix} = 0$$

$$-(2+\lambda)\left[(-\lambda)(1-\lambda) - 12\right] - 2(-2\lambda-6) - 3(-4+(1-\lambda)) = 0$$

$$-(2+\lambda)\left[-\lambda+\lambda^2-12\right] + 4\lambda + 12 - 3(-4+1-\lambda) = 0$$

$$-(2+\lambda)\left[-\lambda+\lambda^2-12\right] + 4\lambda + 12 - 3(-\lambda-3) = 0$$

$$-(2+\lambda)\left[\lambda^2-\lambda-12\right] + 4\lambda + 12 + 3\lambda + 9 = 0$$

$$-(2\lambda^2 - 2\lambda - 24 + \lambda^3 - \lambda^2 - 12\lambda) + 7\lambda + 21 = 0$$

$$-2\lambda^2 + 2\lambda + 24 - \lambda^3 + \lambda^2 + 12\lambda + 7\lambda + 21 = 0$$

$$-\lambda^3 - \lambda^2 + 21\lambda + 45 = 0$$

$$\lambda^3 + \lambda^2 - 21\lambda - 45 = 0$$

$$\lambda = -3 \qquad \begin{array}{r|rrrr} -3 & 1 & 1 & -21 & -45 \\ & 0 & -3 & 6 & 45 \\ \hline & 1 & -2 & -15 & 0 \end{array}$$

$$(\lambda+3)(\lambda^2 - 2\lambda - 15) = 0$$

$$(\lambda+3)(\lambda^2 - 5\lambda + 3\lambda - 15) = 0$$

$$(\lambda+3)(\lambda(\lambda-5) + 3(\lambda-5)) = 0$$

$$(\lambda+3)(\lambda+3)(\lambda-5) = 0$$

$\therefore \lambda = -3, -3, 5$ are the Eigen roots of A

Case I

If $\lambda = -3$ $\quad (A-\lambda I)X = 0$

$$\sim \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 1$; $n = 3$

$n - r = 3 - 1 = 2$ L.I.S

$x + 2y - 3z = 0$

$y = k_1$; $z = k_2$

$x + 2k_1 - 3k_2 = 0$

$x = 3k_2 - 2k_1$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3k_2 - 2k_1 \\ k_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} -2k_1 \\ k_1 \\ 0 \end{bmatrix} + \begin{bmatrix} 3k_2 \\ 0 \\ k_2 \end{bmatrix}$$

$$= k_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

case (II)

If $\lambda = 5$ then $(A - \lambda I)X = 0$

$$\sim \begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} +1 & +2 & +5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_3 \to \frac{R_3}{-1} \\ R_1 \leftrightarrow R_3 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} +1 & 2 & 5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + 7R_1 \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix} \begin{matrix} R_2 \to \frac{R_2}{-8} \\ R_3 \to \frac{R_3}{16} \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 - R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n = 3$

$n - r = 3 - 2 = 1$. L.I.S.

$x + 2y + 5z = 0$; $x + 2(-2k) + 5(k) = 0$

$y + 2z = 0$; $x - 4k + 5k = 0$

$z = k$; $x = -k$

$y + 2k = 0$; $y = -2k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k \\ -2k \\ k \end{bmatrix} = k \begin{bmatrix} -1 \\ -2 \\ 0 \end{bmatrix}$$

Date 8/12/2018

Properties of Eigen Values:

1. The sum of the Eigen values of a square matrix is equal to its trace and product of the Eigen values is equals to its determinant
2. If 'λ' is an Eigen value of A corresponding to the Eigen vector "x" then λ^n is Eigen value of A^n corresponding to the Eigen vector "x"
3. A square matrix "A" and its transpose A^T have the same Eigen values.
4. If A and B are $n \times n$ matrix and if A is invertible then $A^{-1}B$ and BA^{-1} have same Eigen values.
5. If $\lambda_1, \lambda_2, \ldots \lambda_n$ are the Eigen values of matrix A
6. If $k\lambda_1 = k\lambda_2 \ldots, k\lambda_n$ are the Eigen values of matrix kA
7. If "λ" is the Eigen value of the matrix A then

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$\lambda + K$ is an Eigen value of the matrix $A+KI$

8. If "λ" is an Eigen value of a non-singular matrix of A corresponding to the Eigen vector "x", then λ^{-1} is an Eigen value of A^{-1} and the corresponding Eigen values itself.

H.W

2. Find the characteristic roots & characteristic vectors of the following matrices.

1. $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ 2. $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

4. $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ 5. $\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$

solu) 5. Given matrix.

$$A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & -6-\lambda & -4 \\ 0 & 4-\lambda & 2 \\ 0 & -6 & -3-\lambda \end{bmatrix}$$

The characteristic Equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -6 & -4 \\ 0 & 4-\lambda & 2 \\ 0 & -6 & -(3+\lambda) \end{vmatrix} = 0$$

$(1-\lambda)\ [(4-\lambda)(3+\lambda)+12]+6(0)-4(0)=0$

$(1-\lambda)\ [-(12-3\lambda+4\lambda-\lambda^2)+12]=0$

$(1-\lambda)\ [\lambda^2-\cancel{\lambda}-\cancel{12}+\cancel{12}]=0$

$(\lambda^2-\lambda)(1-\lambda)=0$

$\lambda^2-\lambda-\lambda^3+\lambda^2=0$

$\lambda^3-2\lambda^2+\lambda=0$

$$\begin{array}{c|cccc} 1 & 1 & -2 & 1 & 0 \\ & 0 & 1 & -1 & \\ \hline & 1 & -1 & \boxed{0} & \end{array}$$

$(\lambda^2-\lambda)(\lambda-1)=0$

$\lambda=1\ ;\ \lambda^{\cancel{2}}=\cancel{\lambda}$

$\lambda=1$

$\therefore\ \lambda=1,1,0$ are the Eigen values of A

CO3(P)

If $\lambda=1$ then $(A-\lambda I)X=0$

$$\begin{bmatrix} 0 & -6 & -4 \\ 0 & 3 & -2 \\ 0 & -6 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -6 & -4 \\ 0 & 3 & 2 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3-R_1 \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{\rho(A)=2}\ ;\ n=3$

$n-r=3-2$ L.I.S

$-6y-4z=0$ $\quad z=k\ ;\ -6(-\frac{2}{3}k)-4k$

$3y+2z=0;$

$3y+2k=0$

$3y=-2k;$

$y=-\frac{2}{3}k$

$$\begin{bmatrix} 0 & -6 & -4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} R_2 \to 2R_2+R_1 \begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A)=1$; $n=3$

$n-r = 3-1 = 2$, L.I.S

$-6y-4z=0$; $x=k_1$, $z=k_2$

$-6y-4k_2=0$

$-6y = 4k_2$

$y = -\frac{2}{3}k_2$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k_1 \\ -\frac{2}{3}k_2 \\ k_2 \end{bmatrix} = k_1\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + k_2\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$$

case(iii)

If $\lambda=0$ then $(A-\lambda I)X=0$

$$\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_3 \to 4R_3 + 6R_2$$

$\rho(A)=2$; $n=3$

$n-r = 3-2 = 1$. L.I.S

$x-6y-4z=0$; $z=k$

$4y+2z=0$

$4y+2k=0$; $x-6\left(-\frac{1}{2}k\right)-4k=0$

$4y=-2k$; $x+3k-4k=0$

$y=-\frac{1}{2}k$; $x=k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ -1/2\,k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1/2 \\ 1 \end{bmatrix}$$

4. Given matrix

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -9 & 3 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\left[(5-\lambda)(3-\lambda)-1\right] + 1(-3+\lambda+1) + 1(1-5+\lambda) = 0$$

$$(3-\lambda)\left[15 - 3\lambda - 5\lambda + \lambda^2 - 1\right] - 3 + \lambda \cancel{+1} \cancel{+1} - 5 + \lambda = 0$$

$$45 - 9\lambda - 15\lambda + 3\lambda^2 - 3 - 15\lambda + 3\lambda^2 + 5\lambda^2 - \lambda^3 + \lambda + 2$$

$$-3 + \lambda - 5 + \lambda = 0$$

$$-\lambda^3 + 6\lambda^2 + 5\lambda^2 - 36\lambda + 36 = 0$$

$$\lambda^3 - 11\lambda^2 + 36\lambda - 36 = 0$$

$\lambda = 3$

3	1	-11	36	-36
	0	3	-24	36
	1	-8	+12	0

$$(\lambda-3)(\lambda^2 - 8\lambda + 12) = 0$$

$$(\lambda-3)(\lambda-2)(\lambda-6) = 0$$

$$\lambda = 2, 3, 6$$

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Case (i)

If $\lambda = 2$ then $(A-\lambda I)x = 0$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 + R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n = 3$

$n - r = 3 - 2 = 1$; L.I.S

$x - y + z = 0$; $z = k$

$2y = 0$ $x - 0 + k = 0$

$y = 0$ $x = -k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k \\ 0 \\ k \end{bmatrix} = k\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Case (ii)

If $\lambda = 3$ then $(A-\lambda I)x = 0$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 1 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 + 2R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 + R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_2 \to R_2 - R_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-y+z=0$ $\quad \therefore \rho(A)=2$; $n=3$

$n-r=3-2=1$ L.I.S

$-x+y=0$; $z=k$

$-y+k=0$; $-x+k=0$

$-y=-k$ $\quad -x=-k$

$y=k$; $x=k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

case (iii)

If $\lambda=6$ then $(A-\lambda I)X=0$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}\begin{bmatrix} -3 & -1 & 1 \\ 0 & -2 & -4 \\ 0 & -4 & -2 \end{bmatrix} \begin{array}{l} R_2 \to 3R_2 - R_1 \\ R_3 \to 3R_3 + R_1 \end{array} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 & 1 \\ 0 & -2 & -4 \\ 0 & 0 & 14 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to 2R_3 - 4R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A)=3$; $n=3$

$-3x-y+z=0$

$-2y-4z=0$; $-2y-4(0)=0$; $-3x-0+0=0$

$14z=0$ $\quad -2y=0$ $\quad -3x=0$

$z=0$ $\quad y=0$ $\quad x=0$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

1. Given matrix

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$A-\lambda I = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The characteristic matrix of A is

$$A-\lambda I = \begin{bmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 6-\lambda & -2 & 2 \\ -2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)[(3-\lambda)(3-\lambda)-1] + 2[(-2)(3-\lambda)+2] + 2[2-6+2\lambda] = 0$$

$$(6-\lambda)[9-3\lambda-3\lambda+\lambda^2-1] + 2[-6+2\lambda+2] + 2[-4+2\lambda] = 0$$

$$54-18\lambda-18\lambda+6\lambda^2-6-9\lambda+3\lambda^2+3\lambda^2-\lambda^3+\lambda-12+4\lambda+4$$
$$-8+4\lambda = 0$$

$$-\lambda^3+12\lambda^2-36\lambda+32=0$$

$$\lambda^3-12\lambda^2+36\lambda-32=0$$

$\lambda = 2$

$$\begin{array}{r|rrrr} 2 & 1 & -12 & 36 & -32 \\ & 0 & 2 & -20 & 32 \\ \hline & 1 & -10 & 16 & \boxed{0} \end{array}$$

$$(\lambda^2-10\lambda+16)(\lambda-2)=0$$

$$(\lambda^2-2\lambda-8\lambda+16)(\lambda-2)=0$$

$$(\lambda-2)[(\lambda-2)\lambda-8(\lambda-2)]=0$$

$$(\lambda-2)(\lambda-8)(\lambda-2)=0$$

$$\lambda = 2, 2, 8$$

$\therefore \lambda = 2, 2, 8$ are the Eigen values

Case (i)

If $\lambda = 2$ then $(A-\lambda I)x = 0$

$$\begin{bmatrix} 4 & -2 & 2 \\ -2 & 1 & -1 \\ 2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to 2R_2 + R_1 \\ R_3 \to 2R_3 - R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 1$; $n = 3$

$n - r = 3 - 1 = 2$; L.I.S

$4x - 2y + 2z = 0$

$y = k_1$; $z = k_2$

$4x - 2k_1 + 2k_2 = 0$

$4x = 2k_1 - 2k_2$

$x = \frac{k_1}{2} - \frac{k_2}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k_1/2 - k_2/2 \\ k_1 \\ k_2 \end{bmatrix} = k_1\begin{bmatrix} 1/2 \\ 1 \\ 0 \end{bmatrix} + k_2\begin{bmatrix} -1/2 \\ 0 \\ 1 \end{bmatrix}$$

case (ii)

If $\lambda = 8$ then $(A - \lambda I)X = 0$

$$\begin{bmatrix} -2 & -2 & 2 \\ -2 & -5 & -1 \\ 2 & -1 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -2 & 2 \\ 0 & -3 & -3 \\ 0 & -3 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 + R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 1 \\ 0 & -3 & -3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_1 \to \frac{R_1}{2} \\ R_3 \to R_3 - R_2 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x - y + z = 0 \qquad -x-(-k)+k=0$$
$$-3y - 3z = 0 \quad ; z = k \quad ; \qquad -x+2k=0$$
$$-3y - 3k = 0 \qquad -x = -2k$$
$$-3y = 3k \qquad x = k$$
$$y = -k$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ -k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

2. Given matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

The choracteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{bmatrix}$$

The choracteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$(8-\lambda)\left[(7-\lambda)(3-\lambda)-16\right] + 6\left(-6(3-\lambda)+8\right) + 2\left(24 - 2(7-\lambda)\right) = 0$$

$$(8-\lambda)\left[21 - 3\lambda - 7\lambda + \lambda^2 - 16\right] + 6\left[-18 + 6\lambda + 8\right] + 2\left[24 - 14 + 2\lambda\right] = 0$$

$$(8-\lambda)\left[\lambda^2 - 10\lambda + 5\right] + 6\left[6\lambda - 10\right] + 2\left[2\lambda + 10\right] = 0$$

$$8\lambda^2 - 80\lambda + 40 - \lambda^3 + 10\lambda^2 - 5\lambda + 36\lambda - 60 + 4\lambda + 20 = 0$$

$$-\lambda^3 + 18\lambda^2 - 45\lambda = 0 = 0.$$

$\lambda^3 - 18\lambda^2 + 458\lambda + 40 = 0$

$\lambda(\lambda^2 - 18\lambda + 45) = 0$

$\lambda = 0 \; ; \; (\lambda^2 - 15\lambda - 3\lambda + 45) = 0$

$[\lambda(\lambda - 15) - 3(\lambda - 15)]\lambda = 0$

$\lambda(\lambda - 3)(\lambda - 15) = 0$

$\lambda = 0, 3, 15$

$\therefore \lambda = 0, 3, 15$ are the Eigen values

case (i)

If $\lambda = 0$ then $(A - \lambda I)X = 0$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 8 & -6 & 2 \\ 0 & 20 & -20 \\ 0 & -20 & 20 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to 8R_2 + 6R_1 \\ R_3 \to 8R_3 - 2R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 8 & -6 & 2 \\ 0 & 20 & -20 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 + R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2 \; ; \; n = 3$

$n - r = 3 - 2 = 1 \; ; \; L.I.S$

$8x - 6y + 2z = 0 \qquad ; \; z = k$

$20y - 20z = 0$

$20y = 20z$

$y = z \; ; \; y = k$

$8x - 6k + 2k = 0$

$8x - 4k = 0 \; ; \; 8x = 4k \; ; \; x = \frac{k}{2}$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k/2 \\ k \\ k \end{bmatrix} = k \begin{bmatrix} 1/2 \\ 1 \\ 1 \end{bmatrix}$$

case(ii)

If $\lambda = 3$ then $(A-\lambda I)X = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 5 & -6 & 2 \\ 0 & -16 & -8 \\ 0 & -8 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to 5R_2 + 6R_1 \\ R_3 \to 5R_3 - 2R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 5 & -6 & 2 \\ 0 & 2 & 1 \\ 0 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to \frac{R_2}{-8} \\ R_3 \to \frac{R_3}{-4} \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 5 & -6 & 2 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 - R_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n = 3$

$n - r = 3 - 2 = 1$; L.I.S

$5x - 6y + 2z = 0$; $z = k$

$2y + z = 0$; $2y + k = 0$

$2y = -k$

$y = \frac{-k}{2}$

$5x - 6\left(\frac{-k}{2}\right) + 2k = 0$

$5x + 3k + 2k = 0$

$5x = -5k$

$x = -k$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k \\ -k/2 \\ k \end{bmatrix} = k\begin{bmatrix} -1 \\ -1/2 \\ 1 \end{bmatrix}$$

case(iii)

If $\lambda = 15$ then $(A-\lambda I)X = 0$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -7 & -6 & 2 \\ 3 & 4 & 2 \\ -1 & 2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{matrix} R_2 \to \frac{R_2}{-2} \\ R_3 \to \frac{R_3}{-2} \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -7 & -6 & 2 \\ 0 & 10 & 20 \\ 0 & 20 & 30 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{matrix} R_2 \to 7R_2 + 3R_1 \\ R_3 \to 7R_3 - R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -7 & -6 & 2 \\ 0 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{matrix} R_2 \to R_2/10 \\ R_3 \to R_3/10 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} -7 & -6 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad R_3 \to R_3 - 2R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 3$; $n = 3$

$-7x - 6y + 2z = 0$; $y + 2z = 0$; $-z = 0$

$x = 0$ $\quad$ $y = 0$ $\quad$ $z = 0$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

3. Given matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{bmatrix}$$

The characteristic equation of A is.

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$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 1 \\ 1 & 1-\lambda & 1 \\ 1 & 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)(1-\lambda)-1\right] - 1\left[\cancel{1}-\lambda\cancel{-1}\right] + 1\left[\cancel{1}\cancel{-1}+\lambda\right] = 0$$

$$(1-\lambda)\left[\cancel{1}-\lambda-\lambda+\lambda^2\cancel{-1}\right] - 1[-\lambda] + \lambda = 0$$

$$(1-\lambda)\left[-2\lambda+\lambda^2\right] + \lambda + \lambda = 0$$

$$\cancel{-2\lambda} + \lambda^2 + 2\lambda^2 - \lambda^3 \cancel{+\lambda+\lambda} = 0$$

$$-\lambda^3 + 3\lambda^2 = 0$$

$$-\lambda(\lambda^2 \pm 3\lambda) = 0$$

$$\lambda = 0 \quad \lambda^2 = +3\lambda$$

$$\lambda = +3$$

$$\lambda = 0, 0, 3$$

Case (i)

If $\lambda = 0$ then $(A-\lambda I)X = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 1$; $n = 3$

$n - r = 3 - 1 = 2$; L.I.S

$x + y + z = 0$; $y = k_1$; $z = k_2$

$x + k_1 + k_2 = 0$

$x = -(k_1 + k_2)$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -(k_1+k_2) \\ k_1 \\ k_2 \end{bmatrix} = k_1 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + k_2 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

Cayley - Hamilton theorem and quadratic forms:-

Unit - 2

CALEY - HAMILTON THEOREM

Every square matrix satisfies its characteristic equation

1. If $A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$ verify Caley Hamilton theorem and hence find A^{-1}

Sol) Given matrix.

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$$

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The characteristic equation matrix of A

$$A-\lambda I = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 2-\lambda & 1 & 2 \\ 5 & 3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{bmatrix}$$

the characteristic equation of A is $|A-\lambda I|=0$

$$\begin{bmatrix} 2-\lambda & 1 & 2 \\ 5 & 3-\lambda & 3 \\ -1 & 0 & -2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)[(3-\lambda)(-2-\lambda)-0] - 1[5(-2-\lambda)+3] + 2[0+1(3-\lambda)] = 0$$

$$(2-\lambda)[-6-3\lambda+2\lambda+\lambda^2) - (-10-5\lambda+3) + 6-2\lambda = 0$$

$$(2-\lambda)(-6-3\lambda+2\lambda+\lambda^2) - (-10-5\lambda+13) + 6-2\lambda = 0$$

$$(2-\lambda)(\lambda^2-\lambda-6) + 5\lambda + 7 + 6 - 2\lambda = 0$$

$$2\lambda^2 - 2\lambda - 12 - \lambda^3 + \lambda^2 + 6\lambda + 3\lambda + 13 = 0$$

$$-\lambda^3 + 3\lambda^2 + 7\lambda + 1 = 0$$

$$\lambda^3 - 3\lambda^2 - 7\lambda - 1 = 0$$

By Caley Hamilton theorem

$$A^3 - 3A^2 - 7A - I = 0$$

$$A^2 = A \cdot A = \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+5-2 & 2+3+0 & 4+3-4 \\ 10+15-3 & 5+9+0 & 10+9-6 \\ -2+0+2 & -1+0-0 & -2+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & 5 & 3 \\ 22 & 14 & 13 \\ 0 & -1 & 2 \end{bmatrix}$$

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$$A^3 = A^2 A = \begin{bmatrix} 7 & 5 & 3 \\ 22 & 14 & 13 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ 5 & 3 & 3 \\ -1 & 0 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 14+25-3 & 7+15+0 & 14+15-6 \\ 44+70-13 & 22+42+0 & 44+42-26 \\ 0-5-2 & 0-3+0 & 0-3-4 \end{bmatrix}$$

122
12552
24
50208
25104
301248

$$= \begin{bmatrix} 36 & 22 & 23 \\ 101 & 64 & 60 \\ -7 & -3 & -7 \end{bmatrix}$$

$$A^3 - 3A^2 - 7A - I = \begin{bmatrix} 36 & 22 & 23 \\ 101 & 64 & 60 \\ -7 & -3 & -17 \end{bmatrix} - \begin{bmatrix} 21 & 15 & 9 \\ 66 & 42 & 39 \\ 0 & -3 & 6 \end{bmatrix} - \begin{bmatrix} 14 & 7 & 14 \\ 35 & 21 & 21 \\ -7 & 0 & -14 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore$ Caley Hamilton theorem is satisfied

Now

$$A^3 - 3A^2 - 7A - I = 0$$

$$I = A^3 - 3A^2 - 7A$$

Multiplying with A^{-1} o.b.s

$$A^{-1} = A^2 - 3A - 7I$$

$$A^{-1} = \begin{bmatrix} 7 & 5 & 3 \\ 22 & 14 & 13 \\ 0 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 6 & 3 & 6 \\ 15 & 9 & 9 \\ -3 & 0 & -6 \end{bmatrix} - \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 2 & -3 \\ 7 & -2 & 4 \\ 3 & -1 & 1 \end{bmatrix}$$

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2. Find the inverse of the following matrices by using C-H-T. and also verify C-H-T

i) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ ii) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$ iii) $\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}$

iv) $\begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$ v) $\begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ vi) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$

Solu) i) Given matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

characteristic matrix of A is

$$A-\lambda I = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & -1 & 0 \\ 0 & 1-\lambda & 1 \\ 2 & 1 & 2-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -1 & 0 \\ 0 & 1-\lambda & 1 \\ 2 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[(1-\lambda)(2-\lambda)-1] + 1[0-2] + 0 = 0$$

$$(1-\lambda)[2-2\lambda-\lambda+\lambda^2-1] - 2 = 0$$

$$(1-\lambda)[\lambda^2 3\lambda+1] - 2 = 0$$

$$\lambda^2 3\lambda+1 - \lambda^3 + 3\lambda^2 - \lambda - 2 = 0$$

$$-\lambda^3 + 4\lambda^2 - 4\lambda - 1 = 0$$

$$\lambda^3 - 4\lambda^2 + 4\lambda + 1 = 0$$

By caley hamilton theorem

$$A^3 - 4A^2 + 4A + I = 0$$

$$A^2 = A \cdot A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 2 & 3 \\ 6 & 1 & 5 \end{bmatrix}$$

wrong

$$= \begin{bmatrix} 1+0+0 & 0+0+2 & 2+0+4 \\ -1-1+0 & 0+1+1 & -2+1+2 \\ 0-1+0 & 0+1+2 & 0+1+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 6 \\ -2 & 2 & 1 \\ -1 & 3 & 5 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 1 & -2 & -1 \\ 2 & 2 & 3 \\ 6 & 1 & 5 \end{bmatrix}\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-0-2 & -1-2-1 & 0-2-2 \\ 2+0+6 & -2+2+3 & 0+2+6 \\ 6+0+10 & -6+1+5 & 0+1+10 \end{bmatrix} = \begin{bmatrix} -1 & -4 & -4 \\ 8 & 3 & 8 \\ 16 & 0 & 11 \end{bmatrix}$$

$$A^3 - 4A^2 + 4A + I = \begin{bmatrix} -1 & -4 & -4 \\ 8 & 3 & 8 \\ 16 & 0 & 11 \end{bmatrix} - \begin{bmatrix} \cancel{-4} & \cancel{-16} & \cancel{-16} \\ \cancel{32} & \cancel{12} & \cancel{32} \\ \cancel{64} & \cancel{0} & 44 \end{bmatrix}\begin{bmatrix} 4 & -8 & -4 \\ +8 & 8 & 12 \\ 24 & 4 & 20 \end{bmatrix}$$

$$+ \begin{bmatrix} 4 & -4 & 0 \\ 0 & 4 & 4 \\ 8 & 4 & 8 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ caley hamilton theorem is satisfied

$$I = -A^3 + 4A^2 - 4A$$

Multiplying with A^{-1}

$$A^{-1} = -A^2 + 4A - 4I$$

$$A^{-1} = -\begin{bmatrix} 1 & -2 & -1 \\ 2 & 2 & 3 \\ 6 & 1 & 5 \end{bmatrix} + \begin{bmatrix} 4 & -4 & 0 \\ 0 & 4 & 4 \\ 8 & 4 & 8 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1+4-4 & 2-4+0 & 1+0+0 \\ -2+0-0 & -2+4-4 & 3+4+0 \\ -6+8-0 & -1+4+0 & -5+8-4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -2 & 1 \\ -2 & -2 & 7 \\ 2 & 3 & -1 \end{bmatrix}$$

iv) Given matrix

$$A = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$$

characteristic equation of A is matrix

$$A - \lambda I = \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 7-\lambda & 2 & -2 \\ -6 & -1-\lambda & 2 \\ 6 & 2 & -1-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 7-\lambda & 2 & -2 \\ -6 & -1-\lambda & 2 \\ 6 & 2 & -1-\lambda \end{vmatrix} = 0$$

$$(7-\lambda)\left[(-1-\lambda)(-1-\lambda) - 4\right] - 2(-6(-1-\lambda) - 12) - 2(-12 - 6(-1-\lambda))$$

$$(7-\lambda)\left[+1+\lambda+\lambda+\lambda^2-4\right]-2(6+6\lambda-12)-2(-12+6+6\lambda)=0$$

$$(7-\lambda)\left[\lambda^2+2\lambda-3\right]-2(6\lambda-6)-2(6\lambda-6)=0$$

$$7\lambda^2+14\lambda-21-\lambda^3-2\lambda^2+3\lambda-12\lambda+12-12\lambda+12=0$$

$$-\lambda^3+5\lambda^2-7\lambda+3=0$$

$$\lambda^3-5\lambda^2+7\lambda-3=0$$

By calcy Hamilton theorem

$$A^3-5A^2+7A-3I=0$$

$$A^2=\begin{bmatrix}7&2&-2\\-6&-1&2\\6&2&-1\end{bmatrix}\begin{bmatrix}7&2&-2\\-6&-1&2\\6&2&-1\end{bmatrix}$$

$$=\begin{bmatrix}49-12-12&14-2-4&-14+4+2\\-42+6+12&-12+1+4&12-2-2\\42-12-6&12-2-2&-12+4+1\end{bmatrix}$$

$$=\begin{bmatrix}25&8&-8\\-24&-7&8\\24&8&-7\end{bmatrix}$$

$$A^3=A^2\cdot A=\begin{bmatrix}25&8&-8\\-24&-7&8\\24&8&-7\end{bmatrix}\begin{bmatrix}7&2&-2\\-6&-1&2\\6&2&-1\end{bmatrix}$$

$$=\begin{bmatrix}175-48-48&50-8-16&-50+16+8\\-168+42+48&-48+7+16&+48-14-8\\168-48-42&48-8-14&-48+16+7\end{bmatrix}$$

$$=\begin{bmatrix}79&26&-26\\-78&-25&26\\78&26&-25\end{bmatrix}$$

$$7A=7\begin{bmatrix}7&2&-2\\-6&-1&2\\6&2&-1\end{bmatrix}=\begin{bmatrix}49&14&-14\\-42&-7&14\\42&14&-7\end{bmatrix}$$

[illegible]

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$= \begin{bmatrix} 79 & 26 & -26 \\ -78 & -25 & 26 \\ 78 & 26 & -25 \end{bmatrix} - \begin{bmatrix} 125 & 40 & -40 \\ -120 & -135 & 40 \\ 120 & 40 & -35 \end{bmatrix} + \begin{bmatrix} 49 & 14 & -14 \\ -42 & -7 & 14 \\ 42 & 14 & -7 \end{bmatrix}$$

$$- \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 0$$

$$= \begin{bmatrix} 79-125+49-3 & 26-40+14-0 & -26+40-14-0 \\ -78+120-42-0 & -25+35-7-3 & 26-40+14+0 \\ 78-120+42+0 & 26-40+14-0 & -25+35-7+3 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0$$

$\therefore$ Caley Hamilton theorem is ~~not~~ verified

$$A^3 - 5A^2 + 7A - 3I = 0$$

$$A^3 - 5A^2 + 7A = 3I \rightarrow (1)$$

Multiplying (1) with A^{-1}

$$A^2 - 5A + 7I = 3A^{-1}$$

$$3A^{-1} = \begin{bmatrix} 25 & 8 & -8 \\ -24 & -7 & 8 \\ 24 & 8 & -7 \end{bmatrix} - \begin{bmatrix} 35 & 10 & -10 \\ -30 & -25 & 10 \\ 30 & 10 & -5 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$3A^{-1} = \begin{bmatrix} 25-35+7 & 8-10+0 & -8+10+0 \\ -24+30+0 & -7+5+7 & 8-10+0 \\ 24-30+0 & 8-10+0 & -7+5+7 \end{bmatrix}$$

$$A^{-1} = \frac{1}{3}\begin{bmatrix} -3 & -2 & 2 \\ 6 & 5 & -2 \\ -6 & -2 & 5 \end{bmatrix}$$

(iii) Given matrix

$$A = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 5-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 5-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\left[(5-\lambda)(5-\lambda)-1\right] - 1\left[-1(5-\lambda)+1\right] + 1\left[1-(5-\lambda)\right] = 0$$

$$(3-\lambda)\left[25-5\lambda-5\lambda+\lambda^2-1\right] - 1\left[-5+\lambda+1\right] + \left[1-5+\lambda\right] = 0$$

$$(3-\lambda)\left[24-10\lambda+\lambda^2\right] - 1[\lambda-4) + (\lambda-4) = 0$$

$$(\lambda^2-10\lambda+24)(3-\lambda) - \cancel{(\lambda-4)} + \cancel{(\lambda-4)} = 0$$

$$3\lambda^2 - 30\lambda + 72 - \lambda^3 + 10\lambda^2 - 24\lambda = 0$$

$$-\lambda^3 + 13\lambda^2 - 54\lambda + 72 = 0$$

$$\lambda^3 - 13\lambda^2 + 54\lambda - 72 = 0$$

By Caley Hamilton theorem

$$A^3 - 13A^2 + 54A - 72I = 0$$

$$A^2 = \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}\begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix} = \begin{bmatrix} 9-1+1 & 3+5-1 & 3-1+5 \\ -3-5-1 & -1+25+1 & -1-5-5 \\ 3+1+5 & 1-5-5 & 1+1+25 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & 7 & 7 \\ -9 & 25 & -11 \\ 9 & -9 & 27 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 9 & 7 & 7 \\ -9 & 25 & -11 \\ 9 & -9 & 27 \end{bmatrix} \begin{bmatrix} 3 & 1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 27-7+7 & 9+35-7 & 9-7+35 \\ -27-25-11 & -9+125+11 & -9-25-55 \\ 27+9+27 & 9-45-27 & 9+9+135 \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 37 & 37 \\ -63 & 127 & -89 \\ 63 & -63 & 153 \end{bmatrix}$$

$A^3 - 13A^2 + 54A - 72I$

$$= \begin{bmatrix} 27 & 37 & 37 \\ -63 & 127 & -89 \\ 63 & -63 & 153 \end{bmatrix} - \begin{bmatrix} 117 & 91 & 91 \\ -117 & 325 & -143 \\ 117 & -117 & 351 \end{bmatrix} + \begin{bmatrix} 162 & 54 & 54 \\ -54 & 270 & -54 \\ 54 & -54 & 270 \end{bmatrix} - \begin{bmatrix} 72 & 0 & 0 \\ 0 & 72 & 0 \\ 0 & 0 & 72 \end{bmatrix}$$

$$= \begin{bmatrix} 27-117+162-72 & 37-91+54-0 & 37-91+54+0 \\ -63+117-54+0 & 127-325+270-72 & -89-325-54+0 \\ 63-117+54+0 & -63+117-54+0 & 153-351+270-72 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Coley Hamilton theorem is verified

$A^3 - 13A^2 + 54A - 72I = 0$

$A^3 - 13A^2 + 54A = 72I$

$A^2 - 13A + 54I = 72A^{-1}$

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$$72A^{-1} = \begin{bmatrix} 9 & 7 & 7 \\ -9 & 25 & -11 \\ 9 & -9 & 27 \end{bmatrix} - 13\begin{bmatrix} 39 & 13 & 13 \\ -13 & 65 & -13 \\ 13 & -13 & 65 \end{bmatrix} + \begin{bmatrix} 54 & 0 & 0 \\ 0 & 54 & 0 \\ 0 & 0 & 54 \end{bmatrix}$$

$$72A^{-1} = \begin{bmatrix} 9-39+54 & 7-13+0 & 7-13+0 \\ -9+13+0 & 25-65+54 & -11+13+0 \\ 9-13+0 & -9+13+0 & 27-65+54 \end{bmatrix}$$

$$A^{-1} = \frac{1}{72}\begin{bmatrix} 24 & -6 & -6 \\ 4 & 14 & 2 \\ -4 & 4 & 16 \end{bmatrix}$$

ii) Given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$$

The characteristic matrix of A is

$$A-\lambda I = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 & 3 \\ 2 & -1-\lambda & 4 \\ 3 & 1 & -1-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A-\lambda I| = 0 \Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & -1-\lambda & 4 \\ 3 & 1 & -1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[(-1-\lambda)(-1-\lambda) - 4] - 2[2(-1-\lambda) - 12] + 3(2-3(-1-\lambda)) = 0$$

$$(1-\lambda)[(-1-\lambda)(-1-\lambda)-4] - 2[-2-2\lambda-12] + 3(2+3+3\lambda) = 0$$

$$(1-\lambda)[1+\lambda+\lambda+\lambda^2-4] - 2[-2-2\lambda-12]$$

$$(1-\lambda)[\lambda^2+2\lambda-3] - 2(-2\lambda-14) + 3(3\lambda+5) = 0$$

$$\lambda^2+2\lambda-3-\lambda^3-2\lambda^2+3\lambda+4\lambda+28+9\lambda+15 = 0$$

$$-\lambda^3-\lambda^2+18\lambda+40 = 0$$

By caley Hamilton theorem

$$A^3+A^2-18A-40I=0$$

$$A^2=\begin{bmatrix}1&2&3\\2&-1&4\\3&1&-1\end{bmatrix}\begin{bmatrix}1&2&3\\2&-1&4\\3&1&-1\end{bmatrix}$$

$$=\begin{bmatrix}1+4+9&2-2+3&-3+8-3\\2-2+12&4+1+4&6-4-4\\3+2-3&6-1-1&9-4+1\end{bmatrix}=\begin{bmatrix}14&3&8\\12&9&-2\\2&4&14\end{bmatrix}$$

$$A^3=A^2A=\begin{bmatrix}14&3&8\\12&9&-2\\2&4&14\end{bmatrix}\begin{bmatrix}1&2&3\\2&-1&4\\3&1&-1\end{bmatrix}$$

$$=\begin{bmatrix}14+6+24&28-3+8&42-12-8\\12+18-6&24-9-2&36+36+2\\2+8+42&4-4+14&6+16-14\end{bmatrix}=\begin{bmatrix}44&33&22\\24&13&74\\52&14&8\end{bmatrix}$$

$$A^3+A^2-18A-40I$$

$$=\begin{bmatrix}44&33&22\\24&13&74\\52&14&8\end{bmatrix}+\begin{bmatrix}14&3&8\\12&9&-2\\2&4&14\end{bmatrix}-\begin{bmatrix}18&36&54\\36&-18&72\\54&18&-18\end{bmatrix}-\begin{bmatrix}40&0&0\\0&40&0\\0&0&40\end{bmatrix}$$

$$=\begin{bmatrix}44+14-18+40&33+3-36-0&22+8-54+0\\24+12-36-0&13+9+18-40&74-2-72-0\\52+2-54-0&14+4-18+0&8+14+18-40\end{bmatrix}$$

$$=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$\therefore$ since Caley Hamilton theorem is verified

$$A^3+A^2-18A-40I=0$$

$$A^3+A^2-18A=40I$$

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$$A^2 + A - 18I = 40A^{-1}$$

$$40A^{-1} = \begin{bmatrix} 14 & 3 & 8 \\ 12 & 9 & -2 \\ 2 & 4 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix} - \begin{bmatrix} 18 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 18 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 14+1-18 & 3+2-0 & 8+3+0 \\ 12+2-0 & 9-1-18 & -2+4+0 \\ 2+3+0 & 4+1+0 & 14-1-18 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} -3 & 5 & 11 \\ 14 & -10 & 2 \\ 5 & 5 & -5 \end{bmatrix}$$

v) Given matrix

$$A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 8-\lambda & -8 & 2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0 \Rightarrow \begin{vmatrix} 8-\lambda & -8 & 2 \\ 4 & -3-\lambda & -2 \\ 3 & -4 & 1-\lambda \end{vmatrix} = 0$$

$$(8-\lambda)[(-3-\lambda)(1-\lambda)-8] + 8[4(1-\lambda)+6] + 2[-16-3(-3-\lambda)] = 0$$

$$(8-\lambda)[-3-\lambda+3\lambda+\lambda^2-8] + 8[4-4\lambda+6] + 2(-16+9+3\lambda) = 0$$

$$(8-\lambda)[\lambda^2+2\lambda-11] + 8(10-4\lambda) + 2(3\lambda-7) = 0$$

$$8\lambda^2+16\lambda-88-\lambda^3-2\lambda^2+11\lambda+80-32\lambda+6\lambda-14=0$$

$$-\lambda^3+6\lambda^2+\lambda-22=0$$

$$\lambda^3-6\lambda^2-\lambda+22=0$$

By caley Hamilton theorem

$$A^3 - 6A^2 - A + 22I = 0$$

$$A^2 = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 64-32+6 & -64+24-8 & 16+16+2 \\ 32-12-6 & -32+9+8 & 8+6-2 \\ 24-16+3 & -24+12-4 & 6+8+1 \end{bmatrix}$$

$$= \begin{bmatrix} 38 & -48 & 34 \\ 14 & -15 & 12 \\ 11 & -16 & 15 \end{bmatrix}$$

$$A^3 = A^2 A = \begin{bmatrix} 38 & -48 & 34 \\ 14 & -15 & 12 \\ 11 & -16 & 15 \end{bmatrix} \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 304-192+102 & -304+144-136 & 76+96+34 \\ 112-60+36 & 112+45-48 & 28+30+12 \\ 88-64+45 & -88+48-60 & 22+32+15 \end{bmatrix}$$

$$= \begin{bmatrix} 214 & -296 & 206 \\ 88 & 109 & 70 \\ 69 & -100 & 69 \end{bmatrix}$$

$$A^3 - 6A^2 - A + 22I = 0$$

$$= \begin{bmatrix} 214 & -296 & 206 \\ 88 & 109 & 70 \\ 69 & -100 & 69 \end{bmatrix} - \begin{bmatrix} 228 & -288 & 204 \\ 84 & -90 & 72 \\ 66 & -96 & 90 \end{bmatrix} - \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix} + \begin{bmatrix} 22 & 0 & 0 \\ 0 & 22 & 0 \\ 0 & 0 & 22 \end{bmatrix}$$

$$= \begin{bmatrix} 214-228-8+22 & -296+288+8+0 & 206-204-2+0 \\ 88-84-4+0 & 109+90+3+22 & 70-72+2+0 \\ 69-66-3+0 & -100+76+4+6 & 69-90-1+22 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

. cayley Hamilton theorem is verified.

$$A^3 - 6A^2 - A + 22I = 0$$

$$22I = -A^3 + 6A^2 + A$$

Multiplying with A^{-1}

$$22A^{-1} = -A^2 + 6A + I$$

$$22A^{-1} = \begin{bmatrix} -38 & 48 & -34 \\ -14 & 15 & -12 \\ -11 & 16 & -15 \end{bmatrix} + \begin{bmatrix} 48 & -48 & 12 \\ 24 & -18 & -12 \\ 18 & -24 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$22A^{-1} = \begin{bmatrix} -38+48+1 & 48-48+0 & -34+12+0 \\ -14+24+0 & 15-18+1 & -12-12+0 \\ -11+18+0 & 16-24+0 & -15+6+1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{22} \begin{bmatrix} 11 & 0 & -22 \\ 10 & -2 & -24 \\ 7 & -8 & -8 \end{bmatrix}$$

vi) Given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} = \begin{bmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 5 \\ 3 & 5 & 6-\lambda \end{bmatrix}$$

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The characteristic equation of A is

$$|A-\lambda I| = 0 \rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & 4-\lambda & 5 \\ 3 & 5 & 6-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[(4-\lambda)(6-\lambda)-25] - 2[2(6-\lambda)-15] + 3[10-3(4-\lambda)] = 0$$

$$(1-\lambda)[24-6\lambda-4\lambda+\lambda^2-25] - 2[12-2\lambda-15] + 3[10-12+3\lambda] = 0$$

$$(1-\lambda)[\lambda^2-10\lambda-1] - 2[-2\lambda-3] + 3[3\lambda-2] = 0$$

$$\lambda^2-10\lambda-1-\lambda^3+10\lambda^2+\lambda+4\lambda\cancel{+6}+9\lambda\cancel{-6} = 0$$

$$-\lambda^3+11\lambda^2+4\lambda-1 = 0$$

$$\lambda^3-11\lambda^2-4\lambda+1 = 0$$

By calcy Hamilton theorem

$$A^3-11A^2-4A+I = 0$$

$$A^2 = A\cdot A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1+4+9 & 2+8+15 & 3+10+18 \\ 2+8+15 & 4+16+25 & 6+20+30 \\ 3+10+18 & 6+20+30 & 9+25+36 \end{bmatrix} = \begin{bmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{bmatrix}$$

$$A^3 = A^2\cdot A = \begin{bmatrix} 14 & 25 & 31 \\ 25 & 45 & 56 \\ 31 & 56 & 70 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 14+50+93 & 28+100+155 & 42+125+186 \\ 25+90+168 & 50+180+280 & 75+225+336 \\ 31+112+210 & 62+224+350 & 93+280+420 \end{bmatrix}$$

$$= \begin{bmatrix} 157 & 283 & 353 \\ 283 & 510 & 636 \\ 353 & 636 & 793 \end{bmatrix}$$

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$A^3 - 11A^2 - 4A + I$

$$= \begin{bmatrix} 157 & 283 & 353 \\ 283 & 510 & 636 \\ 253 & 636 & 793 \end{bmatrix} - \begin{bmatrix} 154 & 275 & 341 \\ 275 & 495 & 616 \\ 341 & 616 & 770 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 12 \\ 8 & 16 & 20 \\ 12 & 20 & 24 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 157-154-4+1 & 283-275-8+0 & 353-341-12+0 \\ 283-275-8+0 & 510-495-16+1 & 636-616-20+0 \\ 253-341-12+0 & 636-616-20+0 & 793-770-24+1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ coley Hamilton theorem is verified

$$A^3 - 11A^2 - 4A + I = 0$$

$$I = -A^3 + 11A^2 + 4A$$

multiplying with A^{-1}

$$A^{-1} = -A^2 + 11A + 4I$$

$$A^{-1} = \begin{bmatrix} -14 & -25 & -31 \\ -25 & -45 & -56 \\ -31 & -56 & -70 \end{bmatrix} + \begin{bmatrix} 11 & 22 & 33 \\ 22 & 44 & 55 \\ 33 & 55 & 66 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -14+11+4 & -25+22+0 & -31+33+0 \\ -25+22+0 & -45+44+4 & -56+55+0 \\ -31+33+0 & -56+55+0 & -70+66+4 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

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3. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ express $2A^5 - 3A^4 + A^2 - 4I$ as a

Note

2ab[2]t linear polynomial in A

2018

Solu] Given matrix

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

The characteristic matrix of A is

$$(A - \lambda I) = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & 1 \\ -1 & 2-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)(2-\lambda) + 1 = 0$$

$$6 - 2\lambda - 3\lambda + \lambda^2 + 1 = 0$$

$$\lambda^2 - 5\lambda + 7 = 0$$

By caley Hamilton theorem

$$A^2 - 5A + 7I = 0$$

$$A^2 = 5A - 7I$$

$$A^3 = 5A^2 - 7A$$

$$A^4 = 5A^3 - 7A^2$$

$$A^5 = 5A^4 - 7A^3$$

$$2A^5 - 3A^4 + A^2 - 4I = 2[5A^4 - 7A^3] - 3A^4 + A^2 - 4I$$

$$= 7A^4 - 14A^3 + A^2 - 4I$$

$$= 7[5A^3 - 7A^2] - 14A^3 + A^2 - 4I$$

$$= 21A^3 - 48A^2 - 4I$$

$= 21[5A^2 - 7A] - 48A^2 - 4I$

$= 57A^2 - 147A - 4I$

$= 57(5A - 7I) - 147A - 4I$

$= 138A - 403I$

4. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, express $A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$ as a polynomial

A) Given matrix $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

The characteristic matrix of A is

$$A - \lambda I = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & 2 \\ -1 & 3-\lambda \end{bmatrix}$$

the characteristic equation of A is

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ -1 & 3-\lambda \end{vmatrix} = 0$$

$(1-\lambda)(3-\lambda) + 2 = 0$

$3 - 3\lambda - \lambda + \lambda^2 + 2 = 0$

$\lambda^2 - 4\lambda + 5 = 0$

By caley Hamilton theorem

$A^2 - 4A + 5I = 0$

$A^2 = 4A - 5I$

$A^3 = 4A^2 - 5A$

$A^4 = 4A^3 - 5A^2$

$A^5 = 4A^4 - 5A^3$

$A^6 = 4A^5 - 5A^4$

Given equation

$A^6 - 4A^5 + 8A^4 - 12A^3 + 14A^2$

$$A^6 - 14 = (4A^5 - 5A^4) - 4A^5 + 8A^4 - 12A^3 + 14A^2$$
$$= 3A^4 - 12A^3 + 14A^2$$
$$= 3[4A^3 - 5A^2] - \cancel{12A^3} + 14A^2$$
$$= -15A^2 + 14A^2$$
$$= -A^2 = -4A + 5I$$

Quadratic Forms

* A homogeneous expression of the second degree in any no. of variables is called a quadratic form.

Ex: 1. $3x^2 + 5xy - 2y^2$ is a quadratic form in x & y

2. $x^2 + 2y^2 - 3z^2 + 2xy - 3yz + 5zx$ is a quadratic form in three variables.

* An expression of the form $Q = X^T AX = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j$ where A_{ij} are constants is called a quadratic form in n variables.

Matrix of a Quadratic form

Every quadratic form Q can be expressed as $Q = X^T AX$. The symmetric matrix A is called the matrix of the quadratic form Q and $|A|$ is called the discriminant of the quadratic form

Note:

* If $|A| = 0$ the quadratic form is singlur.
* Ex: To write the matrix of quadratic form follow the diagram given below

write the co-efficients of square terms along the diagonal and divide the co-efficients of the product terms, xy, yz, zx by 2 and write them at the appropriate places.

Ex: $Q = 7x^2 + 8xy + 9yz + 2xz + 3y^2 - 5z^2$

$Q = 7xx + 4xy + 4xy + \frac{9}{2}yz + 2z + xz + 3yy - 5zz$

$= 7xx + 4xy + xz$

$+ 4yx + 3yy + \frac{9}{2}yz$

$+ zx + \frac{9}{2}zy - 5zz$

$$\begin{array}{c|ccc} & x & y & z \\ \hline x & x^2 & \frac{xy}{2} & \frac{xz}{2} \\ y & \frac{yx}{2} & y^2 & \frac{yz}{2} \\ z & \frac{zx}{2} & \frac{zy}{2} & z^2 \end{array}$$

$$A = \begin{bmatrix} 7 & 4 & 1 \\ 4 & 3 & 9/2 \\ 1 & 9/2 & -5 \end{bmatrix}$$

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $X^T = [x \;\; y \;\; z]$;

$$Q = X^T A X = [x \; y \; z] \begin{bmatrix} 7 & 4 & 1 \\ 4 & 3 & 9/2 \\ 1 & 9/2 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

write the symmetric matrix of the following q.F

1. $x^2 + 2y^2 - 7z^2 - 4xy - 6xz$
2. $2x^2 - 3y^2 + 5z^2 + 6xy - yz + 4zx$
3. $4xy + 6yz + 8zx$
4. $x^2 - y^2 + z^2 + 7xy + 9yz + 11zx$

4. $Q = x^2 - y^2 + z^2 + 7xy + 9yz + 11zx$

$Q = xx - yy + zz + \frac{7}{2}xy + \frac{9}{2}yz + \frac{11}{2}zx$

$= xx + \frac{7}{2}xy + \frac{11}{2}zx$

$+ \frac{7}{2}yx - yy + \frac{9}{2}yz$

$+ \frac{19}{2}yz + \frac{9}{2}zy + zz$

$$A = \begin{bmatrix} 1 & 7/2 & \frac{11}{2} \\ \frac{7}{2} & -1 & \frac{9}{2} \\ \frac{11}{2} & \frac{9}{2} & 1 \end{bmatrix}$$

3. $Q = 4xy + 6yz + 8zx$

$Q = 2xy + 3yz + 4zx$

$$A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 0 & 3 \\ 4 & 3 & 0 \end{bmatrix}$$

2. $Q = 2x^2 - 3y^2 + 5z^2 - 6xy - yz + 4zx$

$2xx - 3yy + 5zz - 3xy - \frac{1}{2}yz + 2zx$

$Q = 2xx - 3xy + 2zx$

$-3yx - 3yy - \frac{1}{2}yz$

$+2zx - \frac{1}{2}zy + 5zz$

$$A = \begin{bmatrix} 2 & -3 & 2 \\ -3 & -3 & -1/2 \\ 2 & -1/2 & 5 \end{bmatrix}$$

1. $x^2 + 2y^2 - 7z^2 - 4xy - 6xz$

$Q = xx + 2yy - 7zz - 2xy - 3xz$

$Q = xx - 2xy - 3xz$

$-2xy + 2yy + 0 \cdot yz$

$-3xz + 0 \cdot yz - 7zz$

$$A = \begin{bmatrix} 1 & -2 & -3 \\ -2 & 2 & 0 \\ -3 & 0 & -7 \end{bmatrix}$$

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31/12/2018

Write the Quadratic form of corresponding to the matrix

1) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ 2) $\begin{bmatrix} 2 & 1 & 5 \\ 1 & 3 & -2 \\ 5 & -2 & 4 \end{bmatrix}$ 3) $\begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix}$

4) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ 5) $\begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$

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Given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix}, \; X^T = [x \; y \; z] \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

quadratic form $Q = X^T A X$

$$= [x \; y \; z] \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 3 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x+2y+3z \quad 2x+3z \quad 3x+3y+z] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= x(x+2y+3z) + y(2x+3z) + z(3x+3y+z)$$

$$= x^2+2xy+3zx+2xy+3zx+3zy+z^2$$

$$= x^2+z^2+4xy+6zx+3zy$$

3) Given matrix

$$\begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix}, \; X^T = [x \; y \; z] \; ; \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Quadratic form

$$Q = X^T A X$$

$$= [x \; y \; z] \begin{bmatrix} 1 & 2 & 5 \\ 2 & 0 & 3 \\ 5 & 3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x+2y+5z \quad 2x+3z \quad 5x+3y+4z] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= x(x+2y+5z) + y(2x+3z) + z(5x+3y+4z)$$

$$= x^2+2xy+5zx+2xy+3zy+5zx+3zy+4z^2$$

$$= x^2+4z^2+4xy+10zx+6yz$$

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Rank of a Quadratic form

Let X^TAX be a quadratic form. the rank $R(A)$ is called the rank of the quadratic form. If 'r' is less than n, $|A|=0$ (or) A is singular then the quadratic form is called "singular" otherwise "non-singular"

Canonical Form (or) Normal form of a Quadratic form

Let X^TAX be a quadratic form in n variables then there exsist a real non-singular linear transformation $X = PY$ which transforms X^TAX to another quadratic form of type $Y^TDY = \lambda_1 y_1^2 + \lambda_2 y_2^2 + \lambda_3 y_3^2 + \dots + \lambda_n y_n^2$ then Y^TDY is called the canonical form of quadratic form of X^TAX

Here $D = diag(\lambda_1, \lambda_2, \lambda_3 \dots \lambda_n)$

Index of a Real Quadratic Form

The number of positive terms in canonical form of quadratic form is known as the index of the quadratic form and is denoted by 's'

Signature of a Quadratic form

If 'r' is the rank of the quadratic form and 's' is the index of the quadratic form then $2s-r$ is called the signature of the quadratic form X^TAX.

Nature of Quadratic Forms

⇒ Positive Definite

The Quadratic form X^TAX in 'n' variables is said to be positive definite if all the eigen values of A are positive (or) if $r = n$ and $s = n$ i.e., $r = s = n$

⇒ Negative Definite

The quadratic form X^TAX in n variables is said to be negative definite if $r = n$ and $s = 0$ (or) if all the eigen values of A are negative.

⇒ Positive-Semi-Definite

The Quadratic form X^TAX in n variables is said to be positive semi definite if $r < n$ & $s = r$ (or) If all the eigen values of $A \geq 0$ and atleast one eigen value is zero

⇒ Negative-Semi-Definite

The Quadratic form X^TAX in n variables is said to be negative semidefinite if $r < n$ & $s = 0$ (or) If all the eigen values of $A \leq 0$. and atleast one eigen value is zero

⇒ In-Definite

In all other cases, if all the eigen values of A are positive and negative. then the quadratic form is called in-definite

1. Identify the nature of the Quadratic forms

i) $x_1^2+4x_2^2+x_3^2-4x_1x_2+2x_1x_3-4x_2x_3$

ii) $x^2+4xy+6xz-y^2+2yz+4z^2$

iii) $x^2+y^2+2z^2-2xy+2xz$

iv) $2x^2+9y^2+6z^2+8xy+8yz+6zx$

Sol:- i) Given Quadratic form

$$Q = x_1^2+4x_2^2+x_3^2-4x_1x_2+2x_1x_3-4x_2x_3$$

$$Q = X^TAX, \quad A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}$$

characterestic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -2 & 1 \\ -2 & 4-\lambda & -2 \\ 1 & -2 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)[(4-\lambda)(1-\lambda)-4]+2[-2(1-\lambda)+2]+1(4-(4-\lambda))=0$$

$$(1-\lambda)[\cancel{4}-\lambda-4\lambda+\lambda^2\cancel{-4}]+2[\cancel{-2}+2\lambda\cancel{+2}]\cancel{+4}\cancel{-4}+\lambda=0$$

$$(1-\lambda)[\lambda^2-5\lambda]+4\lambda+\lambda=0$$

$$\lambda^2\cancel{-5\lambda}-\lambda^3+5\lambda^2\cancel{+5\lambda}=0$$

$$-\lambda^3+6\lambda^2=0$$

$$\cancel{\lambda^3}=6\cancel{\lambda^2}$$

$$\lambda = 6 \; ; \; \lambda = 0, 0, 6$$

Eigen values two are zeroes and the remaining is positive

Hence given Quadratic form is positive semidefinite

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(iv) Given Quadratic form

$$Q = 2x^2+9y^2+6z^2+8xy+8yz+6zx$$

$$Q = X^T A X \; ; \quad A = \begin{bmatrix} 2 & 4 & 3 \\ 4 & 9 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$

the characteristic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & 4 & 3 \\ 4 & 9-\lambda & 4 \\ 3 & 4 & 6-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(9-\lambda)(6-\lambda)-16]-4[4(6-\lambda)-12]+3[16-3(9-\lambda)]=0$$

$$(2-\lambda)[54-6\lambda-9\lambda+\lambda^2-16]-4[24-4\lambda-12]+3[16-27+3\lambda]=0$$

$$(2-\lambda)[38-15\lambda+\lambda^2]-4[12-4\lambda]+3[3\lambda-11]=0$$

$$(2-\lambda)[38-15\lambda+\lambda^2]-48+16\lambda+9\lambda-33=0$$

$$2\lambda^2-30\lambda+76-\lambda^3+15\lambda^2-38\lambda-48+16\lambda+9\lambda-33=0$$

$$-\lambda^3+17\lambda^2-43\lambda-5=0$$

$$\lambda^3-17\lambda^2+43\lambda+5=0$$

$27-16=11$, $54-16=38$, $16+65=81$, $-68+25=-43$, $81-76=5$

14) Given quadratic form

$$Q = x^2 + 4xy + 6xz - y^2 + 2yz + 4z^2$$

$$Q = X^T A X, \quad A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 1 \\ 3 & 1 & 4 \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 2 & 3 \\ 2 & -1-\lambda & 1 \\ 3 & 1 & 4-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(-1-\lambda)(4-\lambda) - 1\right] - 2\left[2(4-\lambda) - 3\right] + 3\left[2 - 3(-1-\lambda)\right]$$

$$(1-\lambda)\left[-4 - 4\lambda + \lambda + \lambda^2 - 1\right] - 2\left[8 - 2\lambda - 6\right] + 3\left[2 + 3 + 3\lambda\right] = 0$$

$$(1-\lambda)\left[\lambda^2 - 3\lambda - 5\right] - 2\left[-2\lambda + 2\right] + 3\left[3\lambda + 5\right] = 0$$

$$\lambda^2 - 3\lambda - 5 - \lambda^3 + 3\lambda^2 + 5\lambda + 4\lambda - 4 + 9\lambda + 15 = 0$$

$$-\lambda^3 + 4\lambda^2 + 15\lambda + 6 = 0$$

$$\lambda^3 - 4\lambda^2 - 15\lambda - 6 = 0$$

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iii) Given Quadratic form

$$Q = x^2 + y^2 + 2z^2 - 2xy + 2xz$$

$$Q = X^T A X \quad A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & -1 & 1 \\ -1 & 1-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)(2-\lambda) - 0\right] + 1\left[-1(2-\lambda) - 0\right] + 1\left[0 - (1-\lambda)\right] = 0$$

$$(1-\lambda)\left[2 - 2\lambda - \lambda + \lambda^2 - 0\right] + \left[-2 + \lambda\right] + \lambda - 1 = 0$$

$$(1-\lambda)\left[\lambda^2 - 3\lambda + 2\right] - 2 + \lambda + \lambda - 1 = 0$$

$$\lambda^2 - 3\lambda + 2 - \lambda^3 + 3\lambda^2 - \cancel{2\lambda} - 3 + \cancel{2\lambda} = 0$$

$$-\lambda^3 + 4\lambda^2 - 3\lambda - 1 = 0$$

$$\lambda^3 - 4\lambda^2 + 3\lambda + 1 = 0$$

2. Given matrix

$$A = \begin{bmatrix} 2 & 1 & 5 \\ 1 & 3 & -2 \\ 5 & -2 & 4 \end{bmatrix} \quad X^T = [x\ y\ z] \ ; \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Quadratic form $Q = X^T A X$

$$= [x\ y\ z] \begin{bmatrix} 2 & 1 & 5 \\ 1 & 3 & -2 \\ 5 & -2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [2x+y+5z \quad x+3y-2z \quad 5x-2y+4z] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x(2x+y+5z) \quad y(x+3y-2z) \quad z(5x-2y+4z)]$$

$$= 2x^2 + xy + 5zx + xy + 3y^2 - 2zy + 5zx - 2zy + 4z^2$$

$$= 2x^2 + 3y^2 + 4z^2 + 2xy + 10zx - 4zy$$

4. Given matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix} \quad X^T = [x\ y\ z] \ ; \ X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Quadratic form $Q = X^T A X$

$$= [x\ y\ z] \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x+2y+3z \quad 2x+y+3z \quad 3x+3y+z] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x^2 + 2xy + 3zx + 2xy + y^2 + 3zy + 3zx + 3yz + z^2$$

$$= x^2 + y^2 + z^2 + 4xy + 6zx + 6zy.$$

5. Given matrix

$$A = \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix} \quad X^T = [x\ y\ z] \;;\; X \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

Quadratic form $Q = X^TAX$

$$= [x\ y\ z] \begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [5y-z \quad 5x+y+6z \quad -x+6y+2z] \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$= [x5y - zx + 5xy + y^2 + 6zy - xz + 6yz + 2z^2]$$

$$y^2 + 2z^2 + 10xy + 12yz - 2xz$$

Date 2/1/19

1. Reduce the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ to a diagonal part and interrupt the result in terms of Quadratic forms also find the rank signature, Index.

solu) $A = I_3 A I_3$

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 & 2 \\ 0 & 7 & -1 \\ 0 & -1 & 7 \end{bmatrix} \begin{matrix} R_2 \to 3R_2 + R_1 \\ R_3 \to 3R_3 - R_1 \end{matrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 0 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & -3 \\ 0 & -3 & 21 \end{bmatrix} \begin{matrix} C_2 \to 3C_2 + C_1 \\ C_3 \to 3C_3 - C_1 \end{matrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -1 & 0 & 3 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & -3 \\ 0 & 0 & 144 \end{bmatrix} R_3 \to 7R_3 + R_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -6 & 3 & 21 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -1 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 1008 \end{bmatrix} C_3 \to 7C_3 + C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -6 & 3 & 21 \end{bmatrix} A \begin{bmatrix} 1 & 1 & -6 \\ 0 & 3 & 3 \\ 0 & 0 & 21 \end{bmatrix}$$

$D = P^T A P$

$D = \text{dia}(6, 21, 1008)$

$$D = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 21 & 0 \\ 0 & 0 & 1008 \end{bmatrix} \quad P^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 3 & 0 \\ -6 & 3 & 21 \end{bmatrix} \quad P = \begin{bmatrix} 1 & 1 & -6 \\ 0 & 3 & 3 \\ 0 & 0 & 21 \end{bmatrix}$$

Quadratic form $= X^T A X$

$$= [x \ y \ z] \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$= 6x^2 + 3y^2 + 3z^2 - 4xy + 4xz - 2yz$

Non-singular transformation corresponding to the matrix P is $X = PY$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & -6 \\ 0 & 3 & 3 \\ 0 & 0 & 21 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$= [y_1 + y_2 - 6y_3, \ 3y_2 + 3y_3 \ \ 21y_3]$

$x = y_1 + y_2 - 6y_3 ; \ y = 3y_2 + 3y_3 ; \ z = 21y_3$

Canonical form $= Y^T D Y = 6y_1^2 + 21y_2^2 + 1008y_3^2$

Rank of A is $\rho(A) = 3$ (rank of diagonal matrix non zero rows)

Index $= s = 3$ (no. of positive terms)

Signature $= 2s - r = 2(3) - 3 = 3$

2. Find the rank, signature, index of the quadratic form

i) $2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_1x_3$ by reducing it into canonical form also write the linear transformation which brings about the normal reduction

Sol:

Given

Quadratic form

$$= 2x_1^2 + x_2^2 - 3x_3^2 + 12x_1x_2 - 8x_2x_3 - 4x_1x_3$$

Given quadratic form into matrix

$$A = \begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix}$$

$$A = I_3 A I_3$$

$$\begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 6 & -2 \\ 0 & -17 & 2 \\ 0 & 2 & -5 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_3 + R_1 \end{matrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 2 \\ 0 & 2 & -5 \end{bmatrix} \begin{matrix} C_2 \to C_2 - 3C_1 \\ C_3 \to C_3 + C_1 \end{matrix} = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 2 \\ 0 & 0 & 81 \end{bmatrix} R_3 \to -17R_3 - 2R_2 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -11 & -2 & -17 \end{bmatrix} A \begin{bmatrix} 1 & -3 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 0 \\ 0 & 0 & 1377 \end{bmatrix} C_3 \to -17C_3 - 2C_2 = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -11 & -2 & -17 \end{bmatrix} A \begin{bmatrix} 1 & -3 & -11 \\ 0 & 1 & -2 \\ 0 & 0 & -17 \end{bmatrix}$$

$$D = P^T A P$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -17 & 0 \\ 0 & 0 & 1377 \end{bmatrix} \quad P^T = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ -11 & -2 & -17 \end{bmatrix} \quad P = \begin{bmatrix} 1 & -3 & -11 \\ 0 & 1 & -2 \\ 0 & 0 & -17 \end{bmatrix}$$

Quadratic form $= X^TAX$

$$= [x\ y\ z] \begin{bmatrix} 2 & 6 & -2 \\ 6 & 1 & -4 \\ -2 & -4 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 2x^2 + 6xy - 2zx \\ + 6xy + y^2 - 4zy \\ - 2xz - 4yz - 3z^2 \end{matrix}$$

$$= [2x + 6y - 2z \quad 6x + y - 4z \quad -2x - 4y - 3z]$$

$2x^2+y^2=3z^2+12xy-4zx-8zy$ 1.

Non singular transformation corresponding to the matrix P is $x=Py$. Sol

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & -3 & -11 \\ 0 & 1 & -2 \\ 0 & 0 & -17 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= [y_1-3y_2-11y_3 \quad y_2-2y_3 \quad -17y_3]$$

$x=y_1-3y_2-11y_3$; $y=y_2-2y_3$, $z=-17y_3$

Canonical form $=y^TDy = 2y_1^2-17y_2^2+1377y_3^2$

Rank of A is $\rho(A)=3$

Index $s=2$

signature $=2s-r = 2(2)-3=1$

Date 3/1/2019

3. Reduction to Normal form by orthogonal transformation.

Working Rule:

1. Write the co-efficient matrix 'A' associated with the given quadratic form
2. Find the Eigen values of A.
3. Write the canonical form using $\lambda_1 y_1^2+\lambda_2 y_2^2+\dots+\lambda_n y_n^2$
4. Form the matrix P containing the normalized Eigen vectors of A as column vectors. then $x=Py$ gives the required orthogonal transformation which reduces quadratic form to canonical form.

1 Reduce the quadratic form $3x^2+2y^2+3z^2-2xy-2yz$ to the normal form by orthogonal transformation.

Sol: Given Quadratic form

$$Q = 3x^2+2y^2+3z^2-2xy-2yz$$

The matrix form

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

The characteristic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)[(2-\lambda)(3-\lambda)-1]+1[-(3-\lambda)-0]+0=0$$

$$(3-\lambda)[6-3\lambda-2\lambda+\lambda^2-1]+[-3+\lambda]=0$$

$$(3-\lambda)[\lambda^2-5\lambda+5]-3+\lambda=0$$

$$3\lambda^2-15\lambda+15-\lambda^3+5\lambda^2-5\lambda-3+\lambda=0$$

$$-\lambda^3+8\lambda^2-19\lambda+12=0$$

$$\lambda^3-8\lambda^2+19\lambda-12=0$$

$$\begin{array}{r|rrrr} 1 & 1 & -8 & 19 & -12 \\ & 0 & 1 & -7 & 12 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$

$$(\lambda-1)(\lambda^2-7\lambda+12)=0$$

$$(\lambda-1)[\lambda^2-4\lambda-3\lambda+12]=0$$

$$(\lambda-1)[(\lambda(\lambda-4)-3(\lambda-4)]=0$$

$$(\lambda-1)(\lambda-4)(\lambda-3)=0$$

$$\lambda = 1, 4, 3$$

The are the characteristic roots 1, 4, 3

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case (i)

If $\lambda = 1$ $(A - \lambda I)x = 0$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & -1 & 2 \end{bmatrix} R_2 \to 2R_2 + R_1 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 + R_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Rank $= 2$, $n = 3$.

$n - r = 3 - 2 = 1$ L.I.S

$2x - y = 0$; $y - 2z = 0$; $z = k$.

$2x - 2k = 0$ $y = 2k$

$x = k$

$$X_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ 2k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

case (ii)

If $\lambda = 4$ $(A - \lambda I)x = 0$

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} R_2 \to R_2 - R_1 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

R_3

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - R_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2$; $n - r = 3 - 2 = 1$ L.I.S

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$-x-y=0$; $-y-z=0$ $z=k$

$-x+k=0$ $-y-k=0$

$x=+k$ $-y=k$

$y=-k$

$$X_3=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-k\\-k\\k\end{bmatrix}=\begin{bmatrix}+1\\-1\\1\end{bmatrix}$$

Eigen

$$x_1=\begin{bmatrix}1\\2\\1\end{bmatrix}\quad x_2=\begin{bmatrix}-1\\0\\1\end{bmatrix}\quad x_3=\begin{bmatrix}1\\-1\\1\end{bmatrix}$$

case (iii)

If $\lambda=3$; then $(A-\lambda I)x=0$

$$\begin{bmatrix}0&-1&0\\-1&-1&-1\\0&-1&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}0&-1&0\\-1&-1&-1\\0&0&0\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}_{R_3\to R_3-R_1}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$\rho(A)=2$; $n=3$

$n-r=3-2,\ 1\cdot$ L.I.S

$-y=0$; $-x+y-z=0$; $z=k$

$-x-0-k=0$

$-x=k$

$x=-k$

$$X_2=\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}-k\\0\\k\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}k$$

Here we observed that X_1, X_2, X_3 are mutually $\perp$ler

$X_1X_2=X_2\cdot X_3=X_3X_1=0$

The normalized vectors are

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$$e_1 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix} \quad e_2 = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad e_3 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}$$

$$P = [e_1 \ e_2 \ e_3] = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & 0 & -1/\sqrt{3} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & 1/\sqrt{3} \end{bmatrix}$$

$$D = P^T A P$$

$$D = \begin{bmatrix} \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} & 1/\sqrt{6} \\ \frac{-1}{\sqrt{2}} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \\ 2/\sqrt{6} & 0 & -1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$$

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$$= \begin{bmatrix} 3/\sqrt{6} - 2/\sqrt{6} + 0 & -1/\sqrt{6} + 4/\sqrt{6} - 1/\sqrt{6} & 0 - 2/\sqrt{6} + 3/\sqrt{6} \\ -3/\sqrt{2} + 0 + 0 & 1/\sqrt{2} + 0 - 1/\sqrt{2} & 0 + 0 + 3/\sqrt{2} \\ 3/\sqrt{3} + 1/\sqrt{3} + 0 & -1/\sqrt{3} - 2/\sqrt{3} - 1/\sqrt{3} & 0 + 1/\sqrt{3} + 3/\sqrt{3} \end{bmatrix} \begin{bmatrix} 1/\sqrt{6} & \frac{-1}{\sqrt{2}} & 1/\sqrt{3} \\ 2/\sqrt{6} & 0 & -1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$$

solu 2

$$= \begin{bmatrix} 1/\sqrt{6} & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ -3/\sqrt{2} & 0 & 3/\sqrt{2} \\ 4/\sqrt{3} & -4/\sqrt{3} & 4/\sqrt{3} \end{bmatrix} \begin{bmatrix} 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \\ 2/\sqrt{6} & 0 & -1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{6} + \frac{4}{6} + \frac{1}{6} & -1/\sqrt{12} + 0 + 1/\sqrt{12} & 1/\sqrt{18} - 2/\sqrt{18} + 1/\sqrt{18} \\ \frac{-3}{\sqrt{12}} + 0 + 3/\sqrt{12} & 3/2 + 0 + 3/2 & -3/\sqrt{6} + 0 + 3/\sqrt{6} \\ 4/\sqrt{18} - 8/\sqrt{18} + 4/\sqrt{18} & -4/\sqrt{6} + 0 + 4/\sqrt{6} & \frac{4}{3} + \frac{4}{3} + \frac{4}{3} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \mathrm{diag}(1, 3, 4)$$

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orthogonal transformation

$$X = PY$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1/\sqrt{6} & -1/\sqrt{2} & 1/\sqrt{3} \\ 2/\sqrt{6} & 0 & -1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$x = \frac{y_1}{\sqrt{6}} - \frac{y_2}{\sqrt{2}} + \frac{y_3}{\sqrt{3}}$$

$$y = \frac{2y_1}{\sqrt{6}} - y_3/\sqrt{3}$$

$$z = \frac{y_1}{\sqrt{6}} + \frac{y_2}{\sqrt{2}} + \frac{y_3}{\sqrt{3}}$$

H.W Date 4/1/19

2. Reduce the Quadratic form $3x^2+5y^2+3z^2-2yz+2zx-2xy$ to the canonical form by orthogonal reduction

$1/\sqrt{3}$, $-1/\sqrt{3}$, $1/\sqrt{3}$

3. $x_1^2+3x_2^2+3x_3^2-2x_2x_3$

4. $2x^2+2y^2+2z^2-2xy-2yz-2zx$

solu) 2. Given Quadratic form

$$Q.F = 3x^2+5y^2+3z^2-2yz+2zx-2xy$$

The matrix form

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

The characteristic equation of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -1 & 1 \\ -1 & 5-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{vmatrix} = 0$$

$$[(3-\lambda)[3(5-\lambda)-1]+1[-3+1]+1[1-(5-\lambda)]=0$$

$$(3-\lambda)[15-3\lambda-1]+1[-2]+1[1-5+\lambda]=0$$

$$(3-\lambda)[14-3\lambda]-2+\lambda-4=0$$

$$42-14\lambda-9\lambda+3\lambda^2+\lambda-6=0$$

$$3\lambda^2-22\lambda+36=0$$

$$(3-\lambda)[(5-\lambda)(3-\lambda)-1]+1[-(3-\lambda)+1]+1[1-(5-\lambda)]=0$$

$$(3-\lambda)[(5-\lambda)(3-\lambda)-1]+[-3+\lambda+1]+1-5+\lambda=0$$

$$(3-\lambda)[15-3\lambda-5\lambda+\lambda^2-1]+[-3+\lambda+1]+1-5+\lambda=0$$

$$(3-\lambda)[\lambda^2-8\lambda+14]+\lambda-2+\lambda-4=0$$

$$3\lambda^2-24\lambda+42-\lambda^3+8\lambda^2-14\lambda+2\lambda-6=0$$

$$-\lambda^3+11\lambda^2-36\lambda+36=0$$

$$\lambda^3-11\lambda^2+36\lambda-36=0.$$

$$\begin{array}{c|cccc} 3 & 1 & -11 & 36 & -36 \\ & 0 & 3 & -24 & 36 \\ \hline & 1 & -8 & 12 & \boxed{0} \end{array}$$

$$\lambda^2-8\lambda+12=0$$

$$(\lambda-3)[\lambda^2-6\lambda-2\lambda+12]=0$$

$$(\lambda-3)[\lambda(\lambda-6)-2(\lambda-6)]=0$$

$$(\lambda-2)(\lambda-3)(\lambda-6)=0$$

$$\lambda=2,3,6$$

case(i)

If $\lambda=2$ $\quad (A-\lambda I)X=0$

$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \begin{matrix} R_2 \to R_2+R_1 \\ R_3 \to R_3-R_1 \end{matrix} \quad =\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2, \ n=3$

$n-r = 3-2 = 1$, L.I.S

$x-y+z=0$; $2y=0$; let $z=K$

$x-0+K=0$ $\quad y=0$

$x=-K$

$$\therefore X_1 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -K \\ 0 \\ K \end{bmatrix} = K\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

case (iii)

If $\lambda = 6$ $\quad (A-\lambda I)X=0$

$$\begin{bmatrix} -3 & -1 & 1 \\ -1 & -1 & -1 \\ 1 & -1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 & 1 \\ 0 & -2 & -4 \\ 0 & -4 & -8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to 3R_2 - R_1 \\ R_3 \to 3R_3 + R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 & 1 \\ 0 & -2 & -4 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to 2R_3 - 4R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3 & -1 & 1 \\ 0 & +1 & 2 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_2 \to \frac{R_2}{-2} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2, \ n=3$

$\therefore n-r = 3-2 = 1$ L.I.S

$\therefore -3x-y+z=0$; $y+2z=0$; $z=K$

$-3x+2K+K=0$ $\quad y+2K=0$

$\therefore y=-2K$

$-3x=-3K$

$x=K$

$$\therefore X_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = K\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

case(ii)

If $\lambda = 3$ $(A-\lambda I)X = 0$

$$\begin{bmatrix} 0 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_2 \to R_2 + R_1$$

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad R_3 \to R_3 + R_2$$

$\rho(A) = 2$
$n = 3$
$n - r = 3 - 2 = 1$
$\therefore$ L.I.S

$x - y = 0$; $y - z = 0$; $z = k$

$x - k = 0$; $y - k = 0$
$x = k$. ; $y = k$

$$x_1 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} ; x_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} ; x_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

we observed that x_1, x_2, x_3 are mutually perpendicular

The normalized vectors

$$e_1 = \begin{bmatrix} \frac{-1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix} \quad e_2 = \begin{bmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix} \quad e_3 = \begin{bmatrix} \frac{1}{\sqrt{6}} \\ \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$P = \begin{bmatrix} e_1 & e_2 & e_3 \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

$D = P^T A P$

$$D = \begin{bmatrix} -\frac{1}{\sqrt{2}} & 0 & 1/\sqrt{2} \\ 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{6} & \frac{-2}{\sqrt{6}} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} -\frac{1}{\sqrt{2}} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{3}{\sqrt{2}} + 0 + \frac{1}{\sqrt{2}} & \cancel{\frac{1}{\sqrt{2}}} + 0 - \cancel{\frac{1}{\sqrt{2}}} & \frac{-1}{\sqrt{2}} + 0 + \frac{3}{\sqrt{2}} \\ \frac{3}{\sqrt{3}} - \cancel{\frac{1}{\sqrt{3}}} + \cancel{\frac{1}{\sqrt{3}}} & \frac{-1}{\sqrt{3}} + \frac{5}{\sqrt{3}} - \cancel{\frac{1}{\sqrt{3}}} & \cancel{\frac{1}{\sqrt{3}}} - \cancel{\frac{1}{\sqrt{3}}} + \frac{3}{\sqrt{3}} \\ \frac{3}{\sqrt{6}} + \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} - \frac{10}{\sqrt{6}} - \cancel{\frac{1}{\sqrt{6}}} & \frac{1}{\sqrt{6}} + \frac{2}{\sqrt{6}} + \frac{3}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & \frac{-2}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-2}{\sqrt{2}} & 0 & \frac{2}{\sqrt{2}} \\ \frac{3}{\sqrt{3}} & \frac{3}{\sqrt{3}} & \frac{3}{\sqrt{3}} \\ \frac{6}{\sqrt{6}} & \frac{-12}{\sqrt{6}} & \frac{6}{\sqrt{6}} \end{bmatrix} \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ \frac{1}{\sqrt{2}} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{2}{2} + 0 + \frac{2}{2} & \cancel{\frac{-2}{6}} + 0 + \cancel{\frac{2}{6}} & \cancel{\frac{-2}{12}} + 0 + \cancel{\frac{2}{12}} \\ \cancel{\frac{-3}{6}} + 0 + \cancel{\frac{3}{6}} & \frac{3}{9} + \frac{3}{9} + \frac{3}{9} & \cancel{\frac{3}{18}} - \cancel{\frac{6}{18}} + \cancel{\frac{3}{18}} \\ \cancel{\frac{-6}{12}} + \cancel{\frac{0}{12}} + \cancel{\frac{6}{12}} & \cancel{\frac{6}{18}} - \cancel{\frac{12}{18}} + \cancel{\frac{6}{18}} & \frac{6}{36} + \frac{24}{36} + \frac{6}{36} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} = \text{diag}(2, 3, 6)$$

orthogonar transformation

$X = PY$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \\ 0 & 1/\sqrt{3} & -2/\sqrt{6} \\ 1/\sqrt{2} & 1/\sqrt{3} & 1/\sqrt{6} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$\therefore x = -\frac{y_1}{\sqrt{2}} + \frac{y_2}{\sqrt{3}} + \frac{y_3}{\sqrt{6}}$; $y = \frac{y_2}{\sqrt{3}} - \frac{2y_3}{\sqrt{6}}$; $z = \frac{y_1}{\sqrt{2}} + \frac{y_2}{\sqrt{3}} + \frac{y_3}{\sqrt{6}}$

4. Given Quadratic form

$$Q.F = 2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$$

matrix

$$A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

The characteristic equation of A is

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 2-\lambda & -1 & -1 \\ -1 & 2-\lambda & -1 \\ -1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)[(2-\lambda)(2-\lambda)-1] + 1[-(2-\lambda)-1] - 1[1+(2-\lambda)] = 0$$

$$(2-\lambda)[4 - 2\lambda - 2\lambda + \lambda^2 - 1] + [-2+\lambda-1] - [1+2-\lambda] = 0$$

$$(2-\lambda)[\lambda^2 - 4\lambda + 3] + [\lambda - 3] - [3-\lambda] = 0$$

$$(2-\lambda)[\lambda^2 - 4\lambda + 3] + \lambda - 3 - 3 + \lambda = 0$$

$$2\lambda^2 - 8\lambda + \cancel{6} - \lambda^3 + 4\lambda^2 - 3\lambda + 2\lambda - \cancel{6} = 0$$

$$-\lambda^3 + 6\lambda^2 - 9\lambda = 0$$

$$\lambda^3 - 6\lambda^2 + 9\lambda = 0$$

$$\lambda(\lambda^2 - 6\lambda + 9) = 0$$

$$\lambda[\lambda^2 - 3\lambda - 3\lambda + 9] = 0$$

$$\lambda[\lambda(\lambda-3) - 3(\lambda-3)] = 0$$

$$\lambda(\lambda-3)(\lambda-3) = 0$$

$$\lambda = 0, 3, 3$$

case (i)

If $\lambda = 0$ $(A - \lambda I)X = 0$

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & -3 \\ 0 & -3 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to 2R_2 + R_1 \\ R_3 \to 2R_3 + R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & -1 \\ 0 & 3 & -3 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} R_3 \to R_3 + R_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 2 ;\ n = 3$

$n - r = 3 - 2 = 1$ L.I.S

$2x - y - z = 0 ;\ 3y - 3z = 0 ;\ z = k$

$2x - k - k = 0 \qquad 3y - 3k = 0$

$x = k \qquad 3y = 3k$

$y = k$

$$x_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} k$$

Case (ii)

If $\lambda = 3 ;\ (A - \lambda I)X = 0$

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{matrix} R_2 \to R_2 - R_1 \\ R_3 \to R_3 - R_1 \end{matrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\rho(A) = 1 ;\ n = 3$

$n - r = 3 - 1 = 2$ L.I.S

$-x - y - z = 0 ;\ y = k_1 ;\ z = k_2$

$-x - k_1 - k_2 = 0$

$x + (k_1 + k_2) = 0$

$x = -(k_1 + k_2)$

$$X_2 = \begin{bmatrix} -k_1 - k_2 \\ k_1 \\ k_2 \end{bmatrix} \text{(or)} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} k_1 + \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} k_2$$

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imp

DIAGONALISATION

if square matrix of A order n has a linearly indipendent eign vectors then a matrix ~~B~~ can be found such that ~~BA~~ inward ~~A~~ ~~the~~ AB is a diagonal matrix

$$B = [x_1, x_2, x_3]$$

here

x_1, x_2, x_3 are the eign vectors of the given matrix.

Q. diagonalise the matrix

$$A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$$

Sol^n: The C.H. matrix of A is

$$A - \lambda I$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1-\lambda & 1 & -2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{bmatrix}$$

The ch eq^n of A

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 1 & -2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & -1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)$
$-\lambda+1$

$(1-\lambda)\left[(2-\lambda)(-1-\lambda)-1\right] - 1\left[-1(-1-\lambda)-0\right] - 2\left[-1-0\right] = 0$

$(1-\lambda)\left[-2-2\lambda+\lambda+\lambda^2-1\right] - 1(1+\lambda) + 2 = 0$

$(1-\lambda)\left[\lambda^2-\lambda-3\right] - 1 - \lambda + 2 = 0$

$(1-\lambda)(\lambda^2-\lambda-3) - \lambda + 1 = 0 \Rightarrow (1-\lambda)(\lambda^2-\lambda-3) - \lambda + 1$

$\Rightarrow (1-\lambda)(\lambda^2-\lambda-3) + (1-\lambda)$

$\lambda^2 - \lambda - 3 - \lambda^3 + \lambda^2 + 3\lambda - \lambda + 1 = 0$

$(1-\lambda)(\lambda^2-\lambda-3)$

$-\lambda^3 + 2\lambda^2 + \lambda - 2 = 0$

$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$

$\lambda = 1 \Rightarrow 1 - 2 - 1 + 2 = 0$ $\quad \Delta = [x_1, x_2, x_3]$

$$\begin{array}{r|l} & \lambda^2 - \lambda - 2 \\ (\lambda-1) & \lambda^3 - 2\lambda^2 - \lambda + 2 \\ & \underline{\lambda^3 - \lambda^2} \\ & -\lambda^2 - \lambda + 2 \\ & \underline{-\lambda^2 + \lambda} \\ & -2\lambda + 2 \\ & \underline{-2\lambda + 2} \\ & 0 \end{array}$$

$(\lambda-1)(\lambda^2-\lambda-2) = 0$

$\lambda - 1 = 0 \quad \Big| \quad \lambda^2 - \lambda - 2 = 0$

$\qquad\qquad\quad (\lambda-2)(\lambda+1) = 0$

$\lambda = -1, 1, 2.$

The eigen roots of A are $-1, 1, 2$

Case I

if $\lambda = -1$ then $|A - \lambda I| X = 0$

$$\begin{bmatrix} 2 & 1 & -2 \\ -1 & 3 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$2x + y - 2z = 0$ — (1)

$-x + 3y + z = 0$ — (2)

$y = 0$ — (3)

put $y = 0$ in eq^n (1) & eq^n (2),

$2x - 2z = 0$ — (4)

$-x + z = 0$ — (5)

multiply eq^n (5) with (2) & eq^n (4)

$2x - 2z = 0$

$-2x + 2z$

$-x + z = 0$

$x = z$

let $z = k$

$x = k$, $z = k$

$y = 0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}.$$

case - II

if $\lambda = 1$ then $|A - \lambda I| = 0$

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$y - 2z = 0$ — (1)

$-x + y + z = 0$ — (2)

$y - 2z = 0$ — (3)

let

$z = k$

$$\boxed{\begin{matrix} \cancel{y - 2z = 0} \\ y - 2z = 0 \\ - \quad + \end{matrix}}$$

$y - 2z = 0$

$y - 2k = 0$

$+2k = +y$

$y = 2k$

$-x + 2k + k = 0$

$-x + 3k = 0$

$x = 3k$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3k \\ 2k \\ k \end{bmatrix} = k \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}.$$

Case-(iii)

if $\lambda = 2$ then $|A - \lambda I| = 0$

$$\begin{bmatrix} -1 & 1 & -2 \\ -1 & 0 & 1 \\ 0 & 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x + y - 2z = 0$

$-x + z = 0$

$y - 3z = 0$

Let, $z = k$

$-x + k = 0$

$+x = +k$

$x = k$

$-k + y - 2k = 0$

$y - 3k = 0$

$y = 3k$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ 8k \\ k \end{bmatrix} = k \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Hence eign vectors are

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

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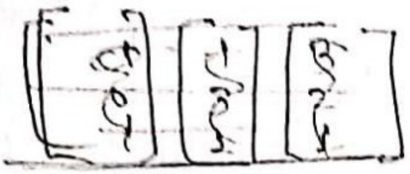

$$B = [x_1, x_2, x_3] = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \operatorname{adj} B$$

$$|B| = \begin{vmatrix} 1 & 3 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix}$$

$$= 1(2-3) - 3(0-3) + 1(0-2)$$

$$= -1 + 9 - 2$$

$$= 6 \neq 0.$$

cofactor of $1 = (2-3) = -1$

" " $3 = -(0-3) = 3$

" " $1 = (0-2) = -2$.

" " $0 = -(3-1) = -2$

" " $2 = (1-1) = 0$

" " $3 = -(1-3) = 2$

" " $1 = (9-2) = 7$

" " $1 = -(3-0) = -3$

" " $1 = 2-0 = 2$.

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co - factor matrix of $B = \begin{bmatrix} -1 & 3 & -2 \\ -2 & 0 & 2 \\ 7 & -3 & 2 \end{bmatrix}$

$$\text{adj } B = \begin{bmatrix} -1 & -2 & 7 \\ 3 & 0 & -3 \\ -2 & 2 & 2 \end{bmatrix}$$

$$B^{-1} = \frac{1}{|B|} \text{adj } B = \frac{1}{6}\begin{bmatrix} -1 & -2 & 7 \\ 3 & 0 & -3 \\ -2 & 2 & 2 \end{bmatrix}$$

$$B^{-1}AB = \frac{1}{6}\begin{bmatrix} -1 & -2 & 7 \\ 3 & 0 & 3 \\ -2 & 2 & 2 \end{bmatrix}\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -1+2+0 & -1+4+7 & 2-2-7 \\ 3-0-0 & 3+0-3 & -6+0+3 \\ -2-2+0 & -2+4+2 & 4+2-2 \end{bmatrix}\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 1 & 2 & -7 \\ 3 & 0 & -3 \\ -4 & 4 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} 1+0-7 & 3+4-7 & 1+6-7 \\ 3+0-3 & 9+0-3 & 3+0-3 \\ -4+0+4 & -12+8+4 & -4+12+4 \end{bmatrix}$$

$$= \frac{1}{6}\begin{bmatrix} -6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}.$$

1/2/18

Diagonalisation by orthogonal transformation.

~~sappase~~ suppose, a is a real symetry matrix then. a characterists matrix of a will not be linearly indipendent. and also waise orthogonal. if we normalise each characteristics vector or eign vectors (X) we devide each component of X by the squqre root of the sum of the squares of all elements, write all normalized eign vectors to form normalized ~~moto~~ model matrix B then it can be easily shown that B is an orthogonal matrix and !

B^{-1} equal to B transparne.

therefore the symelerity transform

$$B^{-1}AB = D$$

Where D is the diogonal matrix.

this tranformation ~~B~~ transpose AB is equal to D, is known as orthogonal transformation.

Q. Calculation of powers of a matrix.

let, A be the given matrix of ~~AB~~ order 3.

We know that

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Using

$$D = B^{-1}AB$$

$$D^2 = (B^{-1}AB)(B^{-1}AB)$$

$$= (B^{-1}A)(BB^{-1})(AB)$$

$$= (B^{-1}A)(I)(AB)$$

$$D^2 = B^{-1}A^2B.$$

III'ly

$$D^3 = B^{-1}A^3B.$$

$\vdots$

$$D^n = B^{-1}A^nB$$

$$(BD^nB^{-1}) = B(B^{-1}A^nB)B^{-1}$$

$$= A^n$$

$$A^n = (BD^nB^{-1})$$

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$D^n = \begin{bmatrix} \lambda_1^n & 0 & 0 \\ 0 & \lambda_2^n & 0 \\ 0 & 0 & \lambda_3^n \end{bmatrix}$$

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

$$X_1 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{x_1}{\sqrt{x_1^2+x_2^2+x_3^2}} \\ \frac{x_2}{\sqrt{x_1^2+x_2^2+x_3^2}} \\ \frac{x_3}{\sqrt{x_1^2+x_2^2+x_3^2}} \end{bmatrix}$$

$$B = [X_1, X_2, X_3].$$

Q.
$$A = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix}$$

Sol^n. The CH matrix of A is

$$A - \lambda I$$

d · mp

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 & -1 \\ 1 & 1-\lambda & -2 \\ -1 & -2 & 1-\lambda \end{bmatrix}$$

The CH. eqn of A is

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} 2-\lambda & 1 & -1 \\ 1 & 1-\lambda & -2 \\ -1 & -2 & 1-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)\left[(1-\lambda)^2 - 4\right] - 1\left[(1-\lambda) - 2\right] - 1\left[-2-(-1+\lambda)\right]$$

$$(2-\lambda)\left[1 - 2\lambda + \lambda^2 - 4\right] - 1\left[-\lambda - 1\right] - 1\left[-2 + 1 - \lambda\right] = 0$$

$$(2-\lambda)\left[\lambda^2 - 2\lambda - 3\right] + \lambda + 1 + 1 + \lambda = 0$$

$$2\lambda^2 - 4\lambda - 6 - \lambda^3 + 2\lambda^2 + 3\lambda + 2\lambda + 2 = 0$$

$$-\lambda^3 + 4\lambda^2 + \lambda - 4 = 0$$

$$\lambda^3 - 4\lambda^2 - \lambda + 4 = 0$$

$$\lambda^3 - \lambda^2 - 3\lambda^2 + 3\lambda - 4\lambda + 4 = 0$$

$$\lambda^2(\lambda - 1) - 3\lambda(\lambda - 1) - 4(\lambda - 1) = 0$$

$$(\lambda - 1)(\lambda^2 - 3\lambda - 4) = 0$$

$$\lambda^2 - 4\lambda + \lambda - 4 = 0$$

$$\lambda(\lambda - 4) + 1(\lambda - 4) = 0$$

$$(\lambda + 1)(\lambda - 4) = 0$$

$$\boxed{\lambda = -1, 1, 4}$$

$\lambda = -1, 1, 4$.

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U.M.S

The CH roots of the eqⁿ is $-1, 1, 4$.

Case ①

if $\lambda = -1$, then

$$A - \lambda I = 0$$

$$\begin{bmatrix} 3 & 1 & -1 \\ 1 & 2 & -2 \\ -1 & -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & 5 & -5 \\ 0 & -5 & 5 \end{bmatrix} \begin{matrix} R_2 \to 3R_2 - R_1 \\ R_3 \to 3R_3 + R_1 \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & -1 \\ 0 & 5 & -5 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 + R_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$3x + y - z = 0$$

$$5y - 5z = 0$$

$\rho(A) = 2$, $n = 3$, $n - r = 3 - 2 = 1$ L.I.S

Let,

$$z = K$$

$$5y - 5K = 0$$

$$5y = 5K$$

$$y = K$$

$$3x + K - K = 0$$

$$x = 0$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ K \\ K \end{bmatrix} = K \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}.$$

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Case (2)

if $\lambda = 1$ then

$$A - \lambda I = 0$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 0 & -2 \\ -1 & -2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix} \begin{matrix} \\ R_2 \to R_2 - R_1 \\ R_3 \to R_3 + R_1 \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - R_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x + y - z = 0$

$-y - z = 0$

$\rho(A) = 2, \quad n = 3, \quad n - r = 3 - 2 = 1$ L.I.S

Let, $z = k_1$

$-y - k_1 = 0$

$-y = k_1$

$y = -k_1$

$x - k_1 - k_1 = 0$

$x - 2k_1 = 0$

$x = 2k_1$

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$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2k_1 \\ -k_1 \\ k_1 \end{bmatrix} = k_1 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Case (3) if $\lambda = 4$ then

$$A - \lambda I = 0$$

$$\begin{bmatrix} -2 & 1 & -1 \\ 1 & -3 & -2 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\begin{bmatrix} -2 & 1 & -1 \\ 0 & -5 & -5 \\ 0 & -5 & -5 \end{bmatrix} \begin{matrix} R_2 \to 2R_2 + R_1 \\ R_3 \to 2R_3 - R_1 \end{matrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 1 & -1 \\ 0 & -5 & -5 \\ 0 & 0 & 0 \end{bmatrix} R_3 \to R_3 - R_2 \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$-2x + y - z = 0$$

$$-5y - 5z = 0$$

$\rho(A) = 2$, $n = 3$, $n - r = 3 - 2 = 1$ L.I.S

let

$$z = k_1$$

$$-5y - 5k_1 = 0$$

$$-5y = 5k_1$$

$$y = -k_1$$

$$-2x - k_1 - k_1 = 0$$

$$-2x - 2k_1 = 0$$

$$-2x = 2k_1$$

$$x = -k_1$$

$$X_3 = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -k_1 \\ -k_1 \\ k_1 \end{bmatrix} = k_1 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} = k_1 \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

eign vector

$B = [x_1, \cancel{x_2}, x_3]$

$$= \begin{bmatrix} 0 & 2 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}$$

$\frac{0}{\sqrt{0^2+1^2+1^2}}$ $\quad \frac{2}{\sqrt{4+1+1}}$ $\quad \frac{1}{\sqrt{1^2+1^2}}$

$B^T = B^{-1}$

We observed that eigen vectors are pair-wise orthogonal.

$$B = \begin{bmatrix} 0 & \frac{2}{\sqrt6} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & \frac{-1}{\sqrt6} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt6} & \frac{-1}{\sqrt3} \end{bmatrix}$$

$\frac{0}{\sqrt{2}}$ $\frac{1}{\sqrt2}$ $\quad \frac{2}{\sqrt6}$ $\frac{1}{\sqrt6}$

$$B^{-1} = B^T = \begin{bmatrix} 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \frac{2}{\sqrt6} & \frac{-1}{\sqrt6} & \frac{1}{\sqrt6} \\ \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & \frac{-1}{\sqrt3} \end{bmatrix}$$

$$B^{-1}AB = \begin{bmatrix} 0 & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} \\ \frac{2}{\sqrt6} & \frac{-1}{\sqrt6} & \frac{1}{\sqrt6} \\ \frac{1}{\sqrt3} & \frac{1}{\sqrt3} & \frac{-1}{\sqrt3} \end{bmatrix} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{bmatrix} \begin{bmatrix} 0 & \frac{2}{\sqrt6} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & \frac{-1}{\sqrt6} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt6} & \frac{-1}{\sqrt3} \end{bmatrix}$$

$$= \begin{bmatrix} 0+\frac{1}{\sqrt2}-\frac{1}{\sqrt2} & 0+\frac{1}{\sqrt2}-\frac{2}{\sqrt2} & 0-\frac{2}{\sqrt2}+\frac{1}{\sqrt2} \\ \frac{4}{\sqrt6}-\frac{1}{\sqrt6}-\frac{1}{\sqrt6} & \frac{2}{\sqrt6}-\frac{1}{\sqrt6}-\frac{2}{\sqrt6} & \frac{-2}{\sqrt6}+\frac{2}{\sqrt6}+\frac{1}{\sqrt6} \\ \frac{2}{\sqrt3}+\frac{1}{\sqrt3}+\frac{1}{\sqrt3} & \frac{1}{\sqrt3}+\frac{1}{\sqrt3}+\frac{2}{\sqrt3} & \frac{-1}{\sqrt3}-\frac{2}{\sqrt3}-\frac{1}{\sqrt3} \end{bmatrix} \begin{bmatrix} 0 & \frac{2}{\sqrt6} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & \frac{-1}{\sqrt6} & \frac{1}{\sqrt3} \\ \frac{1}{\sqrt2} & \frac{1}{\sqrt6} & \frac{1}{\sqrt3} \end{bmatrix}$$

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$$\begin{bmatrix} 0-\frac{1}{2}-\frac{1}{2} & 0+\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}} & 0-\frac{1}{\sqrt{6}}+\frac{1}{\sqrt{6}} \\ 0-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}} & \frac{4}{6}+\frac{1}{6}+\frac{1}{6} & \frac{2}{\sqrt{18}}-\frac{1}{\sqrt{18}}-\frac{1}{\sqrt{18}} \\ 0+\frac{4}{\sqrt{6}}-\frac{4}{\sqrt{6}} & \frac{8}{\sqrt{18}}-\frac{4}{\sqrt{18}}-\frac{4}{\sqrt{18}} & \frac{4}{3}+\frac{4}{3}+\frac{4}{3} \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} = D(-1, 1, 4)$$

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Recall that if A is a symmetric $n \times n$ matrix, then A has real eigenvalues $\lambda_1, \ldots, \lambda_n$ (possibly repeated), and $\mathbb{R}^n$ has an orthonormal basis $v_1, \ldots, v_n$, where each vector v_i is an eigenvector of A with eigenvalue λ_i. Then

$$A = PDP^{-1}$$

where P is the matrix whose columns are $v_1, \ldots, v_n$, and D is the diagonal matrix whose diagonal entries are $\lambda_1, \ldots, \lambda_n$. Since the vectors $v_1, \ldots, v_n$ are orthonormal, the matrix P is orthogonal, i.e. $P^T P = I$, so we can alternately write the above equation as

$$A = PDP^T. \tag{1}$$

A singular value decomposition (SVD) is a generalization of this where A is an $m \times n$ matrix which does not have to be symmetric or even square.

1 Singular values

Let A be an $m \times n$ matrix. Before explaining what a singular value decomposition is, we first need to define the singular values of A.

Consider the matrix $A^T A$. This is a symmetric $n \times n$ matrix, so its eigenvalues are real.

Lemma 1.1. *If λ is an eigenvalue of $A^T A$, then $\lambda \geq 0$.*

Proof. Let x be an eigenvector of $A^T A$ with eigenvalue λ. We compute that

$$\|Ax\|^2 = (Ax) \cdot (Ax) = (Ax)^T Ax = x^T A^T Ax = x^T(\lambda x) = \lambda x^T x = \lambda \|x\|^2.$$

Since $\|Ax\|^2 \geq 0$, it follows from the above equation that $\lambda \|x\|^2 \geq 0$. Since $\|x\|^2 > 0$ (as our convention is that eigenvectors are nonzero), we deduce that $\lambda \geq 0$. □

Let $\lambda_1, \ldots, \lambda_n$ denote the eigenvalues of $A^T A$, with repetitions. Order these so that $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n \geq 0$. Let $\sigma_i = \sqrt{\lambda_i}$, so that $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$.

Definition 1.2. The numbers $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$ defined above are called the **singular values** of A.

Proposition 1.3. *The number of nonzero singular values of A equals the rank of A.*

Proof. The rank of any square matrix equals the number of nonzero eigenvalues (with repetitions), so the number of nonzero singular values of A equals the rank of A^TA. By a previous homework problem, A^TA and A have the same kernel. It then follows from the "rank-nullity" theorem that A^TA and A have the same rank. $\square$

Remark 1.4. In particular, if A is an $m \times n$ matrix with $m < n$, then A has at most m nonzero singular values, because $\operatorname{rank}(A) \leq m$.

The singular values of A have the following geometric significance.

Proposition 1.5. *Let A be an $m \times n$ matrix. Then the maximum value of $\|Ax\|$, where x ranges over unit vectors in $\mathbf{R}^n$, is the largest singular value σ_1, and this is achieved when x is an eigenvector of A^TA with eigenvalue σ_1^2.*

Proof. Let $v_1, \ldots, v_n$ be an orthonormal basis for $\mathbf{R}^n$ consisting of eigenvectors of A^TA with eigenvalues σ_i^2. If $x \in \mathbf{R}^n$, then we can expand x in this basis as

$$x = c_1v_1 + \cdots + c_nv_n \tag{2}$$

for scalars $c_1, \ldots, c_n$. Since x is a unit vector, $\|x\|^2 = 1$, which (since the vectors $v_1, \ldots, v_n$ are orthonormal) means that

$$c_1^2 + \cdots + c_n^2 = 1.$$

On the other hand,

$$\|Ax\|^2 = (Ax) \cdot (Ax) = (Ax)^T(Ax) = x^TA^TAx = x \cdot (A^TAx).$$

By (2), since v_i is an eigenvalue of A^TA with eigenvalue σ_i^2, we have

$$A^TAx = c_1\sigma_1^2v_1 + \cdots + c_n\sigma_n^2v_n.$$

Taking the dot prodoct with (2), and using the fact that the vectors $v_1, \ldots, v_n$ are orthonormal, we get

$$\|Ax\|^2 = x \cdot (A^TAx) = \sigma_1^2c_1^2 + \cdots + \sigma_n^2c_n^2.$$

Since σ_1 is the largest singular value, we get

$$\|Ax\|^2 \leq \sigma_1^2(c_1^2 + \cdots + c_n^2).$$

Equality holds when $c_1 = 1$ and $c_2 = \cdots = c_n = 0$. Thus the maximum value of $\|Ax\|^2$ for a unit vector x is σ_1^2, which is achieved when $x = v_1$. $\square$

One can similarly show that σ_2 is the maximum of $\|Ax\|$ where x ranges over unit vectors that are orthogonal to v_1 (exercise). Likewise, σ_3 is the maximum of $\|Ax\|$ where x ranges over unit vectors that are orthogonal to v_1 and v_2; and so forth.

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2 Definition of singular value decomposition

Let A be an $m \times n$ matrix with singular values $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$. Let r denote the number of nonzero singular values of A, or equivalently the rank of A.

Definition 2.1. A singular value decomposition of A is a factorization

$$A = U\Sigma V^T$$

where:

- U is an $m \times m$ orthogonal matrix.
- V is an $n \times n$ orthogonal matrix.
- Σ is an $m \times n$ matrix whose i^{th} diagonal entry equals the i^{th} singular value σ_i for $i = 1, \ldots, r$. All other entries of Σ are zero.

Example 2.2. If $m = n$ and A is symmetric, let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of A, ordered so that $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_n|$. The singular values of A are given by $\sigma_i = |\lambda_i|$ (exercise). Let $v_1, \ldots, v_n$ be orthonormal eigenvectors of A with $Av_i = \lambda_i v_i$. We can then take V to be the matrix whose columns are $v_1, \ldots, v_n$. (This is the matrix P in equation (1).) The matrix Σ is the diagonal matrix with diagonal entries $|\lambda_1|, \ldots, |\lambda_n|$. (This is almost the same as the matrix D in equation (1), except for the absolute value signs.) Then U must be the matrix whose columns are $\pm v_1, \ldots, \pm v_n$, where the sign next to v_i is $+$ when $\lambda_i \geq 0$, and $-$ when $\lambda_i < 0$. (This is almost the same as P, except we have changed the signs of some of the columns.)

3 How to find a SVD

Let A be an $m \times n$ matrix with singular values $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$, and let r denote the number of nonzero singular values. We now explain how to find a SVD of A.

Let $v_1, \ldots, v_n$ be an orthonormal basis of $\mathbb{R}^n$, where v_i is an eigenvector of $A^T A$ with eigenvalue σ_i^2.

Lemma 3.1. *(a)* $\|Av_i\| = \sigma_i$.

(b) If $i \neq j$ then Av_i and Av_j are orthogonal.

Proof. We compute

$$(Av_i)\cdot(Av_j) = (Av_i)^T(Av_j) = v_i^T A^T A v_j = v_i^T \sigma_j^2 v_j = \sigma_j^2 (v_i \cdot v_j).$$

If $i = j$, then since $\|v_i\| = 1$, this calculation tells us that $\|Av_i\|^2 = \sigma_j^2$, which proves (a). If $i \neq j$, then since $v_i \cdot v_j = 0$, this calculation shows that $(Av_i)\cdot(Av_j) = 0$. □

Theorem 3.2. *Let A be an $m \times n$ matrix. Then A has a (not unique) singular value decomposition $A = U\Sigma V^T$, where U and V are as follows:*

- *The columns of V are orthonormal eigenvectors $v_1, \ldots, v_n$ of $A^T A$, where $A^T A v_i = \sigma_i^2 v_i$.*
- *If $i \leq r$, so that $\sigma_i \neq 0$, then the i^{th} column of U is $\sigma_i^{-1} A v_i$. By Lemma 3.1, these columns are orthonormal, and the remaining columns of U are obtained by arbitrarily extending to an orthonormal basis for $\mathbb{R}^m$.*

Proof. We just have to check that if U and V are defined as above, then $A = U\Sigma V^T$. If $x \in \mathbb{R}^n$, then the components of $V^T x$ are the dot products of the rows of V^T with x, so

$$V^T x = \begin{pmatrix} v_1 \cdot x \\ v_2 \cdot x \\ \vdots \\ v_n \cdot x \end{pmatrix}.$$

Then

$$\Sigma V^T x = \begin{pmatrix} \sigma_1 v_1 \cdot x \\ \sigma_2 v_2 \cdot x \\ \vdots \\ \sigma_r v_r \cdot x \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

When we multiply on the left by U, we get the sum of the columns of U, weighted by the components of the above vector, so that

$$\begin{aligned} U\Sigma V^T x &= (\sigma_1 v_1 \cdot x)\sigma_1^{-1} A v_1 + \cdots + (\sigma_r v_r \cdot x)\sigma_r^{-1} A v_r \\ &= (v_1 \cdot x) A v_1 + \cdots + (v_r \cdot x) A v_r. \end{aligned}$$

Since $Av_i = 0$ for $i > r$ by Lemma 3.1(a), we can rewrite the above as

$$\begin{aligned} U\Sigma V^T x &= (v_1 \cdot x)Av_1 + \cdots + (v_n \cdot x)Av_n \\ &= Av_1 v_1^T x + \cdots + Av_n v_n^T x \\ &= A(v_1 v_1^T + \cdots v_n v_n^T)x \\ &= Ax. \end{aligned}$$

In the last line, we have used the fact that if $\{v_1, \ldots, v_n\}$ is an orthonormal basis for $\mathbb{R}^n$, then $v_1 v_1^T + \cdots + v_n v_n^T = I$ (exercise). □

Example 3.3. (from Lay's book) *Find a singular value decomposition of*

$$A = \begin{pmatrix} 4 & 11 & 14 \\ 8 & 7 & -2 \end{pmatrix}.$$

Step 1. We first need to find the eigenvalues of $A^T A$. We compute that

$$A^T A = \begin{pmatrix} 80 & 100 & 40 \\ 100 & 170 & 140 \\ 40 & 140 & 200 \end{pmatrix}.$$

We know that at least one of the eigenvalues is 0, because this matrix can have rank at most 2. In fact, we can compute that the eigenvalues are $\lambda_1 = 360$, $\lambda_2 = 90$, and $\lambda_3 = 0$. Thus the singular values of A are $\sigma_1 = \sqrt{360} = 6\sqrt{10}$, $\sigma_2 = \sqrt{90} = 3\sqrt{10}$, and $\sigma_3 = 0$. The matrix Σ in a singular value decomposition of A has to be a 2×3 matrix, so it must be

$$\Sigma = \begin{pmatrix} 6\sqrt{10} & 0 & 0 \\ 0 & 3\sqrt{10} & 0 \end{pmatrix}.$$

Step 2. To find a matrix V that we can use, we need to solve for an orthonormal basis of eigenvectors of $A^T A$. One possibility is

$$v_1 = \begin{pmatrix} 1/3 \\ 2/3 \\ 2/3 \end{pmatrix}, \quad v_2 = \begin{pmatrix} -2/3 \\ -1/3 \\ 2/3 \end{pmatrix}, \quad v_3 = \begin{pmatrix} 2/3 \\ -2/3 \\ 1/3 \end{pmatrix}.$$

(There are seven other possibilities in which some of the above vectors are multiplied by -1.) Then V is the matrix with v_1, v_2, v_3 as columns, that is

$$V = \begin{pmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & -1/3 & -2/3 \\ 2/3 & 2/3 & 1/3 \end{pmatrix}.$$

Step 3. We now find the matrix U. The first column of U is

$$\sigma_1^{-1}Av_1 = \frac{1}{6\sqrt{10}}\begin{pmatrix}18\\6\end{pmatrix} = \begin{pmatrix}3/\sqrt{10}\\1/\sqrt{10}\end{pmatrix}.$$

The second column of U is

$$\sigma_2^{-1}Av_2 = \frac{1}{3\sqrt{10}}\begin{pmatrix}3\\9\end{pmatrix} = \begin{pmatrix}1/\sqrt{10}\\-3/\sqrt{10}\end{pmatrix}.$$

Since U is a 2×2 matrix, we do not need any more columns. (If A had only one nonzero singular value, then we would need to add another column to U to make it an orthogonal matrix.) Thus

$$U = \begin{pmatrix}3/\sqrt{10} & 1/\sqrt{10}\\1/\sqrt{10} & -3/\sqrt{10}\end{pmatrix}.$$

To conclude, we have found the singular value decomposition

$$\begin{pmatrix}4 & 11 & 14\\8 & 7 & -2\end{pmatrix} = \begin{pmatrix}3/\sqrt{10} & 1/\sqrt{10}\\1/\sqrt{10} & -3/\sqrt{10}\end{pmatrix}\begin{pmatrix}6\sqrt{10} & 0 & 0\\0 & 3\sqrt{10} & 0\end{pmatrix}\begin{pmatrix}1/3 & -2/3 & 2/3\\2/3 & -1/3 & -2/3\\2/3 & 2/3 & 1/3\end{pmatrix}^T.$$

4 Applications

Singular values and singular value decompositions are important in analyzing data.

One simple example of this is "rank estimation". Suppose that we have n data points $v_1,\ldots,v_n$, all of which live in $\mathbb{R}^m$, where n is much larger than m. Let A be the $m\times n$ matrix with columns $v_1,\ldots,v_n$. Suppose the data points satisfy some linear relations, so that $v_1,\ldots,v_n$ all lie in an r-dimensional subspace of $\mathbb{R}^m$. Then we would expect the matrix A to have rank r. However if the data points are obtained from measurements with errors, then the matrix A will probably have full rank m. But only r of the singular values of A will be large, and the other singular values will be close to zero. Thus one can compute an "approximate rank" of A by counting the number of singular values which are much larger than the others, and one expects the measured matrix A to be close to a matrix A' such that the rank of A' is the "approximate rank" of A.

For example, consider the matrix

$$A' = \begin{pmatrix}1 & 2 & -2 & 3\\-4 & 0 & 1 & 2\\3 & -2 & 1 & -5\end{pmatrix}$$

The matrix A' has rank 2, because all of its columns are points in the subspace $x_1+x_2+x_3=0$ (but the columns do not all lie in a 1-dimensional subspace). Now suppose we perturb A' to the matrix

$$A = \begin{pmatrix} 1.01 & 2.01 & -2 & 2.99 \\ -4.01 & 0.01 & 1.01 & 2.02 \\ 3.01 & -1.99 & 1 & -4.98 \end{pmatrix}$$

This matrix now has rank 3. But the eigenvalues of $A^T A$ are

$$\sigma_1^2 \approx 58.604, \quad \sigma_2^2 \approx 19.3973, \quad \sigma_3^2 \approx 0.00029, \quad \sigma_4^2 = 0.$$

Since two of the singular values are much larger than the others, this suggests that A is close to a rank 2 matrix.

For more discussion of how SVD is used to analyze data, see e.g. Lay's book.

5 Exercises (some from Lay's book)

1. (a) Find a singular value decomposition of the matrix $A = \begin{pmatrix} 2 & -1 \\ 2 & 2 \end{pmatrix}$.

   (b) Find a unit vector x for which $\|Ax\|$ is maximized.

2. Find a singular value decomposition of $A = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}$.

3. (a) Show that if A is an $n \times n$ symmetric matrix, then the singular values of A are the absolute values of the eigenvalues of A.

   (b) Give an example to show that if A is a 2×2 matrix which is not symmetric, then the singular values of A might not equal the absolute values of the eigenvalues of A.

4. Let A be an $m \times n$ matrix with singular values $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$. Let v_1 be an eigenvector of $A^T A$ with eigenvalue σ_1^2. Show that σ_2 is the maximum value of $\|Ax\|$ where x ranges over unit vectors in $\mathbb{R}^n$ that are orthogonal to v_1.

5. Show that if $\{v_1, \ldots, v_n\}$ is an orthonormal basis for $\mathbb{R}^n$, then
$$v_1 v_1^T + \cdots + v_n v_n^T = I.$$

6. Let A be an $m \times n$ matrix, and let P be an orthogonal $m \times m$ matrix. Show that PA has the same singular values as A.

Date. 6/9/18

Bisection Method

1 Find the approximate root of the equation $x^3-x-1=0$ by using bi-section method.

Soln. Given

$f(x) = x^3 - x - 1$

$x=0;\ f(0) = 0-0-1 = -1$ -ve

$x=1;\ f(1) = 1-1-1 = -1$ -ve

$x=2;\ f(2) = 2^3-2-1 = 8-3 = 5$ +ve

the root lies between 1 and 2

$x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$

S.No	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
1	1	2	1.5 (+ve)
2	1	1.5	1.25 (-ve)
3	1.25	1.5	1.375 (+ve)
4	1.25	1.375	1.3125 (-ve)
5	1.3125	1.375	1.3438 (+ve)
6	1.3125	1.3438	1.3282 (+ve)
7	1.3125	1.3282	1.3204 (-ve)
8	1.3204	1.3282	1.3243 (-ve)
9	1.3243	1.3282	1.3263 (+ve)
10	1.3243	1.3263	1.3253 (+ve)
11	1.3243	1.3253	1.3248 (+ve)
12	1.3243	1.3248	1.3246 (-ve)
13	1.3246	1.3248	1.3247 (-ve)
14	1.3247	1.3248	1.3248 (+ve)
15	1.3247	1.3248	1.3248 (

$x_{14} = x_{15} = 1.3248$

2 find the approximate root of the equation $\cos x - xe^x$ by bisection method

Solu] consider

$$f(x) = \cos x - xe^x$$

$x=0$ $f(0) = \cos 0 - 0e^0$

$= 1$ +ve

$x=1$ $f(1) = \cos(1) - 1e^1$

$= 0.540302305 - 2.718281828$

$= -2.177949523$ -ve

$[x_0 =$

S.NO	a(+ve)	b(-ve)	$x_n = \frac{a+b}{2}$
1	0	1	0.5 (+ve)
2	0.5	1	0.75 (-ve)]]

$x=2$. $f(2) = \cos(2) - 2e^2$

$= -0.416146 - 2(7.3905)$

$= -0.416146 - 14.7811$ -ve

$= -15.194256$

$x=3$, $f(3) = \cos(3) - 3e^3$

$= -0.989992496 - 3(20.08553)$

$= -0.989992496 - 60.2566$

$= -61.24659$ -ve

3. Find the root of the equation $x^3-5x+1=0$ by using bisection method

Sol:/ Given

$f(x) = x^3-5x+1=0$

$x=0,\ f(0) = 0-5(0)+1 = 1$ +ve

$x=1,\ f(1) = 1-5+1 = -3$ -ve

$x=2,\ f(2) = 8-5(2)+1 = -1$ -ve

$x=3,\ f(3) = 27-15+1 = 13$ +ve

$x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5$

The root lies between 2 and 3

S.NO	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
1.	2	3	2.5 (+ve)
2	2	2.5	2.25 (+ve)
3.	2	2.25	2.125 (-ve)
4.	2.125	2.25	2.1875 (+ve)
5.	2.125	2.1875	2.15625 (+ve) 2.1563
6.	2.125	2.1563	2.1407 (+ve)
7.	2.125	2.1407	2.1329 (+ve)
8.	2.125	2.1329	2.129 (+ve)
9	2.125	2.129	2.127 (-ve)
10.	2.127	2.129	2.128 (-ve)
11.	2.128	2.129	2.1285 (+ve)
12.	2.128	2.1285	2.1283 (-ve)
13.	2.1283	2.1285	2.1284 (-ve)
14.	2.1284	2.1285	2.1285 (+ve)
15.	2.1284		2.1285

$x_{13} = x_{14} = 2.1285$

$x_{14} = x_{15} = 2.1285$

4. Find the real root of the equation $x\log_{10}^{x} = 1.2$ by using bisection method

Solu] Given

$$f(x) = x\log_{10}^{x} - 1.2$$

$x=0, \ f(0) = 0\log_{10}^{0} - 1.2 = -1.2$ (-ve)

$x=1, \ f(1) = 1\log_{10}^{1} - 1.2$

$= 0 - 1.2 = -1.2$ (-ve)

$x=2, \ f(2) = 2\log_{10}^{2} - 1.2$

$= 2(0.3010) - 1.2$

$= 0.602 - 1.2$

$= -0.598$ (-ve)

$x=3, \ f(3) = 3\log_{10}^{3} - 1.2$ (+ve)

$= 3(0.477) - 1.2$

$= 1.43130 - 1.2$

$= 0.23136$

$$x_0 = \frac{2+3}{2} = 2.5$$

S.NO	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
1	2	3	2.5 (-ve)
2.	2.5	3	2.75 (+ve)
3.	2.5	2.75	2.625 (-ve)
4.	2.625	2.75	2.6875 (-ve)
5.	2.6875	2.75	2.7188 (-ve)
6.	2.7188	2.75	2.7344 (-ve)
7.	2.7344	2.75	2.7422 (+ve)
8.	2.7344	2.7422	2.7383 (-ve)

9.	2.7383	2.7422	2.7403 (-ve)
10.	2.7403	2.7422	2.7413 (+ve)
11.	2.7403	2.7413	2.7408 (+ve)
[12.	2.7408	2.7413	2.7411 (+ve)
13	2.7408	2.7411	2.741 (+ve)
14.	2.7408	2.741	2.7409 (+ve)
15.	2.7408	2.7409	2.7409 (+ve)
16.	2.7408	2.7409	2.7409]
12.	2.7403	2.7408	2.7406 (-ve)
13.	2.7406	2.7408	2.7407 (+ve)
14.	2.7406	2.7407	2.7407 (+ve)

$x_{13} = x_{14} = 2.7407$

5. Find the approximate root of the equation $x - \cos x = 0$ by using bi-section method

Solu) Given

$$f(x) = x - \cos x = 0$$

$x=0,\ f(0) = 0 - \cos 0 = -1$ -ve

$x=1,\ f(1) = 1 - \cos(1) = 1 - 0.5403$ +ve

$= 0.4597$

$$x_0 = \frac{0+1}{2} = 0.5$$

S.NO	a(-ve)	b(+ve)	$x_n = \frac{a+b}{2}$
1.	0	1	0.5 (-ve)
2.	0.5	1	0.75 (+ve)
3.	0.5	0.75	0.625 (-ve)
4.	0.625	0.75	0.6875 (-ve)
5.	0.6875	0.75	0.7188 (-ve)

6	0.7188	0.75	0.7344 (-ve)
7.	0.7344	0.75	0.7422 (+ve)
8.	0.7344	0.7422	0.7383 (-ve)
9.	0.7383	0.7422	0.7403 (+ve)
10.	0.7383	0.7403	0.7393 (+ve)
11.	0.7383	0.7393	0.7388 (-ve)
12.	0.7388	0.7393	0.7391 (+ve)
13	0.7388	0.7391	0.739 (-ve)
14.	0.739	0.7391	0.7391 (+ve)
15.	0.739	0.7391	0.7391 (+ve)

$$x_{14} = x_{15} = 0.7391$$

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Iterative method

1. Find the approximate root of the equation $x^3 - x - 1 = 0$ by using iterative method.

Soln Given

$$f(x) = x^3 - x - 1$$

$x = 0$, $f(0) = 0 - 0 - 1 = -1$ -ve

$x = 1$, $f(1) = 1 - 1 - 1 = -1$ -ve

$x = 2$, $f(2) = 8 - 2 - 1 = 5$ +ve

$\therefore$ The root lies between 1 and 2

$$x_0 = \frac{1+2}{2} = 1.5$$

$$x^3 - x - 1 = 0 \Rightarrow x^3 = 1 + x$$

$$x = \sqrt[3]{1+x} = \phi(x)$$

By iterative method.

$$x_1 = \sqrt[3]{1 + x_0}, \quad x_0 = 1.5$$

$$x_1 = \sqrt[3]{1 + 1.5}$$

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$x_1 = \sqrt[3]{2.5}$

$x_1 = 1.3572$

$x_2 = \sqrt[3]{1+x_1}$

$= \sqrt[3]{1+1.3572}$

$x_2 = 1.3309$

$x_3 = \sqrt[3]{1+x_2}$

$= \sqrt[3]{1+1.3309}$

$= \sqrt[3]{2.3309}$

$x_3 = 1.3259$

$x_4 = \sqrt[3]{1+x_3}$

$= \sqrt[3]{1+1.3259}$

$= \sqrt[3]{2.3259}$

$x_4 = 1.3249$

$x_5 = \sqrt[3]{1+x_4}$

$= \sqrt[3]{1+1.3249}$

$= \sqrt[3]{2.3249}$

$x_5 = 1.3248$

$x_6 = \sqrt[3]{1+x_5}$

$= \sqrt[3]{1+1.3247}$

$= \sqrt[3]{2.3247}$

$x_6 = 1.3247$

$\cancel{x_5} = x_7 = 1.3247$

$x_7 = \sqrt[3]{1+x_6}$

$= \sqrt[3]{1+1.3247}$

$$= \sqrt[3]{2.3247}$$

$$x_7 = 1.3247$$

$$x_6 = x_7 = 1.3247$$

2. find the approximate root of the equation $x^3 - 5x + 1 = 0$

solu/

$$f(x) = x^3 - 5x + 1$$

$x = 0$, $f(0) = 0 - 5(0) + 1 = 1$ +ve

$x = 1$, $f(1) = 1 - 5 + 1 = -3$ -ve

$x = 2$, $f(2) = 8 - 10 + 1 = -1$ -ve

$x = 3$, $f(3) = 27 - 15 + 1 = 13$ +ve

$$\begin{array}{r} 28 \\ 15 \\ \hline 13 \end{array}$$

The root lies between 2 and 3

$$x_0 = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

$$x^3 - 5x + 1 = 0 \Rightarrow x^3 = 5x - 1$$

$$x = \sqrt[3]{5x-1} = \phi(x)$$

By iterative method

$$x_1 = \sqrt[3]{5x_0 - 1}$$

$$x_1 = \sqrt[3]{5(2.5) - 1}$$

$$x_1 = \cancel{3.274} \sqrt[3]{12.5 - 1}$$

$$x_1 = \sqrt[3]{11.5}$$

$$x_1 = 2.2572$$

$$x_2 = \sqrt[3]{5x_1 - 1}$$

$$= \sqrt[3]{5(2.2572) - 1}$$

$$= \sqrt[3]{11.286 - 1}$$

$$= \sqrt[3]{10.286}$$

$$x_2 = 2.1748$$

$x_3 = \sqrt[3]{5x_2 - 1}$

$= \sqrt[3]{5(2.1748) - 1}$

$= \sqrt[3]{10.874 - 1}$

$= \sqrt[3]{9.874}$

$x_3 = 2.1453$

$x_4 = \sqrt[3]{5x_3 - 1}$

$= \sqrt[3]{5(2.1453) - 1}$

$= \sqrt[3]{10.7265 - 1}$

$= \sqrt[3]{9.7265}$

$x_4 = 2.1346$

$x_5 = \sqrt[3]{5x_4 - 1}$

$= \sqrt[3]{5(2.1346) - 1}$

$= \sqrt[3]{10.673 - 1}$

$= \sqrt[3]{9.673}$

$x_5 = 2.1307$

$x_6 = \sqrt[3]{5x_5 - 1}$

$= \sqrt[3]{5(2.1307) - 1}$

$= \sqrt[3]{10.6535 - 1}$

$= \sqrt[3]{9.6535}$

$x_6 = 2.1293$

$x_7 = \sqrt[3]{5x_6 - 1}$

$= \sqrt[3]{5(2.1293) - 1}$

$= \sqrt[3]{9.6465}$

$= 2.1287$

$$x_8 = \sqrt[3]{5x_7 - 1}$$

$$= \sqrt[3]{5(2.1287) - 1}$$

$$= \sqrt[3]{9.6435}$$

$$x_8 = 2.1285$$

$$x_9 = \sqrt[3]{5x_8 - 1}$$

$$= \sqrt[3]{5(2.1285) - 1}$$

$$= \sqrt[3]{9.6425}$$

$$x_9 = 2.1284$$

$$x_{10} = \sqrt[3]{5x_9 - 1}$$

$$= \sqrt[3]{5(2.1284) - 1}$$

$$= \sqrt[3]{9.642}$$

$$x_{10} = 2.1284$$

$$x_9 = x_{10} = 2.1284$$

3. find the approximate root of the equation $\cos x = 3x - 1$

Solu) $f(x) = \cos x - 3x + 1$

$x = 0$, $f(0) = \cos 0 - 3(0) + 1$ +ve

$= 1 + 1 = 2$

$x = 1$, $f(1) = \cos(1) - 3 + 1$ -ve

$= 0.54030 - 3 + 1$

$= -1.459697$

The root lies between 0 and 1

$$x_0 = \frac{1+0}{2} = \frac{1}{2} = 0.5$$

$$\cos x - 3x + 1 = 0 \Rightarrow \cos x + 1 = 3x$$

$$x = \frac{1 + \cos x}{3} = \phi(x)$$

By iterative method

$$x_1 = \frac{1+\cos x_0}{3}$$

$$x_1 = \frac{1+\cos(0.5)}{3}$$

$$x_1 = 0.6259$$

$$x_2 = \frac{1+\cos x_1}{3}$$

$$= \frac{1+\cos(0.6259)}{3}$$

$$= \frac{1+0.810436203}{3}$$

$$x_2 = 0.6035$$

$$x_3 = \frac{1+\cos x_2}{3}$$

$$= \frac{1+\cos(0.6035)}{3}$$

$$= \frac{1+0.8223 54315}{3}$$

$$x_3 = 0.6078$$

$$x_4 = \frac{1+\cos x_3}{3}$$

$$= \frac{1+\cos(0.6078)}{3}$$

$$= \frac{1+0.820906341}{3}$$

$$x_4 = 0.607$$

$$x_5 = \frac{1+\cos x_4}{3}$$

$$= \frac{1+\cos(0.607)}{3}$$

$$= \frac{1+0.821362929}{3}$$

$$x_5 = 0.6071$$

$$x_6 = \frac{1+\cos x_5}{3}$$

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$$= \frac{1+\cos(0.6071)}{3}$$

$$x_6 = 0.6071$$

$$x_5 = x_6 = 0.6071$$

4. Find the approximate value of $x^3+x^2\pm 1=0$

$$f(x) = x^3+x^2-1$$

$x=0$, $f(0) = 0+0-1 = -1$ $-ve$

$x=1$, $f(1) = 1+1-1 = +1$ $+ve$

The root lies between 0 and 1

$$x_0 = \frac{0+1}{2} = \frac{1}{2} = 0.5$$

$[x^3+x^2\pm 1=0$

$x^2(x+1\pm 1)=0$

$x^2(\cdot\ x^3+x^2]$

$$x^3+x^2-1=0$$

$$x^3+x^2=1$$

$$x^2(x+1)=1$$

$$x^2=\frac{1}{1+x}$$

By iteration method

$$x = \frac{1}{\sqrt{1+x}} = \phi(x)$$

$$x_1 = \frac{1}{\sqrt{1+x_0}} = \frac{1}{\sqrt{1+0.5}}$$

$$= \frac{1}{\sqrt{1.5}} = \frac{1}{1.224744871} = 0.8165$$

$$x_2 = \frac{1}{\sqrt{1+x_1}} = \frac{1}{\sqrt{1+0.8165}} = \frac{1}{\sqrt{1.8165}} = 0.742$$

$$x_3 = \frac{1}{\sqrt{1+x_2}} = \frac{1}{\sqrt{1+0.742}} = \frac{1}{\sqrt{1.742}} = 0.7577$$

$$x_4 = \frac{1}{\sqrt{1+x_3}} = \frac{1}{\sqrt{1+0.7577}} = \frac{1}{\sqrt{1.7577}} = 0.7543$$

$$x_5 = \frac{1}{\sqrt{1+x_4}} = \frac{1}{\sqrt{1+0.7543}} = \frac{1}{\sqrt{1.7543}} = 0.7550$$

$$x_6 = \frac{1}{\sqrt{1+x_5}} = \frac{1}{\sqrt{1+0.7550}} = \frac{1}{\sqrt{1.7550}} = 0.755$$

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5. Find a root near 3.8 for the equation $2x - \log_{10}^{x} = 7$ correct to 4 decimal places by the iterative method

solu) $f(x) = 2x - \log_{10}^{x} - 7$

Given $x_0 = 3.8$

$$2x - \log_{10}^{x} = 7$$

$$2x = \log_{10}^{x} + 7$$

$$x = \frac{1}{2}[\log_{10}^{x} + 7] = \phi(x)$$

By iterative method

$$x_1 = \frac{1}{2}\left[\log_{10}^{x_0} + 7\right]$$

$$x_1 = \frac{1}{2}\left[\log_{10}^{(3.8)} + 7\right]$$

$$= 3.789891798$$

$$x_1 = 3.7899$$

$$x_2 = \frac{1}{2}\left[\log_{10}^{x_1} + 7\right]$$

$$= \frac{1}{2}\left[\log_{10}^{(3.7899)} + 7\right]$$

$$= 3.789313875$$

$$x_2 = 3.7893$$

$$x_3 = \frac{1}{2}\left[\log_{10}^{x_2} + 7\right]$$

$$= \frac{1}{2}\left[\log_{10}^{(3.7893)} + 7\right]$$

$$= 3.789279495$$

$$x_3 = 3.7893$$

$$x_2 = x_3 = 3.7893$$

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6. Find the approximate root of the equation $\tan x = x$ by using iterative method

$f(x) = \tan x - x$

$x=0,\ f(0) = \tan 0 - 0 = 0$ +ve

$x=1,\ f(1) = \tan 1 - 1 = 0.557407724$ +ve

$x=2,\ f(2) = \tan 2 - 2 = -4.185039$ -ve

The root lies between 1 ∧ 2

$x_0 = \frac{1+2}{2} = \frac{3}{2} = 1.5$

$\tan x = x = \phi(x)$

$[x_1 = \tan x_0 = \tan(1.5)$

$= 14.10141995$

$= 14.1014$

$x_2 = \tan x_1]$

$x = \tan^{-1}(x)$

$x_1 = \tan^{-1} x_0$

$= \tan^{-1}(1.5)$

$= 0.982793723$

$x_1 = 0.9828$

$x_2 = \tan^{-1}(x_1)$

$= \tan^{-1}(0.9828)$

$= 0.776723779)$

$x_2 = 0.7767$

$x_3 = \tan^{-1}(x_2)$

$= \tan^{-1}(0.7767)$

$= 0.660371299$

$x_3 = 0.6604$

$x_4 = \tan^{-1}(x_3)$

$= \tan^{-1}(0.6604)$

$= 0.583651584$

$x_4 = 0.5837$

$x_5 = \tan^{-1}(x_4)$

$= \tan^{-1}(0.5837)$

$= 0.528347979$

$x_5 = 0.5283$

$x_6 = \tan^{-1}(x_5)$

$= \tan^{-1}(0.5283)$

$= 0.486030454$

$x_6 = 0.4860$

$x_7 = \tan^{-1}(x_6)$

$= \tan^{-1}(0.4860)$

$= 0.452385012$

$x_7 = 0.4524$

$x_8 = \tan^{-1}(x_7)$

$= \tan^{-1}(0.4524)$

$= 0.424847974$

$x_8 = 0.4248$

$x_9 = \tan^{-1}(x_8)$

$= \tan^{-1}(0.4248)$

$= 0.401701233$

$x_9 = 0.4017$

$x_{10} = \tan^{-1}(x_9)$

$= \tan^{-1}(0.4017)$

$= 0.381971034$

$x_{10} = 0.382$

$x_{11} = \tan^{-1}(x_{10})$

$= \tan^{-1}(0.382)$

$= 0.364893489$

$= 0.3649$

$x_{12} = \tan^{-1}(x_{11})$

$= \tan^{-1}(0.3649)$

$= 0.349886608$

$x_{12} = 0.3499$

$x_{13} = \tan^{-1}(x_{12})$

$= \tan^{-1}(0.3499)$

$= 0.336585729$

$x_{13} = 0.3366$

$x_{14} = \tan^{-1}(0.3366) = \tan^{-1}(x_{13})$

$= 0.324687667$

$x_{14} = 0.3247$

$x_{15} = \tan^{-1}(x_{14})$

$= \tan^{-1}(0.3247)$

$= 0.313960535$

$x_{15} = 0.3140$

$x_{16} = \tan^{-1}(x_{15})$

$= \tan^{-1}(0.3140)$

$= 0.304250832$

$x_{16} = 0.3043$

$x_{17} = \tan^{-1}(x_{16})$

$= \tan^{-1}(0.3043)$

$= 0.295397064$

$x_{17} = 0.2954$

$x_{18} = \tan^{-1}(x_{17})$

$= \tan^{-1}(0.2954)$

$= 0.287231286$

$x_{18} = 0.2872$

$x_{19} = \tan^{-1}(x_{18})$

$= \tan^{-1}(0.2872)$

$= 0.279672704$

$x_{19} = 0.2797$

$x_{20} = \tan^{-1}(x_{19})$

$= \tan^{-1}(0.2797)$

$= 0.272730491$

$x_{20} = 0.2727$

$x_{21} = \tan^{-1}(x_{20})$

$= \tan^{-1}(0.2727)$

$= 0.266226664$

$x_{21} = 0.2662$

$x_{22} = \tan^{-1}(x_{21})$

$= \tan^{-1}(0.2662)$

$= 0.260166056$

$x_{22} = 0.2602$

$x_{23} = \tan^{-1}(x_{22})$

$= \tan^{-1}(0.2602)$

$= 0.254555385$

$x_{23} = 0.2546$

$x_{24} = \tan^{-1}(x_{23})$

$= \tan^{-1}(0.2546)$

$= 0.249303367$

$x_{24} = 0.2493$

$x_{25} = \tan^{-1}(x_{24})$

$= \tan^{-1}(0.2493)$

$= 0.244319731$

$x_{25} = 0.2443$

$x_{26} = \tan^{-1}(x_{25})$

$= \tan^{-1}(0.2443)$

$= 0.239606804$

$x_{26} = 0.2396$

$x_{27} = \tan^{-1}(x_{26})$

$= \tan^{-1}(0.2396)$

$x_{27} = 0.2352$

$x_{28} = \tan^{-1}(x_{27})$

$= \tan^{-1}(0.2352)$

$x_{28} = 0.231$

$x_{29} = \tan^{-1}(x_{28})$

$= \tan^{-1}(0.231)$

$x_{29} = 0.2270$

$x_{30} = \tan^{-1}(x_{29})$

$= \tan^{-1}(0.2270)$

$x_{30} = 0.2232$

$x_{31} = \tan^{-1}(x_{30})$

$= \tan^{-1}(0.2232)$

$x_{31} = 0.2196$

$x_{32} = \tan^{-1}(x_{31})$

$= \tan^{-1}(0.2196)$

$x_{32} = 0.2162$

$x_{33} = \tan^{-1}(x_{32})$

$= \tan^{-1}(0.2162)$

$x_{42} = \tan^{-1}(x_{41})$

$= \tan^{-1}(0.1911)$

$x_{42} = 0.1888$

$x_{43} = \tan^{-1}(x_{42})$

$= \tan^{-1}(0.1888)$

$x_{43} = 0.1866$

$x_{44} = \tan^{-1}(x_{43})$

$= \tan^{-1}(0.1866)$

$x_{44} = 0.1845$

$x_{45} = \tan^{-1}(x_{44})$

$= \tan^{-1}(0.1845)$

$x_{45} = 0.1824$

$x_{46} = \tan^{-1}(x_{45})$

$= \tan^{-1}(0.1824)$

$x_{46} = 0.1804$

$x_{47} = \tan^{-1}(x_{46})$

$= \tan^{-1}(0.1804)$

$x_{47} = 0.1785$

$x_{48} = \tan^{-1}(x_{47})$

$= \tan^{-1}(0.1785)$

$x_{48} = 0.1766$

$x_{49} = \tan^{-1}(x_{48})$

$= \tan^{-1}(0.1766)$

$x_{49} = 0.1748$

$= 0.2129$

$x_{34} = \tan^{-1}(x_{33})$

$= \tan^{-1}(0.2129)$

$x_{34} = 0.2098$

$x_{35} = \tan^{-1}(x_{34})$

$= \tan^{-1}(0.2098)$

$x_{35} = 0.2068$

$x_{36} = \tan^{-1}(x_{35})$

$x = \tan^{-1}(0.2068)$

$x_{36} = 0.2037$

$x_{37} = \tan^{-1}(x_{36})$

$= \tan^{-1}(0.2037)$

$x_{37} = 0.2011$

$x_{38} = \tan^{-1}(x_{37})$

$= \tan^{-1}(0.2011)$

$x_{38} = 0.1985$

$x_{39} = \tan^{-1}(x_{38})$

$= \tan^{-1}(0.1985)$

$x_{39} = 0.1960$

$x_{40} = \tan^{-1}(39)$

$= \tan^{-1}(0.1960)$

$x_{40} = 0.1935$

$x_{41} = \tan^{-1}(x_{40})$

$= \tan^{-1}(0.1935)$

$x_{41} = 0.1911$

$x_{50} = \tan^{-1}(x_{49})$

$= \tan^{-1}(0.1748)$

$x_{50} = 0.1731$

$x_{51} = \tan^{-1}(x_{50})$

$= \tan^{-1}(0.1731)$

$x_{51} = 0.1714$

$x_{52} = \tan^{-1}(x_{51})$

$= \tan^{-1}(0.1714)$

$x_{52} = 0.1698$

$x_{53} = \tan^{-1}(x_{52})$

$= \tan^{-1}(0.1698)$

$x_{53} = 0.1682$

$x_{54} = \tan^{-1}(x_{53})$

$= \tan^{-1}(0.1682)$

$x_{54} = 0.1666$

$x_{55} = \tan^{-1}(x_{54})$

$= \tan^{-1}(0.1666)$

$x_{55} = 0.1651$

$x_{56} = \tan^{-1}(x_{55})$

$= \tan^{-1}(0.1651)$

$x_{56} = 0.1636$

$x_{57} = \tan^{-1}(x_{56})$

$= \tan^{-1}(0.1636)$

$x_{57} = 0.1622$

$x_{58} = \tan^{-1}(x_{57})$

$= \tan^{-1}(0.1622)$

$x_{58} = 0.1608$

$x_{59} = \tan^{-1}(x_{58})$

$= \tan^{-1}(0.1608)$

$x_{59} = 0.1594$

$x_{60} = \tan^{-1}(x_{59})$

$= \tan^{-1}(0.1594)$

$x_{60} = 0.1587$

$x_{61} = \tan^{-1}(x_{61})$

$= \tan^{-1}(0.1587)$

$x_{61} = 0.1568$

$x_{62} = \tan^{-1}(x_{61})$

$= \tan^{-1}(0.1568)$

$x_{62} = 0.1555$

$x_{63} = \tan^{-1}(x_{62})$

$= \tan^{-1}(0.1555)$

$x_{63} = 0.1543$

$x_{64} = \tan^{-1}(x_{63})$

$= \tan^{-1}(0.1543)$

$x_{64} = 0.1531$

$x_{65} = \tan^{-1}(x_{64})$

$= \tan^{-1}(0.1531)$

$x_{65} = 0.1519$

$x_{66} = \tan^{-1}(x_{65})$

$= \tan^{-1}(0.1519)$

$x_{66} = 0.1507$

$x_{67} = \tan^{-1}(x_{66})$

$= \tan^{-1}(1507)$

$x_{67} = 0.1496$

$x_{68} = \tan^{-1}(x_{67})$

$= \tan^{-1}(0.1496)$

$x_{68} = 0.1485$

$x_{69} = \tan^{-1}(x_{68})$

$= \tan^{-1}(0.1485)$

$x_{69} = 0.1474$

$x_{70} = \tan^{-1}(x_{69})$

$= \tan^{-1}(0.1474)$

$x_{70} = 0.1463$

$x_{71} = \tan^{-1}(x_{70})$

$= \tan^{-1}(0.1463)$

$x_{71} = 0.1453$

$x_{72} = \tan^{-1}(x_{71})$

$= \tan^{-1}(0.1453)$

$x_{72} = 0.1443$

$x_{73} = \tan^{-1}(x_{72})$

$= \tan^{-1}(0.1443)$

$x_{74} = 0.1433$

$x_{74} = \tan^{-1}(x_{74})$

$= \tan^{-1}(0.1433)$

$x_{74} = 0.1423$

$x_{75} = \tan^{-1}(x_{74})$

$= \tan^{-1}(0.1423)$

$x_{75} = 0.1414$

$x_{76} = \tan^{-1}(x_{76})$

$= \tan^{-1}(0.1414)$

$x_{76} = 0.1405$

$x_{77} = \tan^{-1}(x_{76})$

$= \tan^{-1}(0.1405)$

$x_{77} = 0.1396$

$x_{78} = \tan^{-1}(x_{77})$

$= \tan^{-1}(0.1396)$

$x_{78} = 0.1387$

$x_{79} = \tan^{-1}(x_{78})$

$= \tan^{-1}(0.1387)$

$x_{79} = 0.1378$

$x_{80} = \tan^{-1}(x_{79})$

$= \tan^{-1}(0.1378)$

$x_{80} = 0.1369$

$x_{81} = \tan^{-1}(x_{80})$

$= \tan^{-1}(0.1369)$

$x_{81} = 0.1361$

$x_{82} = \tan^{-1}(x_{81})$

$= \tan^{-1}(0.1361)$

$x_{82} = 0.1353$

$x_{83} = \tan^{-1}(x_{82})$

$= \tan^{-1}(0.1353)$

$x_{83} = 0.1345$

$x_{84} = \tan^{-1}(x_{83})$

$= \tan^{-1}(0.1345)$

$= 0.1337$

$x_{85} = \tan^{-1}(x_{84})$

$= \tan^{-1}(0.1337)$

$x_{85} = 0.1329$

$x_{86} = \tan^{-1}(x_{85})$

$= \tan^{-1}(0.1329)$

$x_{86} = 0.1321$

$x_{87} = \tan^{-1}(x_{86})$

$= \tan^{-1}(0.1321)$

$x_{87} = 0.1313$

$x_{88} = \tan^{-1}(x_{87})$

$= \tan^{-1}(0.1313)$

$x_{88} = 0.1306$

$x_{89} = \tan^{-1}(x_{88})$

$= \tan^{-1}(0.1306)$

$x_{89} = 0.1299$

$x_{90} = \tan^{-1}(x_{89})$

$= \tan^{-1}(0.1299)$

$x_{90} = 0.1292$

$x_{91} = \tan^{-1}(x_{90})$

$= \tan^{-1}(0.1292)$

$x_{91} = 0.1285$

$x_{92} = \tan^{-1}(x_{91})$

$= \tan^{-1}(0.1285)$

$x_{92} = 0.1278$

$x_{93} = \tan^{-1}(92)$

$= \tan^{-1}(0.1278)$

$x_{93} = 0.1271.$

$x_{94} = \tan^{-1}(x_{93})$

$= \tan^{-1}(0.1271)$

$x_{94} = 0.1264$

$x_{95} = \tan^{-1}(x_{94})$

$= \tan^{-1}(0.1264)$

$x_{95} = 0.1257$

$x_{96} = \tan^{-1}(x_{95})$

$= \tan^{-1}(0.1257)$

$x_{96} = 0.1250$

$x_{97} = \tan^{-1}(x_{96})$

$= \tan^{-1}(0.1250)$

$x_{97} = 0.1244$

$x_{98} = \tan^{-1}(x_{97})$

$= \tan^{-1}(0.1244)$

$x_{98} = 0.1238$

$x_{99} = \tan^{-1}(98)$

$= \tan^{-1}(0.1238)$

$x_{99} = 0.1232$

$x_{100} = \tan^{-1}(x_{99})$

$= \tan^{-1}(0.1232)$

$x_{100} = 0.1226$

Date 13/8/18 1. Solutions of Algebraic & Transcendental Equations.

Since the given equation having trignometric functions or logarithmic functions or exponent functions, that type of equations are called 'transcendental' equations

Ex:

1. $x = e^{-x}$ 3. $x = \sin x + 1$

2. $x + 1 = \log x$

In the given linear equation having 'x' is called algebraic equation.

Ex:

1. $x^2 + x + 1 = 0$

2. $x^3 - 2x^2 + x + 1 = 0$

⇒ Newton - Rathson Method (or) Newton's Method.

Consider $f(x) = 0$ be the given curve and x takes the values $x_0, x_1, x_2 \cdots x_n$, and h is the common difference then $x_1 = x_0 + h \rightarrow ①$

By taylor's Series

$$f(x+h) = f(x) + h.f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots$$

Since 'h' is very small quantity and $h^2 . h^3 . h^4, \ldots$ are very small [negligible]

∴ In the above equation we eliminate the product of h^2, $h^3 . h^4, \ldots$ terms. then $f(x+h) = f(x) + h.f'(x)$

If. $x = x_1$ is the solution of the given equation $f(x_1) = 0$

$$\Rightarrow f(x_0 + h) = 0$$

$$\Rightarrow f(x_0 + h) = f(x_0) + hf'(x_0)$$

$$\Rightarrow h.f'(x_0) = -f(x_0)$$

$$\text{then } h = \frac{-f(x_0)}{f'(x_0)} \rightarrow ②$$

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From ① ∧ ②

$$x_1 = x_0 + \left[\frac{-f(x_0)}{f'(x_0)} \right]$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$ similarly:

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_0)} \; ; \; x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$\boxed{\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

The above equation is called "Newton's Formulae".

Geometrical Representation of Newton's formulae

Consider the curve $y = f(x)$ be passing through the points (x_0, y_0), (x_1, y_1).

The slope of the curve $m = \frac{dy}{dx} = f'(x)$

It passing

At (x_0, y_0) $m = f'(x_0) \longrightarrow$ ①

The given line (or) curve passing through (x_0, y_0) and slope $m = f'(x_0)$ then equation to the line

$$y - y_0 = m(x - x_0)$$

$$\Rightarrow y - y_0 = f'(x_0)(x - x_0)$$

it intersect x-axis then it's y-co-ordinate is zero.

$$\therefore 0 - y_0 = f'(x_0)(x_1 - x_0)$$

$$-y_0 = f'(x_0)(x_1 - x_0)$$

$$x_1 - x_0 = \frac{-y_0}{f'(x_0)}$$

$$x_1 = x_0 - \frac{y_0}{f'(x_0)}, \quad y_0 = f(x_0)$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

Similarly $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$; $x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

1. Using Newtons Rathson method, find the real root of the situation $3x = \cos x + 1$ correct to four decimal places

Solu

$$3x = \cos x + 1$$

$$f(x) = 3x - \cos x - 1$$

$x = 0 \Rightarrow f(0) = 3(0) - \cos 0 - 1$

$= 0 - 1 - 1 = -2$ -ve

$x = 1 \Rightarrow f(1) = 3(1) - \cos 1 - 1$

$= 3 - 0.9998 - 1$ +ve

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$$f(x) = 3x - \cos x - 1$$

$$f'(x) = \frac{d}{dx}(3x - \cos x - 1)$$

$$= 3 - (-\sin x)$$

$$= 3 + \sin x$$

By Newton's Method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{(3x_0 - \cos x_0 - 1)}{3 + \sin x_0}$$

$$= \frac{x_0(3 + \sin x_0) - (3x_0 - \cos x_0 - 1)}{3 + \sin x_0}$$

$$= \frac{\cancel{3x_0} + x_0 \sin x_0 - \cancel{3x_0} + \cos x_0 + 1}{3 + \sin x_0}$$

$$x_1 = \frac{x_0 \sin x_0 + \cos x_0 + 1}{3 + \sin x_0}$$

$$x_1 = \frac{0.5 \sin(0.5) + \cos(0.5) + 1}{3 + \sin(0.5)}$$

$$= \frac{2.11729533}{3.4794255386}$$

$$= 0.6085186498$$

$$x_1 = \cancel{0.6662}\ 0.6085$$

$$x_2 = \frac{x_1 \sin x_1 + \cos x_1 + 1}{3 + \sin x_1}$$

$$= \frac{(0.6085)\sin(0.6085) + \cos(0.6085) + 1}{3 + \sin(0.6085)}$$

$$= \frac{2.1956929118}{3.6146956091} \quad \left[= \frac{0.6085[0.010620128] + 0.99994360 4 + 1}{3 + 0.010620128} \right.$$

$$= \frac{0.0064623484 07 + 0.999943604 + 1}{3 + 0.010620128}$$

$$\left. = \frac{2.006405952}{3.010620128} \right]$$

$$x_2 = 0.6071087$$

$$x_3 = \frac{x_2 \sin x_2 + \cos x_2 + 1}{3 + \sin x_2}$$

$$= \frac{(0.6071)\sin(0.6071) + \cos(0.6071) + 1}{3 + \sin(0.6071)}$$

$$= \frac{0.6071 [\overset{0.570488075}{\cancel{0.0105956956 2}}] + 0.821305884 + 1}{3 + 0.570488075}$$

$$= \frac{0.34634331 + 1 + 0.821305884}{3.570488075} = \frac{2.167649194}{3.570488075}$$

$$= 0.607101647$$

$x_2 = x_3 = 0.6071$

The approximate root of the given equation is 0.6071

2. find the real root of the equation $x = e^{-x}$ by using Newton Rathson method

Solu)

$x_1 = x_0 - f$

$x = 0 \Rightarrow f(0) = 0 - e^{-0} = -1 \quad -ve$

$x = 1 \Rightarrow f(1) = 1 - e^{-1} = 0.6321 \quad +ve$

$$x_0 = \frac{a+b}{2} = \frac{0+1}{2} = 0.5$$

$f(x) = x - e^{-x}$

$$f'(x) = \frac{d}{dx}[x - e^{-x}]$$

$$= 1 - e^{-x}(-1)$$

$$= 1 + e^{-x}$$

By Newton's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{(x_0 - e^{-x_0})}{1 + e^{-x_0}}$$

$$= \frac{x_0(1 + e^{-x_0}) - (x_0 - e^{-x_0})}{1 + e^{-x_0}}$$

$$= \frac{\cancel{x_0} + x_0 e^{-x_0} - \cancel{x_0} + e^{-x_0}}{1 + e^{-x_0}}$$

$$x_1 = \frac{e^{-x_0}(x_0 + 1)}{1 + e^{-x_0}} \qquad x_0 = 0.5$$

$$x_1 = \frac{e^{-0.5}(0.5+1)}{1 + e^{-0.5}}$$

$$= \frac{0.606530659(1.5)}{1 + 0.606530659}$$

$$= \frac{0.909795988}{1.606530659}$$

$$= 0.5663110031$$

$$x_1 = 0.5663$$

$$x_2 = \frac{e^{-x_1}(x_1+1)}{1+e^{-x_1}}$$

$$= \frac{e^{-0.5663}(0.5663+1)}{1+e^{-0.5663}}$$

$$= \frac{0.5676217586(1.5663)}{1+0.5676217586}$$

$$= \frac{0.8890659605}{1.567621759}$$

$$= 0.5671$$

$$x_3 = \frac{e^{-x_2}(x_2+1)}{1+e^{-x_2}}$$

$$= \frac{e^{-0.5671}(0.5671+1)}{1+e^{-0.5671}}$$

$$= \frac{0.5671678428(1.5671)}{1+0.5671678428}$$

$$= \frac{0.8888087265}{1.567167843}$$

$$= 0.56714329$$

$$= 0.5671$$

$$x_2 = x_3 = 0.5671$$

The approximate roots of the given Equation 0.5671

Date 18/8/18

3\. Find the approximate root of the equation $x^3-5x+3=0$ by using Newtons method.

Solu) Given

$$x^3-5x+3=0$$

$$f(x) = x^3-5x+3$$

$x=0 \Rightarrow 0-5(0)+3 = 3$ +ve

$x=1 \Rightarrow 1-5(1)+3 = -1$ -ve

$x=2 \Rightarrow 2^3-5(2)+3 = 8-10+3 = 1$ +ve

$$x_0 = \frac{a+b}{2} = \frac{1+2}{2} = \frac{3}{2} = 1.5 \quad ; a=1, b=2$$

root lies between 1 and 2

$$f(x) = x^3-5x+3$$

$$f'(x) = 3x^2-5$$

By Newton's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{[x_0^3-5x_0+3]}{3x_0^2-5}$$

$$= \frac{x_0(3x_0^2-5)-(x_0^3-5x_0+3)}{3x_0^2-5}$$

$$= \frac{3x_0^3-\cancel{5x_0}-x_0^3+\cancel{5x_0}-3}{3x_0^2-5}$$

$$x_1 = \frac{2x_0^3-3}{3x_0^2-5} \qquad x_0 = 1.5$$

$$x_1 = \frac{2(1.5)^3-3}{3(1.5)^2-5} = \frac{2(3.375)-3}{3(2.25)-5} = \frac{6.75-3}{6.75-5}$$

$$= \frac{3.75}{1.75} = 2.142857143$$

$$x_1 = 2.1429$$

$$x_2 = \frac{2x_1^3 - 3}{3x_1^2 - 5} \qquad x_1 = 2.1429$$

$$x_2 = \frac{2(2.1429)^3 - 3}{3(2.1429)^2 - 5} = \frac{2(9.840240537) - 3}{3(4.59202041) - 5}$$

$$= \frac{19.68048107 - 3}{13.77606123 - 5} = \frac{16.68048107}{8.77606123}$$

$$= 1.900679659$$

$$x_2 = 1.9007$$

$$x_3 = \frac{2x_2^3 - 3}{3x_2^2 - 5} \qquad x_2 = 1.9007$$

$$= \frac{2(1.9007)^3 - 3}{3(1.9007)^2 - 5} = \frac{2(6.866583793) - 3}{3(3.61266049) - 5}$$

$$= \frac{13.73316759 - 3}{10.83798147 - 5} = \frac{10.73316759}{5.83798147}$$

$$= 1.838506622$$

$$x_3 = 1.8385$$

$$x_4 = \frac{2x_3^3 - 3}{3x_3^2 - 5}$$

$$= \frac{2(1.8385)^3 - 3}{3(1.8385)^2 - 5} = \frac{2(6.214281217) - 3}{3(3.38008225) - 5}$$

$$= \frac{12.428562433 - 3}{10.14024675 - 5} = \frac{9.428562433}{5.14024675}$$

$$= 1.834262613$$

$$x_4 = 1.8343$$

$$x_5 = \frac{2x_4^3 - 3}{3x_4^2 - 5} = \frac{2(1.8343)^3 - 3}{3(1.8343)^2 - 5}$$

$$= \frac{2(6.1717894) - 3}{3(3.36465649) - 5} = \frac{12.3435788 - 3}{10.09396947 - 5}$$

$= 9.4265$

$= \frac{9.3435788}{5.09396947}$

$= 1.834243188$

$= 1.8342$

$$x_6 = \frac{2x_5^3 - 3}{3x_5^2 - 5}$$

$$= \frac{2(1.8342)^3 - 3}{3(1.8342)^2 - 5}$$

$$= \frac{2(6.170780058) - 3}{3(3.36428964) - 5}$$

$$= \frac{12.34156012 - 3}{10.09286892 - 5}$$

$$= \frac{9.341560116}{5.09286892}$$

$= 1.834243186$

$= 1.8342$

The approximate roots $x_5 = x_6 = 1.8342$

4. Find the real root of the equation $x^3 - 2x - 5 = 0$ by using Newton's method.

Solu] Given Equation :

$$x^3 - 2x - 5 = 0$$

$$f(x) = x^3 - 2x - 5$$

$x = 0 \Rightarrow 0 - 2(0) - 5 = -5$ -ve

$x = 1 \Rightarrow 1 - 2(1) - 5 = -6$ -ve

$x = 2 \Rightarrow 2^3 - 2(2) - 5 = 8 - 4 - 5 = -1$ -ve

$x = 3 \Rightarrow 3^3 - 2(3) - 5 = 27 - 6 - 5 = 16$ +ve

$$x_0 = \frac{a+b}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

$$f(x) = x^3 - 2x - 5$$

$$f'(x) = 3x^2 - 2$$

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$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{[x_0^3 - 2x_0 - 5]}{(3x_0^2 - 2)}$$

$$= \frac{x_0(3x_0^2 - 2) - [x_0^3 - 2x_0 - 5]}{3x_0^2 - 2}$$

$$= \frac{3x_0^3 - \cancel{2x_0} - x_0^3 + \cancel{2x_0} + 5}{3x_0^2 - 2}$$

$$x_1 = \frac{2x_0^3 + 5}{3x_0^2 - 2}$$

$$x_1 = \frac{2(2.5)^3 + 5}{3(2.5)^2 - 2}$$

$$= \frac{2(15.625) + 5}{3(6.25) - 2} = \frac{31.25 + 5}{18.75 - 2}$$

$$= \frac{36.25}{16.75} = 2.164179104$$

$$x_1 = 2.1642$$

$$x_2 = \frac{2x_1^3 + 5}{3x_1^2 - 2} = \frac{2(2.1642)^3 + 5}{3(2.1642)^2 - 2}$$

$$= \frac{2(10.13659694) + 5}{3(4.68376164) - 2}$$

$$= \frac{20.27319388 + 5}{14.05128492 - 2}$$

$$= \frac{25.27319388}{12.05128492}$$

$$= 2.097136865$$

$$= 2.0971$$

$$x_3 = \frac{2x_2^3+5}{3x_2^2-2}$$

$$= \frac{2(2.0971)^3+5}{3(2.0971)^2-2}$$

$$= \frac{2(9.222685959)+5}{3(4.39782841)-2}$$

$$= \frac{18.44537192+5}{13.19348523-2}$$

$$= \frac{23.44537192}{11.19348523}$$

$$= 2.094555131$$

$$x_3 = 2.0946$$

$$x_4 = \frac{2x_3^3+5}{3x_3^2-2}$$

$$= \frac{2(2.0946)^3+5}{3(2.0946)^2-2}$$

$$= \frac{2(9.189741551)+5}{3(4.38734916)-2}$$

$$= \frac{18.379483 1+5}{13.16204748-2}$$

$$= \frac{23.3794831}{11.16204748}$$

$$= 2.094551483$$

$$x_4 = 2.0946$$

The approximate roots are $x_3 = x_4 = 2.0946$

H.W 5. find the real root of the equation $x^4-x-10=0$ which is near to $x=2$.

Solu) Given that

$x^4-x-10=0$ $\quad f(x) = x^4-x-10=0$

$x_0 = 2$ $\quad f'(x) = 4x^3-1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{(x_0^4 - x_0 - 10)}{(4x_0^3 - 1)}$$

$$= \frac{x_0(4x_0^3 - 1) - (x_0^4 - x_0 - 10)}{4x_0^3 - 1}$$

$$= \frac{4x_0^4 - \cancel{x_0} - x_0^4 + \cancel{x_0} + 10}{4x_0^3 - 1}$$

$$x_1 = \frac{3x_0^4 + 10}{4x_0^3 - 1} \qquad x_0 = 2$$

$$x_1 = \frac{3(2)^4 + 10}{4(2)^3 - 1} = \frac{3(16) + 10}{4(8) - 1} = \frac{48 + 10}{32 - 1} = \frac{58}{31}$$

$$= 1.870967742$$

$$x_1 = 1.879$$

$$x_2 = \frac{3x_1^4 + 10}{4x_1^3 - 1} = \frac{3(1.871)^4 + 10}{4(1.871)^3 - 1}$$

$$= \frac{3(12.25448741) + 10}{4(6.549699311) - 1}$$

$$= \frac{36.76346223 + 10}{26.19879724 - 1}$$

$$= \frac{46.76346223}{25.19879724}$$

$$= 1.85578152$$

$$x_2 = 1.8558$$

$$x_3 = \frac{3x_2^4 + 10}{4x_2^3 - 1} = \frac{3(1.8558)^4 + 10}{4(1.8558)^3 - 1}$$

$$= \frac{3(11.86109219) + 10}{4(6.391363397) - 1} = \frac{35.58327658 + 10}{25.56545359 - 1}$$

$$= \frac{45.58327658}{24.56545359}$$

$$= 1.855584568$$

$$x_3 = 1.8556$$

$$x_4 = \frac{3x_3^4+10}{4x_3^3-1} = \frac{3(1.8556)^4+10}{4(1.8556)^3-1}$$

$$= \frac{3(11.85597993)+10}{4(6.389297224)-1}$$

$$= \frac{35.56793978+10}{25.55718889-1}$$

$$= \frac{45.56793978}{24.55718889}$$

$$= 1.855584529$$

$$x_4 = 1.8556$$

The approximate roots are $x_3 = x_4 = 1.8556$

Date 17/8/18 Logarithm functions

6. find the real root of the equation $x\log_{10}^{x} = 1.2$

Solu) Given

$$f(x) = x\log_{10}^{x} - 1.2$$

Put $x=0$

$x=0$, $f(0) = 0\log_{10}^{0} - 1.2 = -1.2$ -ve

$x=1$, $f(1) = 1\log_{10}^{1} - 1.2 = -1.2$ -ve

$x=2$, $f(2) = 2\log_{10}^{2} - 1.2 = 2(0.3010) - 1.2$ -ve

$= 0.602 - 1.2$

$= -0.598$

$x=3$ $f(3) = 3\log_{10}^{3} - 1.2 = 3(0.4717) - 1.2$ +ve

$= 1.4151 - 1.2$

$= 0.2151$

the roots are 2 and 3

$$x_0 = \frac{a+b}{2} = \frac{2+3}{2} = \frac{5}{2} = 2.5$$

$$f(x) = x\log_{10}^{x} - 1.2$$

$$f(x) = x \cdot \frac{\log x}{\log_{10}} - 1.2$$

$$= \frac{x\log x - 1.2\log 10}{\log 10}$$

$$f'(x) = \frac{\left[x\frac{1}{x} + \log x \cdot 1 - 0\right]}{\log 10}$$

$$f'(x) = \frac{1+\log x}{\log 10}$$

By Newton's method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{\left(\frac{x_0\log x_0 - 1.2\log 10}{\log 10}\right)}{\frac{1+\log x_0}{\log 10}}$$

$$= x_0 - \frac{(x_0\log x_0 - 1.2\log 10)}{1+\log x_0}$$

$$= \frac{x_0(1+\log x_0) - (x_0\log x_0 - 1.2\log 10)}{1+\log x_0}$$

$$= \frac{x_0 + \cancel{x_0\log x_0} - \cancel{x_0\log x_0} + 1.2\log 10}{1+\log x_0}$$

$$x_1 = \frac{x_0 + 1.2\log 10}{1+\log x_0}$$

$$= \frac{(2.5) + 1.2\log 10}{1+\log(2.5)} = \frac{2.5+1.2(1)}{1+0.397940008}$$

$$= \frac{3.7}{1.397940008} =$$

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$= 2.646751634$

$x_1 = 2.6468$

$$x_2 = \frac{x_1 + 1.2\log 10}{1+\log x_1}$$

$$= \frac{2.6468+1.2}{1+\log(2.6468)}$$

$$= \frac{3.8468}{1+0.422721126}$$

$$= \frac{3.8468}{1.422721126}$$

$$= 2.703832768$$

$x_2 = 2.7038$

$$x_3 = \frac{x_2+1.2\log 10}{1+\log x_2}$$

$$= \frac{2.7038+1.2}{1+\log(2.7038)}$$

$$= \frac{3.9038}{1+0.431974563}$$

$$= \frac{3.9038}{1.431974563}$$

$$= 2.72383961$$

$$= \frac{5.263102112}{1+\log(2.5)}$$

$$= \frac{5.263102112}{1.916290731}$$

$$= 2.74650502$$

$x_1 = 2.7465$

$$x_2 = \frac{x_1+1.2\log 10}{1+\log x_1}$$

$$= \frac{2.7465+1.2(2.302585093)}{1+\log(2.7465)}$$

$$= \frac{5.509602112}{2.010327374}$$

$$= 2.740649201$$

$x_2 = 2.7407$

$$x_3 = \frac{x_2+1.2\log 10}{1+\log x_2}$$

$$= \frac{2.7407+1.2(2.302585093)}{1+\log(2.7407)}$$

$$= \frac{5.503802112}{1+1.008213362}$$

$$= \frac{5.503802112}{2.008213362}$$

$$= 2.740646097$$

$x_3 = 2.7407$

The approximate value $x_2 = x_3 = 2.7407$

7. Compute one positive root of $2x - \log_{10}^{x} = 7$

solu) Given that

$$f(x) = 2x - \log_{10}^{x} - 7$$

put $x=0$

$$f(0) = 2(0) - \log_{10}^{0} - 7 \quad -ve$$

$$= 0 - 0 - 7$$

$$= -7$$

$$F(1) = 2(1) - \log_{10}^{1} - 7 \quad -ve$$
$$= 2 - 0 - 7$$
$$= -5$$
$$F(2) = 2(2) - \log_{10}^{2} - 7 \quad -ve$$
$$= 4 - 0.3010 - 7$$
$$= -3.301$$
$$F(3) = 2(3) - \log_{10}^{3} - 7 \quad -ve$$
$$= 6 - 0.477121 - 7$$
$$= -1.4771$$
$$F(4) = 2(4) - \log_{10}^{4} - 7 \quad +ve$$
$$= 8 - 0.60205 - 7$$
$$= 0.39795$$
$$x_0 = \frac{a+b}{2} = \frac{3+4}{2} = \frac{7}{2} = 3.5$$
$$F(x) = 2x - \log_{10}^{x} - 7$$
$$F'(x) = 2x - \frac{\log x}{\log 10} - 7$$
$$= \frac{2x\log 10 - \log x - 7\log 10}{\log 10}$$
$$= \frac{2x(\log 10) - \log x - 7\log 10}{\log 10}$$
$$F'(x) = \frac{2\left[x\frac{1}{x} + \log x\right] - \log x - 7\log 10}{\log 10}$$
$$= \frac{2(1+\log x) - \log x - 7\log 10}{\log 10}$$
$$= 2 + 2\log x]$$
$$F'(x) = \frac{1}{\log 10}\left[2\log 10 - \frac{1}{x}\right]$$
$$= \frac{1}{\log 10}\left[\frac{2x\log 10 - 1}{x}\right]$$

$$f'(x) = \frac{2x\log 10 - 1}{x\log 10}$$

By Newton's Iterative method

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= x_0 - \frac{\left[\frac{2x_0\log 10 - \log x_0 - 7\log 10}{\log 10}\right]}{\left[\frac{2x_0\log 10 - 1}{x_0\log 10}\right]}$$

$$= x_0 - \frac{(2x_0\log 10 - \log x_0 - 7\log 10)\,x_0}{2x_0\log 10 - 1}$$

$$= \frac{x_0(2x_0\log 10 - 1) - (2x_0^2\log 10 - x_0\log x_0 - x_0 7\log 10)}{2x_0\log 10 - 1}$$

$$= \frac{\cancel{2x_0^2\log 10} - x_0 - \cancel{2x_0^2\log 10} + x_0\log x_0 + x_0 7\log 10}{2x_0\log 10 - 1}$$

$$x_1 = \frac{x_0[-1 + \log x_0 + 7\log 10]}{(2x_0\log 10 - 1)}$$

$\ln(3.5) = 1.252762968$

$$x_1 = 3.5\frac{[-1 + \log(3.5) + 7\log 10]}{2(3.5)\log 10 - 1}$$

$$= 3.5\frac{[-1 + 1.252762968 + 7(2.302585093)]}{7(2.302585093) - 1}$$

$$= \frac{3.5[16.37085862]}{15.11809565}$$

$$= \frac{57.29800517}{15.11809565}$$

$$= 3.790027957$$

$$= 3.7900$$

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$$x_2 = \frac{x_1\,[-1+\log x_1 + 7\log_{10}]}{2x_1\log 10 - 1}$$

$$= \frac{3.7900\,[-1+\log(3.7900) + 16.11809565]}{2(3.7900)\log 10 - 1}$$

$$= \frac{3.7900\,[-1+1.332366019+16.11809565]}{2(3.7900)(2.302585093)-1}$$

$$= \frac{(16.45046167)\,3.7900}{17.71535\,95-1}$$

$$= \frac{62.34724973}{16.453595}$$

$$= 3.789278284$$

$$x_2 = 3.7893$$

$$x_3 = \frac{x_2\,[-1+\log x_2 + 7\log 10]}{2x_2\log 10 - 1}$$

$$= \frac{(3.7893)\,[-1+\log(3.7893) + 16.11809565]}{2(3.7893)(2.302585093)-1}$$

$$= \frac{3.7893\,[-1+1.332181305) + 16.11809565)}{17.45037139-1}$$

$$= \frac{3.7893(16.45027696)}{16.45037139}$$

$$= \frac{62.33503447}{16.45037139}$$

$$= 3.789278247$$

$$x_3 = 3.7893$$

The approximate value $x_2 = x_3 = 3.7893.$

Date 13/8/18

Regula - falsi Method (or) false position Method.

Consider

$y = f(x)$ be the given curve and the given curve passing through $A(x_1, y_1)$ & $B(x_2, y_2)$ then

$$y_1 = f(x_1) \quad \& \quad y_2 = f(x_2)$$

Then the equation to the curve is

$$y - y_1 = m(x - x_1) \quad \text{where } m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

Since the ~~o-y~~$_1$ = given curve intersect at x-axis so $y = 0$

$$\therefore \; 0 - y_1 = \left[\frac{y_2 - y_1}{x_2 - x_1}\right](x - x_1)$$

$$x - x_1 = \frac{-y_1 (x_2 - x_1)}{y_2 - y_1}$$

$$x = x_1 - \frac{(x_2 - x_1) y_1}{y_2 - y_1}$$

$$x = x_1 - \left[\frac{x_2 - x_1}{f(x_2) - f(x_1)}\right] f(x_1)$$

$$= \frac{x_1 (f(x_2) - f(x_1)) - (x_2 - x_1) f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{x_1 f(x_2) - \cancel{x_1 f(x_1)} - x_2 f(x_1) + \cancel{x_1 f(x_1)}}{f(x_2) - f(x_1)}$$

$$x = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$\therefore$ If $x = x_3$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

similarly $x_4 = \dfrac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$

$x_5 = \dfrac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$

Date
3/8/18

1. find a real root of the equation $x\log_{10}^{x} - 1.2 = 0$ by False position method

solu] $x\log_{10}^{x} - 1.2 = f(x)$

$f(x) = x\log_{10}^{x} - 1.2$

$x=0,\ f(0) = 0\log_{10}^{0} - 1.2 = -1.2$

$x=1,\ f(1) = 1\log_{10}^{1} - 1.2 = 0 - 1.2 = -1.2$

$x=2,\ f(2) = 2\boxed{\log_{10}^{2}} - 1.2$

$= 2(0.3010) - 1.2$

$= 0.6030 - 1.2$

$= -0.5980$

$x=3,\ f(3) = 3\log_{10}^{3} - 1.2$

$= 3(0.477121254) - 1.2$

$= 1.431363764 - 1.2$

$= 0.231363764$

$= 0.2314$

$x_1 = 2\ ;\ f(x_1) = -0.598$

$x_2 = 3\ ;\ f(x_2) = 0.2314$

$x_3 = \dfrac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$

$= \dfrac{2(0.2314) - 3(-0.598)}{0.2314 - (-0.598)}$

$$= \frac{0.4628 + 1.794}{0.8294}$$

$$= \frac{2.2568}{0.8294}$$

$$= 2.721003135$$

$$x_3 = 2.721$$

$$f(x_3) = 2.721 \log_{10}(2.721) - 1.2$$

$$= 2.721(0.434728541) - 1.2$$

$$= 1.182896362 - 1.2$$

$$= -0.017103637$$

$$= -0.0171$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{3(-0.0171) - 2.721(0.2314)}{-0.0171 - 0.2314}$$

$$= \frac{-0.0513 - 0.6296394}{-0.2485}$$

$$= \frac{-0.6809394}{-0.2485}$$

$$= 2.740198793$$

$$x_4 = 2.7402$$

$$f(x_4) = 2.7402 \log_{10}(2.7402) - 1.2$$

$$= 2.7402(0.437782262) - 1.2$$

$$= 1.199610954 - 1.2$$

$$= -0.000389046$$

$$= -0.0004$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{2.721 \times -0.0084 - 2.7402 \times (-0.0171)}{-0.0084 - (-0.0171)}$$

$$= \frac{-0.0010884 + 0.04685742}{0.0167}$$

$$= \frac{0.04576902}{0.0167}$$

$$= 2.74065988$$

$$x_5 = 2.7407$$

$$f(x_5) = 2.7407 \log_{10}(2.7407) - 1.2$$

$$= 2.7407(0.437861499) - 1.2$$

$$= 1.200047012 - 1.2$$

$$= 0.000047012488$$

$$= 0.0001$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{2.7402 \times 0.0001 - 2.7407(-0.0004)}{0.0001 - (-0.0004)}$$

$$= \frac{0.00027402 + 0.00109628}{0.0005}$$

$$= \frac{0.0013703}{0.0005}$$

$$= 2.7406$$

$$f(x_6) = 2.7406 \log(2.7406) - 1.2$$

$$= 2.7406(0.43784565) - 1.2$$

$$= 1.199959798 - 1.2$$

$= -0.0000402023171$

$= -0.0$

$x_7 = x_6 = 2.7406$

The roots of the equation

$x_7 = x_6 = 2.7406$

2. Find the real roots of the equation

$x - e^{-x} = 0$

solu)

$x - e^{-x} = 0$

$f(x) = x - e^{-x}$

$x = 0, \; f(0) = 0 - e^{-0} = -1$

$x = 1, \; f(1) = 1 - e^{-1} = 0.6321205588$

$= 0.6321$ 0.54

$x_1 = 0, \; f(x_1) = -1$

$x_2 = 1, \; f(x_2) = 0.6321$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{0(0.6321) - (1)(-1)}{0.6321 - (-1)}$$

$$= \frac{0 + 1}{0.6321 + 1} = \frac{1}{1.6321} = 0.612707554$$

$= 0.6127$

$f(x_3) = (0.6127) \log_{10}(0.6127) - 1.2 \; e^{-0.6127}$

$[= (0.6127)(-0.212752119) - 1.2 \; 0.5418858$

$= -0.130353223 - 1.2 \; 0.5418858$

$= -1.330353224]$ 0.0708141954

$= 0.0708$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{1 \times 0.0708 - 0.6127 \times 0.6321}{0.0708 - 0.6321}$$

$$= \frac{0.0708 - 0.38728767}{-0.5613}$$

$$= \frac{-0.3164876 7}{-0.5613} = 0.563847621$$

$x_4 = 0.5639$

$f(x_4) = 0.5639 \log_{10}(0.5639) - e^{-0.5639}$

$[0.5639(-0.248797905) - (0.568985686)$

$[0.5639(-0.248797905) - 0.568985686]$

$= 0.140297138 - 0.568985686$

$= -0.00510856860$

$f(x_4) = -0.0051$

$$x_5 = \frac{x_3 f(x_4) - x_4 (f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{0.6127 \times -0.0051 - 0.5639 \cdot (0.0708)}{-0.0051 - 0.0708}$$

$$= \frac{-0.04304889}{-0.0759} = 0.567179051$$

$x_5 = 0.5672$

$f(x_5) = 0.5672 - e^{-0.5672}$

$= 0.5672 - 0.567111128$

$= 0.0000888714156$

$= 0.0001$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{0.5639 \times 0.0001 - 0.56721 \times -0.0051}{0.0001 - (-0.0051)}$$

$$= \frac{0.00294911}{0.0052}$$

$$= 0.567136538$$

$$x_6 = 0.5671$$

$$f(x_6) = 0.5671 - e^{-0.5671}$$

$$= 0.5671 - 0.567167842$$

$$= -0.000067842$$

$$= -0.0009$$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{0.5672(0.0001) - 0.5671(0.5672)}{0.0001 - 0.5672}$$

$$= \frac{0.00005672 - 0.32165912}{-0.5671}$$

$$= \frac{-0.3216024}{-0.5671}$$

$$= 0.567099982$$

$$= 0.5671$$

$$x_6 = x_7 = 0.5671$$

The real roots are $x_6 = x_7 = 0.5671$

3. $x^3 - 5x + 3 = 0$ by using false position method c

solu) Given that

$$0 = x^3 - 5x + 3 = f(x)$$

$x = 0$, $f(0) = 0 - 5(0) + 3 = 3$

$x = 1$, $f(1) = 1 - 5(1) + 3 = -1$

$x = 2$, $f(2) = 2^3 - 5(2) + 3$

$$= 8 - 10 + 3$$

$$= 1$$

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$x_1 = 1, \quad f(x_1) = -1$

$x_2 = 2, \quad f(x_2) = 1$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1(1) - 2(-1)}{1-(-1)} = \frac{1+2}{1+1} = \frac{3}{2} = 1.5$$

$x_3 = 1.5$

$$f(x_3) = (1.5)^3 - 5(1.5) + 3$$

$$= 3.375 - 7.5 + 3$$

$$= -1.125$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2(-1.125) - (1.5)(1)}{-1.125 - 1}$$

$$= \frac{-2.25 - 1.5}{-2.125}$$

$$= \frac{-3.75}{-2.125} = 1.764705882$$

$x_4 = 1.76\cancel{47}5$

$$f(x_4) = (1.765)^3 - 5(1.765) + 3$$

$$= 5.498372125 - 8.825 + 3$$

$$= -0.326627875$$

$f(x_4) = -0.327$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(\cancel{-1.125}\ 1.5)(-0.327) - (1.765)(-1.125)}{-0.327 - (-1.125)}$$

$$= \frac{-0.4905 + 1.985625}{0.798}$$

$$= \frac{1.495125}{0.798}$$

$$= 1.873590226$$

$$= 1.874$$

$$f(x_5) = (1.874)^3 - 5(1.874) + 3$$

$$= 6.581255624 - 9.37 + 3$$

$$= 0.211255624$$

$$= 0.211$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{1.765(0.211) - (1.874)(-0.327)}{0.211 - (-0.327)}$$

$$= \frac{0.372415 + 0.612798}{0.538}$$

$$= \frac{0.985213}{0.538}$$

$$= 1.831250929$$

$$x_6 = 1.831$$

$$f(x_6) = (1.831)^3 - 5(1.831) + 3$$

$$= 6.138539191 - 9.155 + 3$$

$$= -0.01646809$$

$$= -0.016$$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{1.874(-0.016) - 1.831(0.211)}{-0.016 - 0.211}$$

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$$= \frac{-0.029984 - 0.386341}{-0.227}$$

$$= \frac{-0.416325}{-0.227}$$

$$= +1.834030837$$

$$= 1.834$$

$$f(x_8) = (1.834)^3 - 5(1.834) + 3$$

$$= 6.168761704 - 9.17 + 3$$

$$= -0.001238296$$

$$= -0.001$$

$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)}$$

$$= \frac{1.831(-0.001) - (1.834)(-0.016)}{-0.001 - (-0.016)}$$

$$= \frac{-0.001831 + 0.029344}{0.015}$$

$$= \frac{0.027513}{0.015} = 1.8342 = 1.834$$

$f(x_8) = \quad x_7 = x_8 = 1.834$

The real roots are $x_7 = x_8 = 1.834$

Date 1/9/18

4 Find the real root of the Equation $\tan x + \tanh x = 0$ in the introval $[1.6, 3)$

solu Given

$$f(x) = \tan x + \tanh x$$

$$\therefore f(1.6) = \tan(1.6) + \tanh(1.6)$$

$$= -34.2325327 4 + 0.921668554$$

$$= -33.31086418$$

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$f(2) = \tan 2 + \tanh 2$

$= -2.185039863 + 0.96402758$

$= -1.221012283$

$f(2.2) = \tan(2.2) + \tanh(2.2)$

$= -1.373823057 + 0.97574313$

$= -0.398079926$

$f(2.4) = \tan(2.4) + \tanh(2.4)$

$= -0.916014289 + 0.983674857$

$= 0.067660568$

$x_1 = 2.2 \quad f(x_1) = -0.3981$

$x_2 = 2.4 \quad f(x_2) = 0.0677$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(2.2)(0.0677) - (2.4)(-0.3981)}{0.0677 - (-0.3981)}$$

$$= \frac{(2.2)(0.0677) + (2.4)(0.3981)}{0.0677 + 0.3981}$$

$$= \frac{0.14894 + 0.95544}{0.4658} = \frac{1.10438}{0.4658}$$

$= 2.37093173$

$x_3 = 2.3709$

$f(x_3) = \tan(2.3709) + \tanh(2.3709)$

$= -0.971013157 + 0.982705001$

$= 0.011691844$

$= 0.0117$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{(2.4)(0.0117) - 2.3709 \times 0.0677}{0.0117 - 0.0677}$$

$$= \frac{0.02808 - 0.16050993}{-0.056}$$

$$= \frac{-0.13242993}{-0.056} = 2.364820179$$

$x_4 = 2.3648 = 2.3645$

$f(x_4) = \tan(2.3648) + \tan h(2.3648)$

$= -0.982935408 + 0.982494568$

$= -0.000440840$

$= -0.0004$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(2.3709)(-0.0004) - (2.3648)(0.0117)}{-0.0004 - 0.0117}$$

$$= \frac{-0.00094836 - 0.02766816}{-0.0121}$$

$$= \frac{-0.02861652}{-0.0121}$$

$= 2.365001653$

$x_5 = 2.365$

$f(x_5) = \tan(2.365) + \tanh(2.365)$

$= -0.982542253 + 0.982501507$

$= \underline{0.00004074} = 0.0000$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{(2.3648)(0.0000) - (2.365)(-0.0004)}{0.0000 - (-0.0004)}$$

$$= \frac{0.000946}{0.0004}$$

$$= 2.365$$

$x_5 = x_6 = 2.365$ are real roots.

5 Find the root of the given Equation $xe^x = \cos x$ in introval $(0,1)$

solu) Given $f(x) = xe^x - \cos x$

$x = 0.5$. $f(0.5) = (0.5)e^{0.5} - \cos(0.5)$

$= (0.5)(1.648721271) - 0.877582561$

$= 0.824360635 - 0.877582561$

$= -0.053221926$

$x = 0.6$ $f(0.6) = (0.6)e^{0.6} - \cos(0.6)$

$= (0.6)(1.8221188) - 0.825335614$

$= 1.09327128 - 0.825335614$

$= 0.267935666$

$x_1 = 0.5$, $f(x_1) = -0.0532$

$x_2 = 0.6$ $f(x_2) = 0.2679$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{(0.5)(0.2679) - (0.6)(-0.0532)}{0.2679 - (-0.0532)}$$

$$= \frac{0.13395 + 0.03192}{0.3211}$$

$$= \frac{0.16587}{0.3211} = 0.516568047$$

$$= 0.5166$$

$f(x_3) = (0.5166)e^{0.5166} - \cos(0.5166)$

$= -0.003517432952$

$= -0.0035$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{(0.6)(-0.0035) - (0.5166)(0.2679)}{-0.0035 - 0.2679}$$

$$= \frac{-0.0021 - 0.13839714}{-0.2714}$$

$$= \frac{-0.14049714}{-0.2714}$$

$$= 0.517675534$$

$$= 0.5177$$

$$f(x_4) = (0.5177)e^{0.5177} - \cos(0.5177)$$

$$= (0.5177)(1.678163432) - 0.868959707$$

$$= 0.868785208 - 0.868959707$$

$$= -0.0002$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(0.5166)(-0.0002) - (0.5177)(-0.0035)}{-0.0002 + 0.0035}$$

$$= \frac{-0.00010332 + 0.00181195}{0.0033}$$

$$= \frac{0.00170863}{0.0033} = 0.517766666$$

$$= 0.5178$$

$$f(x_5) = (0.5178)e^{0.5178} - \cos(0.5178)$$

$x^3 - 4x + 1$

$$= (0.5178)(1.678331256) - 0.868910215$$

$$= 0.869039924 - 0.868910215$$

$$= 0.00012970958 28$$

$$= 0.0001$$

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$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{(0.5177)(0.0001) - (0.5178)(-0.0002)}{0.0001 - (-0.0002)}$$

$$= \frac{0.00005177 + 0.00010356}{0.0003}$$

$$= \frac{0.00015533}{0.0003}$$

$$= 0.51776666$$

$$= 0.5178$$

$$x_5 = x_6 = 0.5178$$

6. $x^3 - 4x + 1$ 7. $xe^x = 3$

Solu) Given that

$f(x) = x^3 - 4x + 1$

$x = 0$, $f(0) = 0 - 4(0) + 1 = 1$

$x = 1$, $f(1) = 1 - 4(1) + 1 = -2$

$x = 2$, $f(2) = 2^3 - 4(2) + 1$

$= 8 - 8 + 1 = 1$

$x_1 = 1$, $f(x_1) = -2$

$x_2 = 2$, $f(x_2) = 1$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1(1) - 2(-2)}{1 - (-2)} = \frac{1+4}{1+2} = \frac{5}{3}.$$

$$= 1.66666667$$

$x_3 = 1.6667$

$f(x_3) = (1.6667)^3 - 4(1.6667) + 1$

$= 4.629907413 - 6.6668 + 1$

$= -1.036892587$

$= -1.03609$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2(-1.0369) - 1.6667(1)}{-1.0369 - 1}$$

$$= \frac{-2.0738 - 1.6667}{-2.0369} = \frac{-3.7405}{-2.0369}$$

$= 1.836368992$

$x_4 = 1.8364$

$f(x_4) = (1.8364)^3 - 4(1.8364) + 1$

$= 6.193011013 - 7.3456 + 1$

$= -0.152588987$

$= -0.1526$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{1.6667(-0.1526) - (1.8364)(-1.0369)}{-0.1526 + 1.0369}$$

$$= \frac{-0.25433842 + 1.90416316}{0.8843}$$

$$= \frac{1.64982474}{0.8843} = 1.865684428$$

$x_5 = 1.8657$

$f(x_5) = (1.8657)^3 - 4(1.8657) + 1$

$= 6.494196639 - 7.4628 + 1$

$= 0.031396639$

$= 0.0314$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{1.8364(0.0314) - 1.8657(-0.1526)}{0.0314 + 0.1526}$$

$$= \frac{0.05766296 + 0.2847058 2}{0.184}$$

$$= \frac{0.34236878}{0.184}$$

$$= 1.860699891$$

$$x_6 = 1.8607$$

$$f(x_6) = (1.8607)^3 - 4(1.8607) + 1$$

$$= 6.442123895 - 7.4428 + 1$$

$$= -0.000676105$$

$$= -0.0007$$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{1.8657(-0.0007) - (1.8607)(0.0314)}{-0.0007 - 0.0314}$$

$$= \frac{-0.00130599 - 0.05842598}{-0.0321}$$

$$= \frac{-0.05973197}{-0.0321}$$

$$= 1.860809034$$

$$x_7 = 1.8608$$

$$f(x_7) = (1.8608)^3 - 4(1.8608) + 1$$

$$= 6.443162612 - 7.4432 + 1$$

$$= -0.0000373 88$$

$$= -0.000$$

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$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)}$$

$$= \frac{1.8607(-0.000) - (1.8608)(-0.0007)}{-0.000 + 0.0007}$$

$$= \frac{0 + 0.00130256}{0.0007}$$

$x_8 = 1.8608$

$x_7 = x_8 = 1.8608$ the real roots

7 Given that

$f(x) = xe^x - 3$

$x = 0,\ f(0) = 0e^0 - 3$
$= -3$

$x = 1,\ f(1) = 1e^1 - 3$
$= 2.718281828 - 3$
$= -0.281718171$
$= -0.2817$

$x = 2,\ f(2) = 2e^2 - 3$
$= 2(7.389056099) - 3$
$= 14.778122 - 3$
$= 11.778122$
$= 11.7781$

$x_1 = 1,\ f(x_1) = -0.2817$
$x_2 = 2,\ f(x_2) = 11.7781$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)}$$

$$= \frac{1(11.7781) - 2(-0.2817)}{11.7781 + 0.2817}$$

$$= \frac{11.7781 + 0.5634}{12.0598}$$

$$= \frac{12.3415}{12.0598}$$

$$= 1.023358596$$

$$x_3 = 1.0234$$

$$f(x_3) = (1.0234)e^{1.0234} - 3$$

$$= (1.0234)(2.782639673) - 3$$

$$= 2.847753442 - 3$$

$$= -0.152246558$$

$$= -0.1522$$

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$$

$$= \frac{2(-0.1522) - (1.0234)(11.7781)}{-0.1522 - 11.7781}$$

$$= \frac{-0.3044 - 12.05370754}{-11.9303}$$

$$= \frac{-12.35810754}{-11.9303}$$

$$= +1.035858909$$

$$= 1.0359$$

$$f(x_4) = (1.0359)e^{1.0359} - 3$$

$$= (1.0359)(2.817640972) - 3$$

$$= 2.918794283 - 3$$

$$= -0.081205717$$

$$= -0.0812$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$$

$$= \frac{(1.0234)(-0.0812) - (1.0359)(-0.1522)}{-0.0812 + 0.1522}$$

$$= \frac{-0.08310008 + 0.15766398}{0.071}$$

$$= \frac{0.0745639}{0.071}$$

$$= \cancel{0.0745639}\ 1.050195775$$

$$= \cancel{0.0746}\ 1.0502$$

$$f(x_5) = (1.0502) e^{1.0502} - 3$$

$$= (1.0502)(2.858222705) - 3$$

$$= 3.001705485 - 3$$

$$= 0.001705485257$$

$$= 0.0017$$

$$x_6 = \frac{x_4 f(x_5) - x_5 f(x_4)}{f(x_5) - f(x_4)}$$

$$= \frac{1.0359(0.0017) - (1.0501)(-0.0812)}{0.0017 + 0.0812}$$

$$= \frac{0.00176103 + 0.08526812}{0.0829}$$

$$= \frac{0.08702915}{0.0829}$$

$$= 1.049808806$$

$$= 1.0498$$

$$f(x_6) = (1.0498) e^{1.0498} - 3$$

$$= (1.0498)(2.857079645) - 3$$

$$= 2.999362211 - 3$$

$$= -0.0006377886908$$

$= -0.0006$

$$x_7 = \frac{x_5 f(x_6) - x_6 f(x_5)}{f(x_6) - f(x_5)}$$

$$= \frac{(1.0501)(-0.0006) - (1.0498)(0.0017)}{-0.0006 - 0.0017}$$

$$= \frac{0.00063006 - 0.00178466}{-0.0023}$$

$$= \frac{-0.0011546}{-0.0023}$$

$= 0.502$

$f(x_7) = (0.502)e^{0.502} - 3$

$= (0.502)(1.652022013) - 3$

$= 0.82931505 - 3$

$= -2.170684949$

$= -2.1707$

$$x_8 = \frac{x_6 f(x_7) - x_7 f(x_6)}{f(x_7) - f(x_6)}$$

$$= \frac{1.0498(-2.1707) - (0.502)(-0.0006)}{-2.1707 + 0.0006}$$

$$= \frac{-2.27880086 + 0.0003012}{-2.1701}$$

$$= \frac{-2.27849966}{-2.1701}$$

$= +1.049951458$

$= 1.0491$

$f(x_8) = (1.0491)e^{1.0491} - 3$

$= (1.0491)(2.855080389) - 3$

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$= 2.995264836 - 3$

$= -0.004735163839$

$= -0.0047$

$$x_9 = \frac{x_7 f(x_8) - x_8 f(x_7)}{f(x_8) - f(x_7)}$$

$$= \frac{(0.502)(-0.0047) - (1.0491)(-2.1707)}{-0.0047 + 2.1707}$$

$$= \frac{-0.0023594 + 2.27728137}{2.166}$$

$$= \frac{2.27492197}{2.166}$$

$= 1.050287151$

$= 1.0503$

$f(x_9) = (1.0503)e^{1.0503} - 3$

$= (1.0503)(2.858508542) - 3$

$= 3.002291522 - 3$

$= 0.002291521669$

$= 0.0023$

$$x_{10} = \frac{x_8 f(x_9) - x_9 f(x_8)}{f(x_9) - f(x_8)}$$

$$= \frac{(1.0491)(0.0023) - (1.0503)(-0.0047)}{0.0023 + 0.0047}$$

$$= \frac{0.00241293 + 0.00493641}{0.007}$$

$$= \frac{0.00734934}{0.007}$$

$$= 1.049905714$$

$$= 1.0499$$

$$f(x_{10}) = (1.0499)e^{1.0499} - 3$$

$$= (1.0499)(2.857365367) - 3$$

$$= 2.999947899 - 3$$

$$= 0.000052100935 63$$

$$= 0.0000$$

$$x_{11} = \frac{x_9 f(x_{10}) - x_{10} f(x_9)}{f(x_{10}) - f(x_9)}$$

$$= \frac{(1.0503)(0.0000) - (1.0499)(0.0023)}{0.0000 - 0.0023}$$

$$= \frac{-0.00241477}{-0.0023}$$

$$= 1.0499$$

$x_{10} = x_{11} = 1.0499$ real values.

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} k \\ -4k \\ k \end{bmatrix} = k\begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix}$$

Date 13/12/18

Gauss - Seidl Iteration Method

we will consider the system of equations

$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$; $a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$;

$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$; $\rightarrow$ ①

where the diagonal co-efficients are not zero and are large comparae to other co-efficients such a system is called diagonally dominant system

1. Solve $10x + y + z = 12$; $2x + 10y + z = 13$; $2x + 2y + 10z = 14$ by Gauss-Seidl iteration method

Solu) Given equations

$$\left.\begin{aligned} 10x + y + z &= 12 \\ 2x + 10y + z &= 13 \\ 2x + 2y + 10z &= 14 \end{aligned}\right\} \rightarrow \text{(I)}$$

Equation (I) is a diagonally dominent system

$10x + y + z = 12$

$$x = (12 - y - z)\frac{1}{10} \rightarrow ①$$

$2x + 10y + z = 13$

$$y = \frac{1}{10}(13 - 2x - z) \rightarrow ②$$

$2x + 2y + 10z = 14$

$$z = \frac{1}{10}(14 - 2x - 2y) \rightarrow ③$$

Put $y=0,\ z=0$ in eq ①

$$x^{(1)} = \frac{1}{10}(12-0-0) = 1.2$$

Put $x=1.2,\ z=0$ in eq ②

$$y^{(1)} = \frac{1}{10}(13-2(1.2)-0)$$

$$y^{(1)} = 1.06$$

Put $x=1.2,\ y=1.06$ in eq ③

$$z^{(1)} = \frac{1}{10}(14-2(1.2)-2\cdot(1.06))$$

$$z^{(1)} = 0.948$$

$$x^{(1)} = 1.2,\ y^{(1)} = 1.06,\ z^{(1)} = 0.948$$

II - Iteration

Put $y=1.06,\ z=0.948$ in eq ①

$$x^{(2)} = \frac{1}{10}(12-1.06-0.948)$$

$$= 0.9992$$

put $x=0.9992,\ z=0.948$ in eq ②

$$y^{(2)} = \frac{1}{10}(13-2(0.9992)-0.948)$$

$$= 1.0054$$

put $x=0.9992;\ y=1.0054$ in eq ③

$$z^{(2)} = \frac{1}{10}(14-2(0.9992)-2(1.0054))$$

$$= 0.99908 = 0.9991$$

$$x^{(2)} = 0.9992;\ y^{(2)} = 1.0054;\ z^{(2)} = 0.9998$$

III - Iteration

put $y=1.0054,\ z=0.9998$ in eq ①

$$x^{(3)} = \frac{1}{10}(12-1.0054-0.9998)$$

$= 0.99955$

put $x = 0.99955$; $z = 0.9991$ in eq ②

$y^{(3)} = \frac{1}{10}(13 - 2(0.9992) - 0.9991)$

$= 1.0001$

put $x = 0.9995$; $y = 1.0001$ in eq ③

$z^{(3)} = \frac{1}{10}(14 - 2(0.9995) - 2(1.0001))$

$z^{(3)} = 1.0001$; $x^{(3)} = 0.99955$; $y^{(3)} = 1.0001$
$z^{(3)} = 1.0001$

IV - Iteration

put $z^{(3)} = 1.0001$, $y^{(3)} = 1.0001 \rightarrow$ ①

$x = \frac{1}{10}(12 - 1.0001 - 1.0001)$

$x^{(4)} = 0.99998$

put $x^{(4)} = 1$, in eq ② ; $z = 1.0001$

$y = \frac{1}{10}(13 - 2(1) - 1.0001)$

$y^{(4)} = 0.999 = 1$

put $x = 1$, $y = 0.99 = 1$ in eq ③

$z = \frac{1}{10}(14 - 2(1) - 2(1))$

$z^{(4)} = 1$

$x^{(4)} = 0.99998$, $y^{(4)} = 0.999 = 1$; $z^{(4)} = 1$

we have

Variable	I st Approximation	2nd	3rd	4th
x	1.2	0.9992	0.99955	1
y	1.06	1.0054	1.0001	1
z	0.948	0.9991	1.0001	1

H.W

2. Solve

$$27x+6y-z=85$$
$$6x+15y+2z=72$$
$$x+y+54z=110$$

3. $8x_1-3x_2+2x_3=20$
$$4x_1+11x_2-x_3=33$$
$$6x_1+3x_2+12x_3=36$$

4. $x+10y+z=6$
$$10x+y+z=6$$
$$x+y+10z=6$$

Solu 2) Given equations

$$\left.\begin{array}{l}27x+6y-z=85\\ 6x+15y+2z=72\\ x+y+54z=110\end{array}\right] \to (A)$$

Equation (A) is a diagonally dominant system.

$$27x+6y-z=85$$
$$x=(85-6y+z)\frac{1}{27} \to (1)$$
$$6x+15y+2z=72$$
$$y=(72-6x-2z)\frac{1}{15} \to (2)$$
$$x+y+54z=110$$
$$z=(110-x-y)\frac{1}{54} \to (3)$$

$\Rightarrow$ put $y=0$, $z=0$ in eq (1)

$$x^{(1)}=(85-0+0)\frac{1}{27}$$
$$x^{(1)}=3.14815$$

$\Rightarrow$ put $x=3.14815$; $z=0$ in eq (2)

$$y^{(1)}=(72-6(3.14815)-2(0))\frac{1}{15}$$
$$y^{(1)}=3.54074$$

$\Rightarrow$ put $x=3.14815$; $y=3.54074$ in eq (3)

$z^{(1)} = (110 - 3.14815 - 3.54074) \frac{1}{54}$

$z^{(1)} = 1.9135$

$\therefore x^{(1)} = 3.14815 ; \; y^{(1)} = 3.54074 ; \; z^{(1)} = 1.9135$

* $\underline{\text{II - Iteration}}$

$\Rightarrow$ put $[x = 3.14815]$; $z = 1.9135$; $y = 3.54074$ in ①

$x^{(2)} = (85 - 6(3.54074) + 1.9135) \frac{1}{27}$

$x^{(2)} = 2.4322$

$\Rightarrow$ put $x = 2.4322$; $z = 1.9135$ in eq ②

$y^{(2)} = (72 - 6(2.4322) - 2(1.9135)) \frac{1}{15}$

$y^{(2)} = 3.572$

$\Rightarrow$ put $x = 2.4322$; $y = 3.572$ in eq ③

$z^{(2)} = (110 - 2.4322 - 3.572) \frac{1}{54}$

$z^{(2)} = 1.9258$

$\therefore x^{(2)} = 2.4322 ; \; y^{(2)} = 3.572 ; \; z^{(2)} = 1.9258$

* III - Iteration

$\Rightarrow$ put $y = 3.572$; $z = 1.9258$ in eq ①

$x^{(3)} = (85 - 6(3.572) + 1.9258) \frac{1}{27}$

$x^{(3)} = 2.4257$

$\Rightarrow$ put $x = 2.4257$; $z = 1.9258$ in eq ②

$y^{(3)} = (72 - 6(2.4257) - 2(1.9258)) \frac{1}{15}$

$y^{(3)} = 3.573$

$\Rightarrow$ put $x = 2.4257$; $y = 3.573$ in eq ③

$z^{(3)} = (110 - 2.4257 - 3.573) \frac{1}{54}$

$= 1.92595$

$x^{(3)} = 2.4257$; $y^{(3)} = 3.573$; $z^{(3)} = 1.92595$.

IV - Iteration

$\Rightarrow$ put $(x = 2.4257)$; $y = 3.573$; $z = 1.926$ in eq ①

$$x^{(4)} = (85 - 6(3.573) + 1.926)\frac{1}{27}$$

$$x^{(4)} = 2.4255$$

$\Rightarrow$ put $x = 2.4255$; $z = 1.926$ in eq ②

$$y^{(4)} = (72 - 6(2.4255) - 2(1.926))\frac{1}{15}$$

$$y^{(4)} = 3.573$$

$\Rightarrow$ put $x = 2.4255$; $y = 3.573$; in eq ③

$$z^{(4)} = (110 - 2.4255 - 3.573)\frac{1}{54}$$
$$= 1.92595$$
$$= 1.926$$

$\therefore x^{(4)} = 2.4255$; $y^{(4)} = 3.573$; $z^{(4)} = 1.926$

V - Iteration

$\Rightarrow$ put $y = 3.573$; $z = 1.926$ in eq ①

$$x^{(5)} = (85 - 6(3.573) + 1.926)\frac{1}{27}$$
$$= 2.4255$$

$\Rightarrow$ put $x = 2.4255$; $z = 1.926$ in eq ②

$$y^{(5)} = (72 - 6(2.4255) - 2(1.926))\frac{1}{15}$$
$$= 3.573$$

$\Rightarrow$ put $x = 2.4255$; $y = 3.573$ in eq ③

$$z^{(5)} = (110 - 2.4255 - 3.573)\frac{1}{54}$$
$$= 1.926$$

$\therefore x^{(5)} = 2.4255 ; y^{(5)} = 3.573 ; z^{(5)} = 1.926$

variable	1st	2nd	3rd	4th	5th
x	3.14815	2.4322	2.4257	2.4255	2.4255
y	3.54075	3.572	3.573	3.573	3.573
z	1.9135	1.9258	1.92595	1.926	1.926

4. Given Equations

$x + 10y + z = 6$

$10x + y + z = 6$

$x + y + 10z = 6$

$$\left.\begin{aligned} 10x + y + z &= 6 \\ x + 10y + z &= 6 \\ x + y + 10z &= 6 \end{aligned}\right\} \rightarrow (A)$$

Equation (A) is a diagonally dominant system

$10x + y + z = 6$

$$x = (6 - y - z)\frac{1}{10} \rightarrow (1)$$

$x + 10y + z = 6$

$$y = (6 - x - z)\frac{1}{10} \rightarrow (2)$$

$x + y + 10z = 6$

$$z = (6 - x - y)\frac{1}{10} \rightarrow (3)$$

I - Iteration

$\Rightarrow$ put $y = 0 ; z = 0$ in eq (1)

$$x^{(1)} = (6 - 0 - 0)\frac{1}{10} = 0.6$$

$\Rightarrow$ put $x = 0.6 ; z = 0$ in eq (2)

$$y^{(1)} = (6 - 0.6 - 0)\frac{1}{10}$$

$$= 0.54$$

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$\Rightarrow$ put $x = 0.6$; $y = 0.54$ in eq ③

$$z^{(1)} = (6 - 0.6 - 0.54)\frac{1}{10}$$

$$= 0.486$$

$x^{(1)} = 0.6$; $y^{(1)} = 0.54$; $z^{(1)} = 0.486$

II - Iteration

$\Rightarrow$ put $y = 0.54$; $z = 0.486$ in eq ①

$$x^{(2)} = (6 - 0.54 - 0.486)\frac{1}{10}$$

$$x^{(2)} = 0.4974$$

$\Rightarrow$ put $x = 0.4974$; $z = 0.486$ in eq ②

$$y^{(2)} = (6 - 0.4974 - 0.486)\frac{1}{10}$$

$$= 0.502$$

$\Rightarrow$ put $x = 0.4974$; $y = 0.502$ in eq ③

$$z^{(2)} = (6 - 0.4974 - 0.502)\frac{1}{10}$$

$$z^{(2)} = 0.50006$$

$\therefore x^{(2)} = 0.4974$; $y^{(2)} = 0.502$; $z^{(2)} = 0.50006$

III - Iteration

$\Rightarrow$ put $y = 0.502$; $z = 0.50006$ in eq ①

$$x^{(3)} = (6 - 0.502 - 0.50006)\frac{1}{10}$$

$$= 0.4998$$

$\Rightarrow$ put $x = 0.4998$; $z = 0.50006$ in eq ②

$$y^{(3)} = (6 - 0.4998 - 0.50006)\frac{1}{10}$$

$$= 0.500014$$

$\Rightarrow$ put $x = 0.4998$; $y = 0.500014$ in eq ③

$$z^{(3)} = (6 - 0.4998 - 0.500014)$$

$$= 0.500019$$

$x^{(3)} = 0.4998$; $y^{(3)} = 0.500014$; $z^{(3)} = 0.500019$

IV - Iteration

$\Rightarrow$ put $(x = 0.4998)$ $y = 0.500014$; $z = 0.500019$ in eq ①

$$x^{(4)} = (6 - 0.500014 - 0.500019)\frac{1}{10}$$

$$= 0.49910$$

$\Rightarrow$ put $x = 0.49910$; $z = 0.500019$ in eq ②

$$y^{(4)} = (6 - 0.49910 - 0.500019)\frac{1}{10}$$

$$= 0.50009$$

$\Rightarrow$ put $x = 0.49910$; $y = 0.50009$ in eq ③

$$z^{(4)} = (6 - 0.49910 - 0.50009)\frac{1}{10}$$

$$= 0.500081$$

$x^{(4)} = 0.49910$; $y^{(4)} = 0.50009$; $z^{(4)} = 0.500081$

V - Iteration

$\Rightarrow$ put $y = 0.50009$; $z = 0.500081$ in eq ①

$$x^{(5)} = (6 - 0.50009 - 0.500081)\frac{1}{10}$$

$$= 0.49910$$

$\Rightarrow$ put $x = 0.49910$; $z = 0.500081$ in eq ②

$$y^{(5)} = (6 - 0.49910 - 0.500081)\frac{1}{10}$$

$$= 0.500082$$

$\Rightarrow$ put $x = 0.49910$; $y = 0.500082$ in eq ③

$$z^{(5)} = (6 - 0.49910 - 0.500082)\frac{1}{10}$$

$$= 0.500082$$

Variable	1st	2nd	3rd	4th	5th
x	0.6	0.4974	0.4998	0.4999	0.4999
y	0.54	0.502	0.500014	0.5000	0.5000
z	0.486	0.50066	0.500019	0.5000	0.5000

3. Given Equation

$$\left.\begin{aligned} 8x_1 - 3x_2 + 2x_3 &= 20 \\ 4x_1 + 11x_2 - x_3 &= 33 \\ 6x_1 + 3x_2 + 12x_3 &= 36 \end{aligned}\right\} \rightarrow (A)$$

Equation (A) is a diagonally dominant system

$$8x_1 - 3x_2 + 2x_3 = 20$$

$$x_1 = (20 + 3x_2 - 2x_3)\frac{1}{8} \rightarrow (1)$$

$$4x_1 + 11x_2 - x_3 = 33$$

$$x_2 = (33 - 4x_1 + x_3)\frac{1}{11} \rightarrow (2)$$

$$6x_1 + 3x_2 + 12x_3 = 36$$

$$x_3 = (36 - 6x_1 - 3x_2)\frac{1}{12} \rightarrow (3)$$

I - Iteration

$\Rightarrow$ put $x_2 = 0$; $x_3 = 0$ in eq (1)

$$x_1^{(1)} = (20 + 3(0) - 2(0))\frac{1}{8}$$

$$= \frac{20}{8} = 2.5$$

$\Rightarrow$ put $x_1 = 2.5$; $x_3 = 0$ in eq (2)

$$x_2^{(1)} = (33 - 4(2.5) + 0)\frac{1}{11}$$

$$= 2.091$$

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$\Rightarrow$ put $x_1 = 2.5$; $x_2 = 2.091$ in eq ③

$$x_3^{(1)} = (36 - 6(2.5) - 3(2.091)) \frac{1}{12}$$

$= 1.22725$

$= 1.23$ $\quad x_1 = 2.5$; $x_2 = 2.091$; $x_3 = 1.23$

II - Iteration

$\Rightarrow$ put $x_2 = 2.091$; $x_3 = 1.23$ in eq ①

$$x_1^{(2)} = [20 + 3(2.091) - 2(1.23)] \frac{1}{8}$$

$= 2.976625$

$x_1 = 2.977$

$\Rightarrow$ put $x_1 = 2.977$; $x_3 = 1.23$ in eq ②

$$x_2^{(2)} = (33 - 4(2.977) + 1.23) \frac{1}{11}$$

$= 2.0293$

$\Rightarrow$ put $x_1 = 2.977$; $x_2 = 2.0293$ in eq ③

$$x_3^{(2)} = [(36 - 6(2.977) - 3(2.0293)] \frac{1}{12}$$

$= 1.004175$

$= 1.0042$

$\therefore x_1^{(2)} = 2.977$; $x_2^{(2)} = 2.0293$; $x_3^{(2)} = 1.0042$

III - Iteration

$\Rightarrow$ put $x_2 = 2.0293$; $x_3 = 1.0042$ in eq ①

$$x_1^{(3)} = [20 + 3(2.0293) - 2(1.0042)] \frac{1}{8}$$

$= 3.009$

$\Rightarrow$ put $x_1 = 3.001$; $x_3 = 1.0042$ in eq ②

$$x_2^{(3)} = (33 - 4(3.001) + 1.0042) \frac{1}{11}$$

$= 2.000018$

$\Rightarrow$ put $x_1 = 3.001$; $x_2 = 2.000$ in eq ③

$$x_3^{(3)} = [(36 - 6(3.001) - 3(2.000)] \frac{1}{12}$$

$$= 0.9995$$

$\therefore x_1^{(3)} = 3.001$; $x_2^{(3)} = 2.000$; $x_3^{(3)} = 0.9995$

IV - Iteration

$\Rightarrow$ put $x_2 = 2.000$; $x_3 = 0.9995$ in eq ①

$$x_1^{(4)} = (20 + 3(2.000) + \cancel{0.9995} - 2(0.9995)) \frac{1}{8}$$

$$= 3.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_3 = 0.9995$ in eq ②

$$x_2^{(4)} = (33 - 4(3.000) + 0.9995) \frac{1}{11}$$

$$= 1.9990 = 2.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_2 = 1.9910$ in eq ③

$$x_3^{(4)} = (36 - 6(3.000) - 3(1.9910)] \frac{1}{12}$$

$= 1.00225$ $\therefore x_1^{(4)} = 3.000$; $x_2^{(4)} = 1.9910$ $= 2.000$

$x_3^{(4)} = 1.00225$

V - Iteration

$\Rightarrow$ put $x_2 = 1.9910$; $x_3 = 1.00225$ in eq ①

$$x_1^{(5)} = [20 + 3(1.9910) - 2(1.00225)] \frac{1}{8} = 3.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_3 = 1.00225$ in eq ②

$$x_2^{(5)} = (33 - 4(3.000) + 1.00225) \frac{1}{11} = 2.000$$

$\Rightarrow$ put $x_1 = 3.000$; $x_2 = 2.000$ in eq ③

$$x_3^{(5)} = [(36 - 6(3.000) - 3(2.000)] \frac{1}{12} = 1$$

$\therefore x_1^{(5)} = 3.000$; $x_2^{(5)} = 2.000$; $x_3^{(5)} = 1$

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variable	1st	2nd	3rd	4th	5th
x	2.5	2.977	3.001	3.000	3.000
y	2.091	2.0293	2.000	2.000	2.000
z	1.23	1.0042	0.9995	1.00205	1.000

Date 14/12/2018

5. Solve $10x_1-2x_2-x_3-x_4=3$; $-2x_1+10x_2-x_3-x_4=15$; $-x_1-x_2+10x_3-2x_4=15$; $-x_1-x_2-2x_3+10x_4=-9$ by Gauss-Seidl method correct to three decimal places.

Sol) Given Equations

$$\left.\begin{aligned}10x_1-2x_2-x_3-x_4&=3\\ -2x_1+10x_2-x_3-x_4&=15\\ -x_1-x_2+10x_3-2x_4&=15\\ -x_1-x_2-2x_3+10x_4&=-9\end{aligned}\right]\rightarrow (A)$$

Equation (A) is a diagonally dominant system

$$10x_1-2x_2-x_3-x_4=3$$

$$x_1=\frac{1}{10}(3+2x_2+x_3+x_4)\longrightarrow (1)$$

$$-2x_1+10x_2-x_3-x_4=15$$

$$x_2=(15+2x_1+x_3+x_4)\frac{1}{10}\rightarrow (2)$$

$$x_3=\frac{1}{10}[15+x_1+x_2+2x_4]\rightarrow (3)$$

$$x_4=\frac{1}{10}[-9+x_1+x_2+2x_3]\rightarrow (4)$$

I-Iteration

$\Rightarrow$ put $x_2=0$; $x_3=0$; $x_4=0$ in eq (1)

$$x_1=\frac{1}{10}(3+2(0)+0+0)$$
$$=0.3$$

$\Rightarrow$ put $x_1=0.3$; $x_3=0$; $x_4=0$ in eq (2)

$$x_2=\frac{1}{10}(15+2(0.3)+0+0)$$
$$=1.56$$

$\Rightarrow$ put $x_1 = 0.3$, $x_2 = 1.56$; $x_4 = 0$ in eq ③

$$x_3^{(1)} = \frac{1}{10}[15 + 0.3 + 1.56 + 0) = 1.686$$

$\Rightarrow$ put $x_1 = 0.3$; $x_2 = 1.56$, $x_3 = 1.686$ in eq ④

$$x_4^{(1)} = \frac{1}{10}(-9 + 0.3 + 1.56 + 2(1.686)]$$

$$= -0.377$$

$\therefore x_1^{(1)} = 0.3$; $x_2^{(1)} = 1.56$; $x_3^{(1)} = 1.686$; $x_4^{(1)} = -0.377$

II - Iteration

$\Rightarrow$ Put $x_2 = 1.56$; $x_3 = 1.686$; $x_4 = -0.377$ in eq ①

$$x_1^{(2)} = \frac{1}{10}(3 + 2(1.56) + 1.686 - 0.377)$$

$$= 0.74223$$

$\Rightarrow$ put $x_1 = 0.74223$; $x_3 = 1.686$; $x_4 = -0.377$ in eq ②

$$x_2^{(2)} = \frac{1}{10}(15 + 2(0.74223) + 1.686 - 0.377)$$

$$= 1.778695$$

$\Rightarrow$ put $x_1 = 0.74223$; $x_2 = 1.7795$; $x_4 = -0.377$ in eq ③

$$x_3^{(2)} = \frac{1}{10}(15 + 0.743 + 1.7795 + 2(-0.377))$$

$$= 1.6768$$

$\Rightarrow$ put $x_1 = 0.743$; $x_2 = 1.7795$; $x_3 = 1.6768$ in eq ④

$$x_4^{(2)} = \frac{1}{10}(-9 + 0.743 + 1.7795 + 2(1.6768))$$

$$= -0.31239$$

$x_1^{(2)} = 0.743$; $x_2^{(2)} = 1.779$; $x_3^{(2)} = 1.6768$; $x_4^{(2)} = -0.31239$

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III - Iteration

$\Rightarrow$ put $x_2 = 1.779$; $x_3 = 1.6768$; $x_4 = -0.3124$ in eq ①

$$x_1^{(3)} = \frac{1}{10}(3+2(1.779)+1.6768 + -0.3124)$$

$$= 0.7922$$

$\Rightarrow$ put $x_1 = 0.7922$; $x_3 = 1.6768$; $x_4 = -0.3124$ in eq ②

$$x_2^{(3)} = \frac{1}{10}(15+2(0.7922)+1.6768-0.3124)$$

$$= 1.79488 = 1.795$$

$\Rightarrow$ put $x_1 = 0.792$; $x_2 = 1.795$; $x_4 = -0.3124$ in eq ③

$$x_3^{(3)} = \frac{1}{10}[(15+0.792+1.795 \pm 2(0.3124))$$

$$= 1.696$$

$\Rightarrow$ put $x_1 = 0.792$; $x_2 = 1.795$; $x_3 = 1.696$ in eq ④

$$x_4^{(3)} = \frac{1}{10}[(-9+0.792+1.795+2(1.696)]$$

$$= -0.3021 = -0.302$$

$\therefore x_1^{(3)} = 0.792$; $x_2^{(3)} = 1.795$; $x_3^{(3)} = 1.696$; $x_4^{(3)} = -0.302$

IV - Iteration

$\Rightarrow$ put $x_2 = 1.795$; $x_3 = 1.696$; $x_4 = -0.302$ in eq ①

$$x_1^{(4)} = \frac{1}{10}(3+2(1.795)+1.696-0.302)$$

$$= 0.7984 = 0.798$$

$\Rightarrow$ put $x_1 = 0.798$; $x_3 = 1.696$; $x_4 = -0.302$ in eq ②

$$x_2^{(4)} = \frac{1}{10}(15+2(0.798)+1.696-0.302)$$

$$= 1.799$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_4 = -0.302$ in eq ③

$$x_3^{(4)} = \frac{1}{10}(15+0.798+1.799 \pm 2(0.302)]$$

$$= 1.6993 = 1.699$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_3 = 1.699$ in eq ④

$$x_4^{(4)} = \frac{1}{10}[-9 + 0.798 + 1.799 + 2(1.699)]$$

$$= -0.3005$$

$$= -0.300$$

$\therefore x_1^{(4)} = 0.798$; $x_2^{(4)} = 1.799$; $x_3^{(4)} = 1.699$; $x_4^{(4)} = -0.30$

V - Iteration

$\Rightarrow$ put $x_2 = 1.799$; $x_3 = 1.699$; $x_4 = -0.300$ in eq ①

$$x_1^{(5)} = \frac{1}{10}(3 + 2(1.799) + 1.699 - 0.300)$$

$$= 0.7997 = 0.799$$

$\Rightarrow$ put $x_1 = 0.798$; $x_3 = 1.699$; $x_4 = -0.300$ in eq ②

$$x_2^{(5)} = \frac{1}{10}(15 + 2(0.798) + 1.699 - 0.300)$$

$$= 1.7995 = 1.799$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_4 = -0.300$ in eq ③

$$x_3^{(5)} = \frac{1}{10}[15 + 0.798 + 1.799 + 2(0.300)]$$

$$= 1.6997 = 1.699$$

$\Rightarrow$ put $x_1 = 0.798$; $x_2 = 1.799$; $x_3 = 1.699$ in eq ④

$$x_4^{(5)} = \frac{1}{10}[-9 + 0.798 + 1.799 + 2(1.699)]$$

$$= -0.3005 = -0.300$$

variable	1st	2nd	3rd	4th	5th
x_1	0.3	0.743	0.792	0.798	0.799
x_2	1.56	1.779	1.795	1.799	1.799
x_3	1.686	1.6768	1.696	1.699	1.699
x_4	-0.377	-0.3124	-0.302	-0.300	-0.300

Gauss -

Solutions of Linear systems Direct Methods

1) Gaussian Elimination Method

This method of solving system of n linear equations in 'n' unknowns consists of eliminating the co-efficients in such a way that the system reduces to upper triangular system which may be solved by backward substitution.

1. Solve the Equations $2x+y+z=10$; $3x+2y+3z=18$; $x+4y+9z=16$; by using Gauss elimination method.

Solu) Given Equations

$$\left.\begin{array}{l} 2x+y+z=10 \\ 3x+2y+3z=18 \\ x+4y+9z=16 \end{array}\right\} \rightarrow (1)$$

system (1) can be expressed in the form $AX=B$

where

$$A=\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}; \quad X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad B=\begin{bmatrix} 10 \\ 18 \\ 16 \end{bmatrix}$$

Argumented matrix

$$[AB]=\begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$$\sim\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix} \begin{array}{l} R_2 \rightarrow 2R_2-3R_1 \\ R_3 \rightarrow 2R_3-R_1 \end{array}$$

$$\sim\begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix} R_3 \rightarrow R_3-7R_2$$

70
21
59

22
22
20

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which is a upper triangular matrix

$2x+y+z=10$; $y+3z=6$

$-4z=-20$

$z=5$

$y+3(5)=6$; $2x-9+5=10$

$y=6-15$; $2x=14$

$y=-9$; $x=7$;

$x=7$; $y=-9$; $z=5$

2. Solve $3x+y-z=3$; $2x-8y+z=-5$; $x-2y+9z=8$ by Gaussian elimination method

Given Equations

$$\left.\begin{array}{l} 3x+y-z=3 \\ 2x-8y+z=-5 \\ x-2y+9z=8 \end{array}\right] \rightarrow ①$$

system ① can be expressed in the form $AX=B$

$$A=\begin{bmatrix} 3 & 1 & -1 \\ 2 & -8 & 1 \\ 1 & -2 & 9 \end{bmatrix} ; X=\begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B=\begin{bmatrix} 3 \\ -5 \\ 8 \end{bmatrix}$$

Argumented matrix

$$[AB]=\begin{bmatrix} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{bmatrix}$$

$$\sim\begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & -7 & 28 & 21 \end{bmatrix} \begin{array}{l} R_2 \rightarrow 3R_2-2R_1 \\ R_3 \rightarrow 3R_3-R_1 \end{array}$$

-24
-2

$$\sim\begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & -1 & 4 & 3 \end{bmatrix} R_3 \rightarrow \frac{R_3}{7}$$

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$$\sim \begin{bmatrix} 3 & 1 & -1 & 3 \\ 0 & -26 & 5 & -21 \\ 0 & 0 & 99 & 99 \end{bmatrix} R_3 \rightarrow 26R_3 + R_2$$

$\begin{array}{r} 26 \\ 3 \\ \hline 78 \\ 21 \\ \hline 99 \end{array}$ $\begin{array}{r} 2 \\ 26 \\ 4 \\ \hline 104 \\ 5 \\ \hline 99 \end{array}$

which is a upper triangular matrix

$$3x + y - z = 3$$

$$-26y + 5z = -21$$

$$99z = 99$$

$$z = 1$$

$$-26y + 5 = -21$$

$$-26y = -21 - 5$$

$$-26y = -26$$

$$y = 1$$

$$3x + 1 - 1 = 3$$

$$x = 1$$

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$\therefore$ $x = 1,\ y = 1,\ z = 1$

3. Solve $2x + y + z = 10$; $3x + 2y + 3z = 18$; $x + 4y + 9z = 16$ by using Gauss-Jordan-Method (only row operations)

Solu) Given Equations

$$\left.\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 3z &= 18 \\ x + 4y + 9z &= 16 \end{aligned}\right\} \rightarrow (1)$$

system (1) can be expressed in the form $AX = B$ where

$$[A\ B] = \begin{bmatrix} 2 & 1 & 1 & 10 \\ 3 & 2 & 3 & 18 \\ 1 & 4 & 9 & 16 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 7 & 17 & 22 \end{bmatrix} \begin{matrix} R_2 \rightarrow 2R_2 - 3R_1 \\ R_3 \rightarrow 2R_3 - R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & -4 & -20 \end{bmatrix} R_3 \rightarrow R_3 - 7R_2$$

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$$\sim \begin{bmatrix} 2 & 1 & 1 & 10 \\ 0 & 1 & 3 & 6 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_3 \to R_3/-4$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 5 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad \begin{matrix} R_1 \to R_1 - R_3 \\ R_2 \to R_2 - 3R_3 \end{matrix}$$

$$\sim \begin{bmatrix} 2 & 0 & 0 & 14 \\ 0 & 1 & 0 & -9 \\ 0 & 1 & 1 & 5 \end{bmatrix} \quad R_1 \to R_1 - R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 7 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & 5 \end{bmatrix} \quad R_1 \to R_1/2$$

$x=7$; $y=-9$; $z=5$

H.W.

4. Solve the equations $x+y+z=6$; $3x+3y+4z=20$; $2x+y+3z=13$; using partial pivoting Gaussian elimination method.

Solu) Given Equations

$$\left.\begin{matrix} x+y+z=6 \\ 3x+3y+4z=20 \\ 2x+y+3z=13 \end{matrix}\right\} \to (1)$$

system (1) can be expressed in the form

$AX=B$ where

$$A=\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix};\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix};\ B=\begin{bmatrix} 6 \\ 20 \\ 13 \end{bmatrix}.$$

Argumented matrix

$$[AB] = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 3 & 3 & 4 & 20 \\ 2 & 1 & 3 & 13 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 0 & 1 & 2 \\ 0 & -1 & 1 & 1 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 3R_1 \\ R_3 \to R_2 - 2R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -1 & 1 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad R_2 \leftrightarrow R_3$$

which is a upper triangular matrix

$x+y+z=6$; $x+1+2=6$

$x=3$

$-y+z=1$; $-y+2=1$

$z=2$; $-y=-1$

$y=1$;

$\therefore x=3;\ y=1;\ z=2$

5. Solve the Equations $3x+y+2z=3$; $2x-3y-z=-3$; $x+2y+z=4$ by using Gauss Elimination method

Solu) Given Equations

$$\left.\begin{matrix} 3x+y+2z=3 \\ 2x-3y-z=-3 \\ x+2y+z=4 \end{matrix}\right\} \to ①$$

System ① can be expressed in the form $AX=B$.

where $A=\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$; $X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B=\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$

Argumented matrix

$$[AB] = \begin{bmatrix} 3 & 1 & 2 & 3 \\ 2 & -3 & -1 & -3 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & -3 & -1 & -3 \\ 3 & 1 & 2 & 3 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & -5 & -1 & -9 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -7 & -3 & -11 \\ 0 & 0 & 8 & -8 \end{bmatrix} \quad R_3 \to 7R_3 - 5R_2.$$

which is an upper triangular matrix

$x + 2(2) - 1 = 4$; $x + 2y + z = 4$

$x + 4 - 1 = 4$ $\qquad -7y - 3z = -11$; $-7y - 3(-1) = -11$

$x = 1$ $\qquad 8z = -8$ $\qquad -7y + 3 = -11$

$z = -1$ $\qquad -7y = -14$

$y = 2$

$\therefore x = 1$; $y = 2$; $z = -1$

6. Solve the Equations $10x + y + z = 12$; $2x + 10y + z = 13$ and $x + y + 5z = 7$ by Gauss - Jordan Method

Solu) Given Equations

$$\left.\begin{aligned} 10x + y + z &= 12 \\ 2x + 10y + z &= 13 \\ x + y + 5z &= 7 \end{aligned}\right\} \to (1)$$

system (1) can be expressed in the form

$AX = B$

$$A = \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix}; \; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \; B = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

Argumented matrix

$$[AB] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 2 & 10 & 1 & 13 \\ 1 & 1 & 5 & 7 \end{bmatrix}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & 12 \\ 0 & 49 & 4 & 53 \\ 0 & 9 & 49 & 58 \end{bmatrix} \quad \begin{aligned} &R_2 \to 5R_2 - R_1 \\ &R_3 \to 10R_3 - R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 10 & 1 & 1 & 1 \\ 0 & 49 & 4 & 53 \\ 0 & 0 & 2365 & 2365 \end{bmatrix} \quad R_3 \to 49R_3 - 9R_2$$

$$\sim \begin{bmatrix} 20 & 1 & 1 & 1 \\ & & & \\ & & & \end{bmatrix} \quad R_2 \leftrightarrow$$

441 $\quad \dfrac{65}{12}$ $\quad 53$ $\quad \dfrac{70}{12}$

2401 $\quad 58$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix} \quad \begin{array}{l} R_2 \leftrightarrow R_2 - 2R_1 \\ R_3 \to R_3 - 10R_1 \end{array}$$

$$\sim \begin{bmatrix} 1 & -8 & -44 & -51 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{bmatrix} \quad R_1 \to R_1 + R_3$$

$$\sim \begin{bmatrix} -1 & +8 & +44 & +51 \\ 0 & 8 & -9 & -1 \\ 0 & 9 & 49 & 58 \end{bmatrix} \quad \begin{array}{l} R_1 \to \frac{R_1}{-1} \\ R_3 \to \frac{R_3}{-1} \end{array}$$

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464

$$\sim \begin{bmatrix} -1 & 8 & 44 & 51 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 473 & 473 \end{bmatrix} \quad R_3 \to 8R_3 - 9R_2$$

$$\sim \begin{bmatrix} -1 & 8 & 44 & 51 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 \to \frac{R_3}{473}$$

$$\sim \begin{bmatrix} -1 & 0 & 53 & 52 \\ 0 & 8 & 0 & 8 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 \to R_1 - R_2 \\ R_2 \to R_2 + 9R_3 \end{array}$$

$$\sim \begin{bmatrix} -1 & 0 & 53 & 52 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_2 \to \frac{R_2}{8}$$

$$\sim \begin{bmatrix} +1 & 0 & 0 & +1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_1 \to \frac{R_1 - 53R_3}{-1}$$

$\therefore x = 1 ; \ y = 1 ; \ z = 1$

7. Solve the Equations

$10x_1 + x_2 + x_3 = 12$; $x_1 + 10x_2 - x_3 = 10$ and $x_1 - 2x_2 + 10x_3 = 9$ by Gauss - Jordan method

Sol: Given Equations

$$\left.\begin{aligned} 10x_1 + x_2 + x_3 &= 12 \\ x_1 + 10x_2 - x_3 &= 10 \\ x_1 - 2x_2 + 10x_3 &= 9 \end{aligned}\right\} \rightarrow (1)$$

system (1) can be expressed in the form $AX = B$

$$A = \begin{bmatrix} 10 & 1 & 1 \\ 1 & 10 & -1 \\ 1 & -2 & 10 \end{bmatrix} ; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; B = \begin{bmatrix} 12 \\ 10 \\ 9 \end{bmatrix}$$

Argumented matrix

$$[AB] = \begin{bmatrix} 10 & 1 & 1 & 12 \\ 1 & 10 & -1 & 10 \\ 1 & -2 & 10 & 9 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 1 & 10 & -1 & 10 \\ 10 & 1 & 1 & 12 \end{bmatrix} \quad R_1 \leftrightarrow R_3$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 21 & -99 & -78 \end{bmatrix} \quad \begin{aligned} R_2 &\rightarrow R_2 - R_1 \\ R_3 &\rightarrow R_3 - 10R_1 \end{aligned}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 7 & -33 & -26 \end{bmatrix} \quad R_3 \rightarrow \frac{R_3}{3}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 0 & -319 & -319 \end{bmatrix} \quad R_3 \rightarrow 12R_3 - 7R_2$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & -11 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_3 \rightarrow \frac{R_3}{-319}$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 12 & 0 & 12 \\ 0 & 0 & 1 & 1 \end{bmatrix} \quad R_2 \rightarrow R_2 + 11R_3$$

$$\sim \begin{bmatrix} 1 & -2 & 10 & 9 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_2 \to \frac{R_2}{12}$$

$$\sim \begin{bmatrix} 1 & 0 & 10 & 11 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_1 \to R_1 + 2R_2$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} R_1 \to R_1 - 10R_3$$

$x_1 = 1$; $x_2 = 1$; $x_3 = 1$

8. Solve the system of Equations by Gauss-seidel method $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$

Solu) Given Equations

$$\left.\begin{aligned} 20x + y - 2z &= 17 \\ 3x + 20y - z &= -18 \\ 2x - 3y + 20z &= 25 \end{aligned}\right\} \to (A)$$

Equation (A) can be expressed in the form $AX = B$

where $A = \begin{bmatrix} 20 & 1 & -2 \\ 3 & 20 & -1 \\ 2 & -3 & 20 \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$; $B = \begin{bmatrix} 17 \\ -18 \\ 25 \end{bmatrix}$

Arg] equation (A) is a diagonally dominant

$$x = (17 - y + 2z)\frac{1}{20} \to (1)$$

$$y = (-18 - 3x + z)\frac{1}{20} \to (2)$$

$$z = (25 - 2x + 3y)\frac{1}{20} \to (3)$$

I- Iteration

$\Rightarrow$ put $y = 0$; $z = 0$ in eq (1)

$$x^{(1)} = (17 - 0 + 2(0))\frac{1}{20} = 0.85$$

$\Rightarrow$ put $x = 0.85$; $z = 0$ in eq ②

$$y^{(1)} = (-18 - 3(0.85) + 0)\frac{1}{20}$$

$$= -1.0275$$

$\Rightarrow$ put $x = 0.85$; $y = -1.0275$ in eq ③

$$z^{(1)} = (25 - 2(0.85) + 3(-1.0275))\frac{1}{20}$$

$$= 1.010875$$

$$= 1.0109$$

$$x^{(1)} = 0.85 \; ; \; y^{(1)} = -1.0275 \; ; \; z^{(1)} = 1.0109$$

II - Iteration

$\Rightarrow$ put $y = -1.0275$; $z = 1.0109$ in eq ①

$$x^{(2)} = (17 + 1.0275 + 2(1.0109))\frac{1}{20}$$

$$= 1.002465$$

$$= 1.0025$$

$\Rightarrow$ put $x = 1.0025$; $z = 1.0109$ in eq ②

$$y^{(2)} = (-18 - 3(1.0025) + 1.0109)\frac{1}{20}$$

$$= -0.99983$$

$$= -0.9998 = -0.9910$$

$\Rightarrow$ put $x = 1.0025$; $y = -0.9910$ in eq ③

$$z^{(2)} = [(25 - 2(1.0025) \pm 3(0.9910)]\frac{1}{20}$$

$$= 1.0011$$

$$\therefore x^{(2)} = 1.0025 \; ; \; y^{(2)} = -0.9910 \; ; \; z^{(2)} = 1.0011$$

III - Iteration

$\Rightarrow$ put $y = -0.9910$; $z = 1.0011$ in eq ①

$$x^{(3)} = (17 + 0.9910 + 2(1.0011))\frac{1}{20}$$

$= 0.99966$

$= 0.999 = 1.00$

$\Rightarrow$ put $x=1$; $z = 1.0011$ in eq ②

$$y^{(3)} = (-18 - 3(1) + 1.0011)\frac{1}{20}$$

$$= -0.999945$$

$$= -1.000$$

$\Rightarrow$ put $x=1$; $y=-1$ in eq ③

$$z^{(3)} = (25 - 2(1) - 3(1))\frac{1}{20}$$

$$= 1$$

$\therefore x^{(3)} = 1$; $y^{(3)} = -1$; $z^{(3)} = 1$

IV - Iteration

$\Rightarrow$ put $y=1$; $z=1$ in eq ①

$$x^{(4)} = (17 \pm 1 + 2(1))\frac{1}{20}$$

$$= 0.99 = 1$$

$\Rightarrow$ put $x=1$; $z=1$ in eq ②

$$y^{(4)} = (-18 - 3(1) + 1)\frac{1}{20}$$

$$= -1$$

$\Rightarrow$ put $x=1$; $y=-1$ in eq ③

$$z^{(4)} = (25 - 2(1) - 3)\frac{1}{20}$$

$$= 1$$

$\therefore x^{(4)} = 1$; $y^{(4)} = -1$; $z^{(4)} = 1$

Variable	1st	2nd	3rd	4th
x	0.85	1.0025	1	1
y	-1.0275	-0.9910	-1	-1
z	1.0109	1.0011	1	1

9. Solve the following system of equations by using Gauss - Seidel method correct to three decimal places. $8x-3y+2z=20$; $4x+11y-z=33$; $6x+3y+12z=35$

Solu) Given Equations

$$\left.\begin{aligned} 8x-3y+2z &= 20 \\ 4x+11y-z &= 33 \\ 6x+3y+12z &= 35 \end{aligned}\right\} \rightarrow (A)$$

system (A) is a diagonally dominant system

where

$$x = \frac{1}{8}(20+3y-2z) \rightarrow (1)$$

$$y = \frac{1}{11}(33-4x+z) \rightarrow (2)$$

$$z = \frac{1}{12}(35-6x-3y) \rightarrow (3)$$

I - Iteration

$\Rightarrow$ put $y=0$; $z=0$ in eq (1)

$$x^{(1)} = \frac{1}{8}(20+3(0)-2(0))$$

$$= 2.5$$

$\Rightarrow$ put $x=2.5$; $z=0$ in eq (2)

$$y^{(1)} = \frac{1}{11}(33-4(2.5)+0) = 2.09090$$

$$= 2.091$$

$\Rightarrow$ put $x=2.5$; $y=2.091$ in eq (3)

$$z^{(1)} = \frac{1}{12}(35-6(2.5)-3(2.091))$$

$$= 1.14439166 = 1.1444$$

$\therefore x^{(1)} = 2.5$; $y^{(1)} = 2.091$; $z^{(1)} = 1.444$

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II - Iteration

$\Rightarrow$ put $(x = 2.5)$ $y = 2.091$; $z = 1.444$ in eq ①

$$x^{(2)} = \frac{1}{8}(20 + 3(2.091) - 2(1.444))$$
$$= 2.923125$$
$$= 2.923$$

$\Rightarrow$ put $x = 2.923$; $z = 1.444$ in eq ②

$$y^{(2)} = \frac{1}{11}(33 - 4(2.923) + 1.444)$$
$$= 2.0683636$$
$$= 2.068$$

$\Rightarrow$ put $x = 2.923$; $y = 2.068$ in eq ③

$$z^{(2)} = \frac{1}{12}(35 - 6(2.923) - 3(2.068))$$
$$= 0.938166$$
$$= 0.938$$

$\therefore x^{(2)} = 2.923$; $y^{(2)} = 2.068$; $z^{(2)} = 0.938$

III - Iteration

$\Rightarrow$ put $y = 2.068$; $z = 0.938$ in eq ①

$$x^{(3)} = \frac{1}{8}(20 + 3(2.068) - 2(0.938))$$
$$= 3.041$$

$\Rightarrow$ put $x = 3.041$; $z = 0.938$ in eq ②

$$y^{(3)} = \frac{1}{11}(33 - 4(3.041) + 0.938)$$
$$= 1.9794545 = 1.979$$

$\Rightarrow$ put $x = 3.041$; $y = 1.979$ in eq ③

$$z^{(3)} = \frac{1}{12}(35 - 6(3.041) - 3(1.979))$$
$$= 0.9014166 = 0.901$$

$\therefore x^{(3)} = 3.041 ; y^{(3)} = 1.979 ; z^{(3)} = 0.901$

IV - Iteration

$\Rightarrow$ put $y = 1.979 ; z = 0.901$ in eq ①

$$x^{(4)} = \frac{1}{8}(20 + 3(1.979) - 2(0.901))$$

$$= 3.016875$$

$$= 3.017$$

$\Rightarrow$ put $x = 3.017 ; z = 0.901$ in eq ②

$$y^{(4)} = \frac{1}{11}(33 - 4(3.017) + 0.901)$$

$$= 1.984818$$

$$= 1.985$$

$\Rightarrow$ put $x = 3.017 ; y = 1.985$ in eq ③

$$z^{(4)} = \frac{1}{12}(35 - 6(3.017) - 3(1.985))$$

$$= 0.9119166$$

$$= 0.912$$

$x^{(4)} = 3.017 ; y^{(4)} = 1.985 ; z^{(4)} = 0.912$

V - Iteration

$\Rightarrow$ put $y = 1.985 ; z = 0.912$ in eq ①

$$x^{(5)} = \frac{1}{8}(20 + 3(1.985) - 2(0.912))$$

$$= 3.016375 = 3.016$$

$\Rightarrow$ put $x = 3.016 ; z = 0.912$ in eq ②

$$y^{(5)} = \frac{1}{11}(33 - 4(3.016) + 0.912)$$

$$= 1.9861818$$

$$= 1.986$$

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$\Rightarrow$ put $x = 3.016$; $y = 1.986$; in eq ③

$$z^{(5)} = \frac{1}{12}(35 - 6(3.016) - 3(1.986))$$

$$= 0.9121666$$

$$= 0.912$$

$\therefore x^{(5)} = 3.016$; $y^{(5)} = 1.986$; $z^{(5)} = 0.912$

VI - Iteration

$\Rightarrow$ put $y = 1.986$; $z = 0.912$ in eq ①

$$x^{(6)} = \frac{1}{8}(20 + 3(1.986) - 2(0.912))$$

$$= 3.01675 = 3.016$$

$\Rightarrow$ put $x = 3.016$; $z = 0.912$ in eq ②

$$y^{(6)} = \frac{1}{11}(33 - 4(3.016) + (0.912))$$

$$= \cancel{1.654545} \quad 1.98618$$

$$= \cancel{1.655} \quad 1.986$$

$\Rightarrow$ put $x = 3.016$; $y = 1.986$ in eq ③

$$z^{(6)} = \frac{1}{12}(35 - 6(3.016) - 3(1.986))$$

$$= 0.9121666$$

$$= 0.912$$

$x^{(6)} = 3.016$; $y^{(6)} = 1.986$; $z^{(6)} = 0.912$

Variable	I	II	III	IV	V	VI
x	2.5	2.923	3.041	3.017	3.016	3.016
y	2.091	2.068	1.979	1.985	1.986	1.986
z	1.444	0.938	0.901	0.912	0.912	0.912

Date 28/6/18 UNIT-IV

Unit- IV Interpolation

* Introduction :

Since $y=f(x)$ be the given function. The given function defined in the interval (a,b) then it is called "interpolation".

Consider 'x' takes the values $x_0, x_1, x_2, x_3, x_4, \ldots, x_n$ the corresponding y-values are $y_0, y_1, y_2, y_3, y_4, \ldots, y_n$ respectively. And the differences of x are is 'h' then

$x_1 - x_0 = h,\ x_2 - x_1 = h,\ x_3 - x_2 = h, \ldots, x_n - x_{n-1} = h$

$\Rightarrow x_1 = x_0 + h$.

$\Rightarrow x_2 = x_1 + h \Rightarrow x_2 = (x_0+h)+h$

$$\boxed{x_2 = x_0 + 2h}$$

$\Rightarrow x_3 = x_2 + h \Rightarrow x_3 = (x_0+2h)+h$

$$\boxed{x_3 = x_0 + 3h}$$

$\Rightarrow x_n - x_{n-1} = h \Rightarrow \boxed{x_n = x_0 + nh}$

Given, $y = f(x)$

$y_0 = f(x_0)$

$y_1 = f(x_1)$

$y_1 = f(x_0+h)$

$y_2 = f(x_2)$

$= f(x_0+2h)$

$y_3 = f(x_3)$

$= f(x_0+3h)$

$y_n = f(x_n)$

$$\boxed{y_n = f(x_0+nh)}$$

The differences

$y_1 - y_0,\ y_2 - y_1,\ y_3 - y_2,\ y_4 - y_3, \ldots$ are represented by

$\Delta y_0, \Delta y_1, \Delta y_2, \Delta y_3, \ldots$ respectively are called first order forward differences and Δ is called forward difference operator.

The differences

$\Delta y_1 - \Delta y_0, \Delta y_2 - \Delta y_1, \Delta y_3 - \Delta y_2, \ldots$ are represented by $\Delta^2 y_0, \Delta^2 y_1, \Delta^2 y_2, \ldots$ are called second order "forward differences"

The differences

$\Delta^2 y_1 - \Delta^2 y_0, \Delta^2 y_2 - \Delta^2 y_1, \Delta^2 y_3 - \Delta^2 y_2, \ldots$ are represented by $\Delta^3 y_0, \Delta^3 y_1, \Delta^3 y_2, \ldots$ respectively are called third order forward differences.

The differences $y_1 - y_0, y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$ are represented by $\nabla y_1, \nabla y_2, \nabla y_3, \nabla y_4, \ldots$ respectively are called first order backward differences and '∇' is called Backward difference operator.

The differences $\nabla y_2 - \nabla y_1, \nabla y_3 - \nabla y_2, \nabla y_4 - \nabla y_3, \ldots$ are represented by $\nabla^2 y_2, \nabla^2 y_3, \nabla^2 y_4, \ldots$ respectevely are called second order backward differences.

The differences $\nabla^2 y_3 - \nabla^2 y_2, \nabla^2 y_4 - \nabla^2 y_3, \ldots$ are represented by $\Delta^3 y_3, \Delta^3 y_4, \nabla^3 y_5, \ldots$ respectively are called third order backward differences.

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The differences $y_1 - y_0, y_2 - y_1, y_3 - y_2, y_4 - y_3, \ldots$, are represented by small (δ) $\delta y_{1/2}, \delta y_{3/2}, \delta y_{5/2}, \delta y_{7/2} \ldots$ respectively are called central differences and 'δ' is called Central difference operator.

The differences $\delta y_{3/2} - \delta y_{1/2}, \delta y_{5/2} - \delta y_{3/2}, \delta y_{7/2} - \delta y_{5/2} \ldots$ are represented by $\delta^2 y_1, \delta^2 y_2, \delta^2 y_3, \ldots$ respectively are called second order central differences

$\frac{3}{2} + \frac{1}{2} =$

Similarly $\delta^2 y_2 - \delta^2 y_1$, $\delta^2 y_3 - \delta^2 y_2$, $\delta y_4^2 - \delta_3^2 \cdot \delta^2$ - are represented by $\delta^3 y_{3/2}$, $\delta^3 y_{5/2}$, $\delta^3 y_{7/2}$, ... respectively are called the third order central differences

Shifting Operator

Since 'E' is called shifting operator. It shifts the given function into the next level.

Cons Therefore

$$Ey_0 = y_1 \Rightarrow Ef(x_0) = f(x_1)$$

$$\boxed{Ef(x_0) = f(x_0+h)}$$

$$Ey_1 = y_2 \Rightarrow Ef(x_1) = f(x_2)$$

$$E\cdot f(x_0+h) = f(x_0+2h)$$

$$E\cdot Ef(x_0) = f(x_0+2h)$$

$$\boxed{E^2 f(x_0) = f(x_0+2h)}$$

$$\therefore E^n f(x_0) = f(x_0+nh)$$

Similarly $E^3 f(x_0) = f(x_0+3h)$

Therefore $\boxed{E^n f(x) = f(x+nh)}$

Note

Since $E^n f(x) = f(x+nh)$

put $n = -n \Rightarrow E^{-n} f(x) = f(x+(-n)h)$

$$\boxed{E^{-n} f(x) = f(x-nh)}$$

Book Work

Since we know the $y_1 - y_0 = \Delta y_0 \rightarrow ①$

and $Ey_0 = y_1 \rightarrow ②$

from ① & ②

$Ey_0 - y_0 = \Delta y_0$

$(E-1) y_0 = \Delta y_0$

$E - 1 = \Delta$

$\boxed{E = 1 + \Delta}$ *

Relation between S.O and forward differences

Since we know that $y_1 - y_0 = \nabla y_1 \rightarrow ①$

we know and $Ey_0 = y_1$

$\Rightarrow y_0 = E^{-1} y_1 \rightarrow ②$

from ① & ②

$y_1 - E^{-1} y_1 = \nabla y_1$

$y_1 (1 - E^{-1}) = \nabla y_1$

$1 - E^{-1} = \nabla$

$\boxed{E^{-1} = 1 - \nabla}$ *

Relation between shifting operator and backward differences

Since we know that

$y_1 - y_0 = \delta y_{1/2} \rightarrow$

$\Rightarrow y_{\frac{1}{2}+\frac{1}{2}} - y_{\frac{1}{2}-\frac{1}{2}} = \delta y_{1/2}$ $\because$

$E^{1/2} y_{1/2} - E^{-1/2} y_{1/2} = \delta y_{1/2}$ $E^1 y_3 = y_{3+1}$

$E^1 y_0 = y_{0+1}$

$y_{1/2} [E^{1/2} - E^{-1/2}] = \delta y_{1/2}$

$\boxed{E^{1/2} - E^{-1/2} = \delta}$ *

Relation between central difference and shifting operator

Average Operator

'μ' is called Average operator such that

$$\mu y_n = \frac{y_{n+1/2} + y_{n-\frac{1}{2}}}{2}$$

$$\mu y_n = \frac{E^{1/2} y_n + E^{-1/2} y_n}{2}$$

$$\mu y_n = \left[\frac{E^{1/2} + E^{-1/2}}{2}\right] y_n$$

$$\boxed{\mu = \frac{E^{1/2} + E^{-1/2}}{2}}$$

The above equation is the relation between Average operator and shifting operator

Pascal's Triangle.

					0					
				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

$$\Delta^4 y_0 = 1y_1 - 4y_2 + 6y_3 - 4y_4 + 1y_5$$

Date 2/07/18

Newtons Forward interpolation formulae

Consider $y = f(x)$ be the given function.

x creates the values, $x_0, x_1, x_2 - - x_n$ and the common difference between 'x' is 'h'.

The corresponding 'y' values are $y_0, y_1, y_2, - - - y_n$ respectively then

$$y_n = f(x_0 + nh)$$
$$= E^n f(x_0)$$

$(1+\Delta)^n y_0 = y_n$

$\because (1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$

$$y_n = (1+\Delta)^n = \left[1 + n\Delta + \frac{n(n-1)}{2!}\Delta^2 + \frac{n(n-1)(n-2)}{3!}\Delta^3 + \cdots\right] y_0$$

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n+1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

Newton's Backward Interpolation Formulae

At arbitrary value $x = x_n$ the corresponding y values is y_n then $y_n = f(x_n)$

$$\Rightarrow y_n = f(x_n + nh)$$
$$= E^n f(x_n)$$
$$= (E^{-1})^{-n} f(x_n)$$
$$= (1-\nabla)^{-n} y_n$$

$$\because (1-x)^{-n} = \left[1 + nx + \frac{n(n+1)}{2!}x^2 + \frac{n(n+1)(n+2)}{3!}x^3 + \frac{n(n+1)(n+2)(n+3)}{4!}\right.$$

$$y_n = (1-\nabla)^{-n} = \left[1 + n\nabla + \frac{n(n+1)}{2!}\nabla^2 + \frac{n(n+1)(n+2)}{3!}\nabla^3 + \frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 + \right.$$

$$y_n = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$

Problems

1. find $\Delta f(x)$, $f(x) = x^3 - x^2 + x + 10$, $h = 1$

Solu) Since we know that

$$\Delta f(x) = f(x+h) - f(x)$$
$$= f(x+1) - f(x)$$
$$= (x+1)^3 - (x+1)^2 + (x+1) + 10 - [x^3 - x^2 + x + 10]$$
$$= x^3 + 1^3 + 3x^2 1 + 3x - [x^2 + 1 + 2x] + x + 1 + 10 - x^3 + x^2 - x - 10$$
$$= \cancel{x^3} + 1 + 3x^2 + 3x - \cancel{x^2} \cancel{-1} - 2x + \cancel{x} \cancel{+1} \cancel{+10} \cancel{-x^3} + \cancel{x^2} \cancel{-x} \cancel{-10}$$
$$= 3x^2 + 3x + 1 - 2x$$

$\therefore \Delta f(x) = 3x^2 + x + 1$

2. find $\Delta^2 f(x)$, given $f(x) = e^{2x}$, $h=1$

solu) Since $\Delta f(x) = f(x+h) - f(x)$

we know that

$\Delta f(x) = f(x+1) - f(x)$

$= e^{2(x+1)} - e^{2x}$

$= e^{2x+2} - e^{2x}$

$= e^{2x} \cdot e^2 - e^{2x}$

$\Delta f(x) = e^{2x}(e^2 - 1)$

$\Delta e^{2x} = e^{2x}(e^2-1) \rightarrow (1)$

$\Delta^2 f(x) = \Delta[\Delta f(x)]$

$= \Delta[e^{2x}(e^2-1)]$

$= (e^2-1)[\Delta e^{2x}]$

$= (e^2-1)[e^{2x}(e^2-1)]$ from (1)

$\therefore \Delta^2 f(x) = (e^2-1)^2 e^{2x}$

3. If $f(x) = \frac{10}{x!}$ find $\Delta f(x)$ and $h=1$

solu) $\Delta f(x) = f(x+h) - f(x)$

$= f(x+1) - f(x)$

$= \frac{10}{(x+1)!} - \frac{10}{x!} \Rightarrow \frac{10}{(x+1)!x!} - \frac{10}{x!}$

$= \frac{10 - 10(x+1)}{(x+1)!\,x!}$

$= \frac{10[1-x-1]}{(x+1)!}$

$= \frac{-10x}{(x+1)!}$

4. Find $\Delta^2 \ldots$

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7 Show that $\delta^2 E = \Delta^2$

Solu) $\delta^2 E = \Delta^2$

$\delta = E^{1/2} - E^{-1/2}$

$\Delta = E - 1$

$L.H.S = (E^{1/2} - E^{-1/2})^2 E$

$= \left((E^{1/2})^2 + (E^{-1/2})^2 - 2E^{1/2}E^{-1/2}\right) E$

$= [E + E^{-1} - 2] E$

$= E^2 + E^{-1} \cdot E - 2E$

$= [E^2 + 1 - 2E \cdot 1]$

$= [E-1]^2$

$= \Delta^2 = R.H.S$

$L.H.S = R.H.S$

Hence proved.

8 Show that $\mu\delta = \frac{E - E^{-1}}{2}$

Solu) $\mu = \frac{E^{1/2} + E^{-1/2}}{2}$ $\quad \delta = E^{1/2} - E^{-1/2}$

$L.H.S = \left[\frac{E^{1/2} + E^{-1/2}}{2}\right]\left[E^{1/2} - E^{-1/2}\right]$

$= \frac{(E^{1/2})^2 - (E^{-1/2})^2}{2}$

$= \frac{E^1 - E^{-1}}{2}$

$= R.H.S$

$E = 1 + \Delta$

$E^{-1} = 1 - \nabla$

$\mu = \frac{E^{1/2} + E^{-1/2}}{2}$

$\delta = \frac{E^{1/2} - E^{-1/2}}{2}$

9 Show that $\Delta = \nabla(1-\nabla)^{-1}$

Solu) $\Delta = \nabla(1-\nabla)^{-1}$

$1 - \nabla = E^{-1}$

$R.H.S = \nabla(E^{-1})^{-1}$

$= \nabla E, \quad \nabla = \boxed{1 - E^{-1}}$

$= (1-E^{-1})E$

$= E - E^{-1} \cdot E$

$= E - 1$

$= \Delta$

$= R.H.S$

10. Write Forward difference table for

x :	10	20	30	40
y :	1.1	2.0	4.4	7.9

Solu) Forward Difference Table

x	y	Δ	Δ^2	Δ^3
10	1.1			
		$= 2.0 - 1.1 = 0.9$		
20	2.0		$= 2.4 - 0.9 = 1.5$	
		$= 4.4 - 2.0 = 2.4$		$= 1.1 - 1.5 = -0.4$
30	4.4		$= 3.5 - 2.4 = 1.1$	
		$= 7.9 - 4.4 = 3.5$		
40	7.9			

why we took only (3) when

11. Constiuct the difference table for the given data and evaluate $\Delta^2 f(2)$

x :	0	1	2	3	4
f(x) :	1.0	1.5	2.2	3.1	4.6

Solu) Difference table

x	f(x)	1st	2nd	3rd	4th
0	1.0				
		$1.5 - 1.0 = 0.5$			
1	1.5		$0.7 - 0.5 = 0.2$		
		$2.2 - 1.5 = 0.7$		$0.2 - 0.2 = 0.0$	
2	2.2		$0.9 - 0.7 = 0.2$		$0.4 - 0.0 = 0.4$
		$3.1 - 2.2 = 0.9$		$0.6 - 0.2 = 0.4$	
3	3.1		$1.5 - 0.9 = 0.6$		
		$4.6 - 3.1 = 1.5$			
4	4.6				

why we should do forward

In the above question Δ is given so that

From the difference table $\Delta^2 f(2) = 0.6$

Forward starts with y_0

Note in backward starts y_0, y_1, y_2, y_3

From the difference table $\nabla^2 f(2) = 0.2$

12. Find the missing value of the following data.

x : 1 2 3 4 5

$f(x)$: 7 - 13 21 37

Solu) Difference table

x	$f(x)$	1st	2nd	3rd	4th
1	7				
		$y-7$			
2	y		$(13-y)-(y-7) = 20-2y$		
		$13-y$		$y-5-20+2y = 3y-25$	
3	13		$8-(13-y) = y-5$		$13-y-3y+25 = 38-4y$
		$21-13 = 8$		$8-y+5 = 13-y$	
4	21		$16-8 = 8$		
		$37-21 = 16$			
5	37				

[$(13-y)(y-7)$

$13y - y^2 + 7y - 91$

$13\ 6y - y^2 - 91$

$y^2 - 6y + 91$]

From the Difference table

$38 - 4y = 0$

$38 = 4y$

$y = 9.5$

4)38(9.5
36
20

13
7
91

13. Prove that $U_4 = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$

Solu) $R.H.S = u_3 + \Delta u_2 + \Delta^2 u_1 + \Delta^3 u_1$

$= u_3 + \Delta u_2 + \Delta^2 u_1 + (\Delta^2 u_2 - \Delta^2 u_1)$

$= u_3 + \Delta u_2 + \cancel{\Delta^2 u_1} + \Delta^2 u_2 - \cancel{\Delta^2 u_1}$

$= u_3 + \Delta u_2 + \Delta^2 u_2$

$= u_3 + \cancel{\Delta u_2} + (\Delta u_3 - \cancel{\Delta u_2})$

$= u_3 + \Delta u_3$

$= \cancel{u_3} + u_4 - \cancel{u_3}$

$= U_4 = R = L.H.S$

14. Evaluate $u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$

Solu) $[= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$

$= u_0 + 4(u_1 - u_0) + 6(\Delta u_0 - \Delta u_{-1}) + 10(\Delta^2 u_0 - \Delta u_{-1})$

$= u_0 + 4u_1 - 4u_0 + 6\Delta u_0 - 6\Delta u_{-1} + 10\Delta^2 u_0 - 10\Delta u^2_{-1}$

$= u_0 + 6\Delta u_0 + 10\Delta^2 u_0 + 4u_1 - 6\Delta u_{-1} - 10\Delta u_{-1}$

$= u_0 + 6\Delta u_0 + 10\Delta^2 u_0 + 4u_1 - 10\Delta u^2_{-1} - 6\Delta u_{-1}]$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^3 u_{-1}$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10[\Delta^2 u_0 - \Delta^2 u_{-1}]$

$= u_0 + 4\Delta u_0 + 6\Delta^2 u_{-1} + 10\Delta^2 u_0 - 10\Delta^2 u_{-1}$

$= u_0 + 4\Delta u_0 + 10\Delta^2 u_0 - 4\Delta^2 u_{-1}$

$= u_0 + 4\Delta u_0 + 10\Delta^2 u_0 - 4(\Delta u_0 - \Delta u_{-1})$

$= u_0 + 4\cancel{\Delta u_0} + 10\Delta^2 u_0 - 4\cancel{\Delta u_0} + 4\Delta u_{-1}$

$= u_0 + 10\cdot\Delta^2 u_0 + 4\Delta u_{-1}$

$= u_0 + 10[\Delta u_1 - \Delta u_0) + 4(u_0 - u_{-1})$

$= u_0 + 10\Delta u_1 - 10\Delta u_0 + 4u_0 - 4u_{-1}$

$= u_0 + 10[u_2 - u_1] - 10[u_1 - u_0] + 4u_0 - 4u_{-1}$

$= 10u_2 - 20u_1 + 15u_0 - 4u_{-1}$

15. Evaluate $\Delta(e^{ax}\log(bx))$

solu) $\Delta f(x) = f(x+h) - f(x)$

$= e^{a(x+h)}\log b(x+h) - e^{ax}\log(bx)$

16. u_x is a function of x for which 5th differences are constant and $u_1 + u_7 = -786$; $u_2 + u_6 = 686$. $u_3 + u_5 = 1088$ find u_4

solu) Since Given that 5th differences are constants

$\therefore \Delta^6 u_1 = 0$.

Since we know that $\Delta = E - 1$ why we took Δ (Deu) here

$\therefore (E-1)^6 u_1 = 0$

$[1.E^6 - 6c_1 E^5 + 6c_2 E^4 - 6c_3 E^3 + 6c_4 E^2 - 6c_5 E + 6c_6\, 1]u_1 = 0$

$E^6 u_1 - 6Eu_1^5 + \frac{6\times5}{1\times2}Eu_1^4 - \frac{6\times5\times4}{1\times2\times3}Eu_1^3 + \frac{6\times5\times4\times3}{1\times2\times3\times4}Eu_1^2 - \frac{6\cdot5\cdot4\cdot3\cdot2}{1\cdot2\cdot3\cdot4\cdot5}Eu + {}^6_1 u_1 = 0$

$E^6 u_1 - 6E^5 u_1 + 15E^4 u_1 - 20E^3 u_1 + 15E^2 u_1 - 6Eu_1 + u_1 = 0$

$u_7 - 6u_6 + 15u_5 - 20u_4 + 15u_3 - 6u_2 + u_1 = 0$

$(u_7 + u_1) - 6(u_6 + u_2) + 15(u_5 + u_3) - 20u_4 = 0$

$-786 - 6(686) + 15(1088) - 20u_4 = 0$

$-786 - 4116 + 16320 - 20u_4 = 0$

$-4902 + 16320 = 20u_4$

$20u_4 = 11418$

$\therefore u_4 = \frac{11418}{20}$

$u_4 = 570.9$

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Note:

$$e^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots+$$

Since we know that $Ef(x) = f(x+h)$ — by Taylor's series formula

From what we know

$$= f(x)+hf'(x)+\frac{h^2}{2!}f''(x)+\frac{h^3}{3!}f'''(x)+\frac{h^4}{4!}f''''(x)\dots$$

$$= f(x)+h\frac{d}{dx}f(x)+\frac{h^2}{2!}\frac{d^2}{dx^2}f(x)+\frac{h^3}{3!}\frac{d^3}{dx^3}f(x)+\frac{h^4}{4!}\frac{d^4}{dx^4}f(x)+\dots$$

$$= f(x)\left[1+h\frac{d}{dx}+\frac{h^2}{2!}\frac{d^2}{dx^2}+\frac{h^3}{3!}\frac{d^3}{dx^3}+\frac{h^4}{4!}\frac{d^4}{dx^4}+\dots\right]$$

$$= f(x)\left[1+hD+\frac{h^2}{2!}D^2+\frac{h^3D^3}{3!}+\frac{h^4D^4}{4!}\right]$$

$$= f(x)\left[1+hD+\frac{(hD)^2}{2!}+\frac{(hD)^3}{3!}+\frac{(hD)^4}{4!}+\dots\right]$$

$\left[\because \text{Here } \frac{d}{dx}=D\right]$

$$\therefore Ef(x) = f(x).e^{hD}$$

$\boxed{\therefore E = e^{hD}}$ (or) $E = 1+\Delta \Rightarrow 1+\Delta = e^{hD}$

$\Delta = e^{hD}-1$

17. Show that $\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n n!\, h^n}{x(x+h)(x+2h)\dots(x+nh)}$

Soln $\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n n!\, h^n}{x(x+h)(x+2h)\dots(x+nh)}$

$\therefore \Delta f(x) = f(x+h)-f(x)$.

Now $n=1$

$$\Delta\left[\frac{1}{x}\right] = \frac{1}{x+h}-\frac{1}{x}$$

$$= \frac{x-(x+h)}{x(x+h)}$$

$$= \frac{x-x-h}{x(x+h)}$$

$$= \frac{(-1)h}{x(x+h)} \longrightarrow (1)$$

$n=2$

$$\Delta^2\left[\frac{1}{x}\right] = \Delta\left[\Delta\left[\frac{1}{x}\right]\right]$$

$$= \Delta\left[\frac{(-1)h}{x(x+h)}\right]$$

$$= h(-1)\left[\Delta\frac{1}{x(x+h)}\right]$$

$$= h(-1)\left[\frac{1}{(x+h)(x+h+h)}-\frac{1}{x(x+h)}\right]$$

$$= (-1)h\left[\frac{x-(x+2h)}{x(x+h)(x+2h)}\right]$$

$$= (-1)h\left[\frac{\cancel{x}-\cancel{x}-2h}{x(x+h)(x+2h)}\right]$$

$$= \frac{(-1)^2 2 h^2}{x(x+h)(x+2h)}$$

$$= \frac{(-1)^2 2.1 h^2}{x(x+h)(x+2h)}$$

$$= \frac{(-1)^2 2! h^2}{x(x+h)(x+2h)} \longrightarrow (2)$$

If $n=3$

$$\Delta^3\left[\frac{1}{x}\right] = \Delta\left[\Delta^2\left[\frac{1}{x}\right]\right]$$

$$= \Delta\left[\frac{(-1)^2 2! h^2}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2 2! h^2 \left[\Delta\left(\frac{1}{x(x+h)(x+2h)}\right)\right]$$

$$= (-1)^2 2! h^2\left[\frac{1}{(x+h)(x+h+h)(x+h+2h)} - \frac{1}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2 2! h^2\left[\frac{1}{(x+h)(x+2h)(x+3h)} - \frac{1}{x(x+h)(x+2h)}\right]$$

$$= (-1)^2 2! h^2\left[\frac{x-(x+3h)}{x(x+h)(x+2h)(x+3h)}\right]$$

$$= (-1)^2 2! h^2\left[\frac{\cancel{x}-\cancel{x}-3h}{x(x+h)(x+2h)(x+3h)}\right]$$

$$= \frac{(-1)^2 2! h^2 (-1) 3h}{x(x+h)(x+2h)(x+3h)}$$

$$= \frac{(-1)^3 \; 1\times2\times3 \; h^3}{x(x+h)(x+2h)(x+3h)}$$

$$\therefore \Delta^3\left[\frac{1}{x}\right] = \frac{(-1)^3 \, 3! \, h^3}{x(x+h)(x+2h)(x+3h)} \longrightarrow (3)$$

Hence from (1), (2) & (3)

$$\Delta^n\left[\frac{1}{x}\right] = \frac{(-1)^n n! h^n}{x(x+h)(x+2h)\cdots(x+nh)}$$

Date 5/8/18

18. Given, $u_0+u_8 = 1.9243$, $u_1+u_7 = 1.9590$. $u_2+u_6 = 1.9823$
$u_3+u_5 = 1.9956$ then find u_4.

solu) Since
$\Delta^8 u_0 = 0$

$(E-1)^8 u_0 = 0$

$u_0[1 \cdot E^8 - 8c_1E^7 + 8c_2E^6 + 8c_3E^5 + 8c_4E^4 + 8c_5E^3 + 8c_6E^2 + 8c_7E + 8c_8] = 0$

$u_0E^8 - 8E^7u_0 + \frac{8\times 7}{1\times 2}E^6u_0 + \frac{8\times 7\times 6}{1\times 2\times 3}E^5u_0 + \frac{8\times 7\times 6\times 5}{1\times 2\times 3\times 4}E^4u_0$

$+ \frac{8\times 7\times 6\times 5\times 4}{1\times 2\times 3\times 4\times 5}E^3u_0 + \frac{8\times 7\times 6\times 5\times 4\times 3}{1\times 2\times 3\times 4\times 5\times 6}E^2u_0 + \frac{8\times 7\times 6\times 5\times 4\times 3\times 2}{1\times 2\times 3\times 4\times 5\times 6\times 7}E\,u_0$

$+ 8u_0 = 0$ $+ u_0 = 0$

$u_8 - 8u_7 + 28u_6 - 56u_5 + 70u_4 - 56u_3 + 28u_2 - 8u_1 + u_0 = 0$

$70u_4 + (u_0+u_8) - 8(u_1+u_7) + 28(u_2+u_6) - 56(u_3+u_5) = 0$

$70u_4 + 1.9243 - 8(1.9590) + 28(1.9823) - 56\cdot(1.9956) = 0$

$70u_4 + 1.9243 - 15.672 + 55.5044 - 111.7536 = 0$

$69.9969 = 70u_4$

$u_4 = \frac{69.9969}{70}$

$u_4 = 0.999955714$

$\therefore u_4 = 1$

19. find the missing term in the following

x:	0	5	10	15	20	25
y:	6 (y_0)	10 (y_1)	— (y_2)	17 (y_3)	— (y_4)	31 (y_5)

solu) Consider $\Delta^4 y_0 = 0 \to ①$ $\Delta^4 y_1 = 0 \to ②$ and we know that
$\Delta = E-1$ sub in ① & ②

$(E-1)^4 y_0 = 0$ and $(E-1)^4 y_1 = 0$

$\Rightarrow [1\cdot E^4 - 4c_1E^3 + 4c_2E^2 + 4c_3E + 4c_4] y_0 = 0$ &

$[1\cdot E^4 - 4c_1E^3 + 4c_2E^2 + 4c_3E + 4c_4] y_1 = 0$

$[1\cdot E^4 - 4c_1E^3 + 4c_2E^2 + 4c_3E + 4] y_0 = 0$ &]

$[\Rightarrow) [u_4 - 4a_3 + \frac{4\times 3}{1\times 2}u_2 + \frac{4\times 3\times 2}{1\times 2\times 3}u_1 + 4$

$\Rightarrow E^4y_0 - 4E^3y_0 + \frac{4\times 3}{1\times 2}y_0E^2 - \frac{4\times 3\times 2}{1\times 2\times 3}Ey_0 + 1\cdot y_0 = 0$ &

$E^4y_1 - 4E^3y_1 + \frac{4\times 3}{1\times 2}y_1E^2 - \frac{4\times 3\times 2}{1\times 2\times 3}Ey_1 + 1\cdot y_1 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0 \rightarrow (3)$

$y_5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0 \rightarrow (4)$

from (3)

$[17 - 4y_3 + 6(10) - 4(6) +]$

$\Rightarrow y_4 - 4(17) + 6y_2 - 4(10) + 6 = 0$

$\Rightarrow y_4 + 6y_2 - 68 - 40 + 6 = 0$

$\Rightarrow y_4 + 6y_2 - 102 = 0$

$\Rightarrow y_4 + 6y_2 = 102 \rightarrow (5)$

from (4)

$31 - 4y_4 + 6(17) - 4y_2 + 10 = 0$

$31 + 102 + 10 = y_4 + 4y_2$

$143 = 4y_4 + 4y_2 \rightarrow (6)$

$\left.\begin{array}{l} 6y_2 + y_4 - 102 = 0 \\ 4y_2 + 4y_4 - 143 = 0 \end{array}\right\}$ By 2312

$\therefore y_2 = 13.25, \quad y_4 = 22.5$

$20)265(13$ $\quad 20 \quad 65 \quad 60 \quad 50$

$102 \quad 4 \quad 408$ $\quad 858 \quad 408 \quad 450$

$y_2 \quad y_4$

$1 \quad -102 \quad 6 \quad 1$

$4 \quad -143 \quad 4 \quad 4$

$\frac{y_2}{-143 + 408} = \frac{y_4}{-408 + 858} = \frac{1}{24-4}$

$\frac{y_2}{265} = \frac{y_4}{450} = \frac{1}{20}$

$y_2 = \frac{265}{20} = 13.25; \quad y_4 = \frac{450}{20} = 22.5$

$408 \quad 143 \quad 265$

20: find the missing value of the following table

x	1	2	3	4	5
y	7 y_0	x y_1	13 y_2	21 y_3	37 y_4

solu) $\Delta^4 y_0 = 0$ since we know that

$\therefore E = 1 + \Delta$

$\Delta = E - 1$

$(E-1)^4 y_0 = 0$

$[1 \cdot E^4 - 4c_1 E^3 + 4c_2 E^2 - 4c_3 E + 4c_4] y_0 = 0$

$E^4 y_0 - 4E^3 y_0 + \frac{4\times 3}{1\times 2} E^2 y_0 - \frac{4\times 3\times 2}{1\times 2\times 3} E y_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$37 - 4(21) + 6(13) - 4x + 4(7) = 0$

$37 - 84 + 68 - 4x + 28 = 0$

$65 + 68 - 84 = 4x$

$3 - 81 = 4x$

$-78 = 4x$

$7 - 4x =$

$7 = 4x$

$x = \frac{7}{4}$

$4)7(1.7 \quad 4 \quad 30 \quad 28$

$84 \quad 54 \quad 30$

$37 \quad 28 \quad 65$

$37 - 4\times21 + 6\times13 - 4y_1 + 7 = 0$

$-84 \quad -4y_1 + 37 + 7 + 78 = 0$

$-4y_1 + 38 = 0$

$4y_1 = 38$

$y_1 = 9.5$

21. Estimate the missing term in the following table.

x	1	2	3	4	5	6	7
y	2	4	8	–	32	64	128
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

solu) Since we know that

$\Delta^6 y_0 = 0 \qquad E = 1+\Delta$

$\qquad\qquad\qquad \Delta = E-1$

$(E-1)^6 y_0 = 0$

$[E^6.1 - 6c_1E^5 + 6c_2E^4 - 6c_3E^3 + 6c_4E^2 - 6c_5E + 6c_6]\, y_0 = 0$

$E^6y_0 - 6E^5y_0 + 15E^4y_0 - \frac{6.5.4}{1.2.3}E^3y_0 + \frac{6.5.4.3}{1.2.3.4}E^2y_0 - \frac{6.5.4.3.2}{1.2.3.4.5}Ey_0 + y_0$

$y_6 - 6y_5 + 15y_4 - 20y_3 + 15y_2 - 6y_1 + y_0 = 0$

$128 - 6(64) + 15(32) - 20(y_3) + 15(8) - 6(4) + 2 = 0$

$128 - 384 + 480 - 20y_3 + 120 - 24 + 2 = 0$

$[730 - 284 - 24 - 20y_3 = 0 \qquad 322 = 20y_3$

$730 - 308 = 20y_3 \qquad y_3 = \frac{322}{20}$

$422 = 20y_3 \qquad y_3 = 16.1$

$y_3 = 21.1]$

22. Given $\log 100 = 2$; $\log 101 = 2.0043$; $\log 103 = 2.0128$; $\log 104 = 2.0170$ and find $\log 102$.

solu) Here given

x	100	101	102	103	104
$y = \log x$	2	2.0043	–	2.0128	2.0170
	y_0	y_1	y_2	y_3	y_4

Since we know that

$\Delta^4 y_0 = 0$

$\Delta = E-1$

From $E = 1+\Delta$

$(E-1)^4 y_0 = 0$

$[1.E^4 - 4c_1E^3 + 4c_2^2E - 4c_3E + 4c_4]y_0 = 0$

$E^4y_0 - 4E^3y_0 + \frac{4\times3}{1\times2}E^2y_0 - \frac{4\times3\times2}{1\times2\times3}Ey_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$2.0170 - 4(2.0128) + 6y_2 - 4(2.0043) + 2 = 0$

$2.0170 - 8.0512 + 6y_2 - 8.0172 + 2 = 0$

$4.0170 - 16.0684 + 6y_2 = 0$

$6y_2 = \frac{12.0514}{6}$

$\therefore y_2 = 2.0086$

$\therefore \log_{10}2 = 2.0086$

2.0170
2
4.0170
0.0340
8.0512
8.0172
0.0340
16.0684

23. find the missing values of the following.

x	10	15	20	25	30	35
y	43 y_0	- y_1	29 y_2	32 y_3	- y_4	77 y_5

Solu) Since we know that

$\Delta^4 y_0 = 0$; $E = 1+\Delta \Rightarrow \Delta = E-1$

$(E-1)^4 y_0 = 0$

$[1.E^4 - 4c_1E^3 + 4c_2E^2 + 4c_3E + 4c_4]y_0 = 0$

$E^4y_0 - 4E^3y_0 + 6E^2y_0 - 4Ey_0 + y_0 = 0$

$y_4 - 4y_3 + 6y_2 - 4y_1 + y_0 = 0$

$y_4 - 4(32) + 6(29) - 4y_1 + 43 = 0$

$y_4 - 128 + 174 - 4y_1 + 43 = 0$

$y_4 - 4y_1 = 128 - 174 - 43$

$y_4 - 4y_1 = 128 - 217$

$y_4 - 4y_1 = -89 \to (1)$ (or) $4y_1 - y_4 = 89 \to (1)$

$-(-y_4 + 4y_1) = -89$

$4y_1 - y_4 = 89$

174
43
217

217
128
89

$\Delta^4 y_1 = 0$

$(E-1)^4 y_1 = 0$

$[1.E^4 - 4c_1E^3 + 4c_2E^2 - 4c_3E + 4c_4]y_1 = 0$

$E^4y_1 - 4E^3y_1 + 6E^2y_1 - 4Ey_1 + y_1 = 0$

$y^5 - 4y_4 + 6y_3 - 4y_2 + y_1 = 0$.

$77 - 4y_4 + 6(32) - 4(29) + y_1 = 0$

$y_1 - 4y_4 + 77 + 192 - 116 = 0$

$y_1 - 4y_4 = 116 - 192 - 77$

$y_1 - 4y_4 = 116 - 269$

$y_1 - 4y_4 = -153 \rightarrow (2)$

y_1	y_4	1
-1	-89	-4	-1
-4	153	1	-4

$$\frac{y_1}{-153-356} = \frac{y_4}{-89-612} = \frac{1}{-16+1}$$

$$\frac{y_1}{-509} = \frac{y_4}{-701} = \frac{1}{-15}$$

$y_1 = \frac{-509}{-15}$; $y_4 = \frac{-701}{-15}$

$y_1 = 33.9334$; $y_4 = 46.7334$

192 + 77 = 269; 269 − 116 = 153; 153 × 4 = 612; 89 × 4 = 356; 356 + 153 = 509; 89 + 612 = 1502

Date 6/8/18

24\. Estimate the production for 1964 and 1968 from the following data.

years (x)	1961	1962	1963	1964	1965	1966	1967
production (y)	200 (y_0)	220 (y_1)	260 (y_2)	— (y_3)	350 (y_4)	— (y_5)	430 (y_6)

solu) Given that years x = 1961, 1962, 1963, 1964, 1965, 1966, 1967

y = 200, 220, 260 — 350 — 430

$\Delta^5 y_0 = 0 \rightarrow (1)$ $E = 1 + \Delta$

$(E-1)^5 y_0 = 0$

$[1 \cdot E^5 - 5C_1 E^4 + 5C_2 E^3 - 5C_3 E^2 + 5C_4 E - 5C_5]\, y_0 = 0$

$y_0 E^5 - 5 y_0 E^4 + 5C_2 E y_0 - 5C_3 E y_0 + 5C_4 E y_0 - 5C_5 y_0$

$$y_5 - 5y_4 + \frac{5 \times 4}{1 \times 2} y_3 - \frac{5 \times 4 \times 3}{1 \times 2 \times 3} y_2 + \frac{5 \times 4 \times 3 \times 2}{1 \cdot 2 \cdot 3 \cdot 4} y_1 + y_0 = 0$$

$y_5 - 5y_4 + 10y_3 - 10y_2 + 5y_1 + y_0 = 0$

$y_5 - 5(350) + 10y_3 - 10(260) + 5(220) + 200 = 0$

$y_5 - 1750 + 10y_3 - 2600 + 1100 + 200 = 0$

$y_5 + 10y_3 - \cancel{13050}\ 3450 = 0 \rightarrow (2)$

13050 − 200 = 12850

Scanned with CamScanner

$\Delta^5 y_1 = 0 \rightarrow (3)$

$y_6 \cdot (E-1)^5 y_1 = 0$

$[1 \cdot E^5 - 5c_1 E^4 + 5c_2 E^3 - 5c_3 E^2 + 5c_4 E - 5c_5] y_1 = 0$

$E^5 y_1 - 5c_1 E^4 y_1 + 5c_2 E^3 y_1 - 5c_3 E^2 y_1 + 5c_4 E y_1 - 5c_5 y_1 = 0$

$y_6 - 5y_5 + 10y_4 - 10y_3 + 5y_2 - y_1 = 0$

$430 - 5y_5 + 10(350) - 10y_3 + 5(260) - 220 = 0$

$430 - 5y_5 + 3500 - 10y_3 + 1300 - 220 = 0$

$5010 - 5y_5 - 10y_3 = 0$

$5y_5 + 10y_3 = 5010 \rightarrow (4)$

From (2) and (4)

$y_5 + 10y_3 = 3450$

$5y_5 + 10y_3 = 5010$

$-4y_5 = -1560$

$y_5 = \frac{1560}{4} = 390$

$y_5 + 10y_3 = 3450$

$390 + 10y_3 = 3450$

$10y_3 = 3450 - 390$

$10y_3 = 3060$

$y_3 = 306$

From (1) & (2)

$y_3 \quad y_5 \quad 1$

$6 \quad -706 \quad 20 \quad 6$

$15 \quad -1196 \quad 15 \quad 15$

$\frac{y_3}{-7176+10590} = \frac{y_5}{-10590+2392} = \frac{1}{300-90}$

$\frac{y_3}{3414} = \frac{y_5}{13330} = \frac{1}{210}$

$y_3 = \frac{3414}{210}$; $y_5 = \frac{13330}{210}$

$y_3 = 16.2571$ $y_5 = 6.33$

63.47

31. Fit a polynomial of degree 3 and hence determine $y(3.5)$ for the following data.

x: 3 4 5 6

y: 6 24 60 120

Difference table.

x	y	1st	2nd	3rd
3	6			
		18		
4	24		18	
		36		6
5	60		24	
		60		
6	120			

By Newton's forward Interpolation Formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x - x_0}{h} = \frac{x-3}{1} = x-3$$

$X = x \quad x_0 = 3 \quad h = 1$

$$y(\overset{x}{3.5}) = 6 + (x-3)\,18 + \frac{(x-3)(x-3-1)}{2!}\,18 + \frac{(x-3)(x-3-1)(x-3-2)}{3!}\,6$$

$$= 6 + 18x - 54 + \frac{(x-3)(x-4)}{\cancel{2}} \times \cancel{18}^{\,9} + \frac{(x-3)(x-4)(x-5)}{\cancel{6}} \times \cancel{6}$$

$$= 6 + 18x - 54 + (x^2 - 3x - 4x + 12)\,9 + [x^2 - 3x - 4x + 12](x-5)$$

$$= \cancel{6} + 18x - \cancel{54} + 9x^2 - 27x - 36x + \cancel{108} + x^3 - 3x^2 - 4x^2 + 12x - 5x^2 + 15x + 20x - \cancel{60}$$

$$= x^3 - 3x^2 + 2x$$

put x 3.5

$$y(3.5) = (3.5)^3 - 3(3.5)^2 + 2(3.5)$$

$$= 42.875 - 3(12.25) + 7$$

$$= 42.875 - 36.75 + 7$$

$$\therefore y(3.5) = 13.125$$

$\frac{18}{3}\,2$ 34 27 $\frac{36}{-63}$ 18 12 35 65 $\frac{63}{2}$ 47 $\frac{36}{+11}$ $\frac{27}{4}$ 42.875 $\frac{36.75}{6.125}$ 7

32. Find the cubic polynomial which takes the following values.

$y(0) = 1, \quad y(1) = 0, \quad y(2) = 1, \quad y(3) = 10.$

Hence obtain $y(u)$

$y(0)=1, \quad y(1)=0 \quad y(2)=1. \quad y(3)=10$

Difference table

x	y	1st	2nd	3rd
0	1			
		-1		
1	0		2	
		1		6
2	1		8	
		9		
3	10			

Newtons forward interpolation formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x - x_0}{h} = \frac{x-0}{1} = x$$

$x = x \quad x_0 = 0, \quad h = 1$

$$y_n = 1 + x(-1) + \frac{x(x-1)\cancel{2}}{\cancel{2}} + \frac{x(x-1)(x-2)}{\cancel{6}}\cancel{6}$$

$$= 1 - x + x^2 - x + (x^2 - x)(x-2)$$

$$= 1 - \cancel{x} + x^2 \cancel{-x} + x^3 - x^2 - 2x^2 \cancel{+2x}$$

$$= x^3 - 2x^2 + 1$$

put $x = 4$

$$y(u) = u^3 - 2(u)^2 + 1$$

$$= 64 - 32 + 1$$

$\therefore y(u) = 33 \quad [0, 3]$ interval 'u' is out of intravel so it is called extrapolation.

33 find the polynomial interpolating the data

x : 0 1 2

y : 0 5 2 Difference Table

x	y	1st	2nd
0	0		
		5	
1	5		-8
		-3	
2	2		

Newton's forward Interpolation formula

$$y_n = y_n + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{X - X_0}{h} = \frac{x-0}{1} = x$$

$X = x \quad X_0 = 0 \quad h = 1$

$$[y_n = x + x(0) + \frac{x(x-1)}{2} 5 + \frac{x(x-1)(x-2)}{\cancel{6}\,3} \overset{4}{\cancel{(-8)}})$$

$$= x + 0 + \frac{(x^2-x)5}{2} + \frac{(x^2-x)(x-2)}{3}(-4)$$

$$= x + \frac{5x^2-5x}{2} + \frac{(x^3-x^2-2x^2+2x)}{3}(-4)$$

$$= x + 5x^2 - 5x]$$

$$y_n = 0 + x.5 + \frac{x(x-1)}{\cancel{2}} x(\cancel{-8})4$$

$$= 5x - (x^2-x)4$$

$$= 5x - 4x^2 + 4x$$

$$\therefore y_n = -4x^2 + 9x$$

34. Find the polynomial of deg(4) which takes the following values

x :	2	4	6	8	10
y :	0	0	9	0	0

35. Use Newtons Forward Difference formula to obtain the interpolating polynomial $f(x)$ satisfying the following data

x :	1	2	3	4
y :	26	18	4	1

and find and $x = 5$

solu| Form the Difference table

$$\begin{array}{r}18\\2\end{array} \qquad \begin{array}{r}26\\18\\\hline 8\end{array}$$

35.

x	y	1st	2nd	3rd
1	26			
		-8		
2	18		-6	
		-14		16
3	4		11	
		-3		
4	1			

17 Mistake Practice well

From Newton's Interpolation forward formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0$$

$$n = \frac{x-x_0}{h} = \frac{x-1}{1} = x-1$$

$x = x;\ x_0 = 1;\ h = 1$

$$y_n = 26 + (x-1)(-8) + \frac{(x-1)(x-1-1)}{\cancel{2}}\overset{3}{(-\cancel{6})} + \frac{(x-1)(x-1-1)(x-2-1)}{\cancel{6}\,3}$$

$$= 26 - 8x + 8 - (3x-3)(x-1-1) + \frac{(x-1)(x-2)(x-3)8}{3}$$

$$= 26 - 8x + 8 - (3x-3)(x-2) + \frac{[x^2 - x - 2x + 2](x-3)}{3} \times 8$$

$$= 26 - 8x + 8 - [3x^2 - 3x - 6x + 6] + [x^3 - x^2 - 2x^2 + 2x - 3x^2 + 3x + 6x - 6] \times \frac{8}{3}$$

$$= 26 - 8x + 8 - 3x^2 + 9x - 6 + [x^3 - 6x^2 + 11x - 6] \times \frac{8}{3}$$

$$= 26 - 8x + 8 - 3x^2 + 9x - 6 + [x^3 - 6x^2 + 11x - 6] \times \frac{8}{3}$$

$$= 78 - 24x + 24 - 9x^2 + 27x - 18 + (x^3 - 6x^2 + 11x - 6)8$$

$$= 84 - 24x - 9x^2 + 27x + 8x^3 - 48x^2 + 88x - 48$$

$$y_n = 8x^3 - 57x^2 + 91x + 36$$

$26 \times 3 = 78$; $84 - 48 = 36$; $78 + 24 = 102$; 18

put $x = 5$

$$y(5) = 8(5)^3 - 57(5)^2 + 91(5) + 36$$

$$= 8(125) - 57(25) + 455 + 36$$

$$= 1000 - 1425 + 455 + 36$$

$$\therefore y(5) = 66$$

34. Forming the difference table

x	y	1st	2nd	3rd	4th
2	0				
		0			
4	0		1		
		1		-3	
6	1		-2		+6
		-1		3	
8	0		1		
		0			
10	0				

From Newtons Forward interpolation formula

$$y_n = y_n + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4$$

$$n = \frac{x - x_0}{h} = \frac{x-2}{2}$$

$$y_n = 0 + \frac{x-2}{2}\,0 + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)}{2}\,1 + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)\left(\frac{x-2}{2}-2\right)}{6}(-3)$$

$$+ \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-2}{2}-1\right)\left(\frac{x-2}{2}-2\right)\left(\frac{x-2}{2}-3\right)}{24}\,6$$

$$y_n = \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)}{2} - \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)}{2} + \frac{\left(\frac{x-2}{2}\right)\left(\frac{x-4}{2}\right)\left(\frac{x-6}{2}\right)(\ldots)}{4}$$

$$y_n = \frac{(x-2)(x-4)}{8} - \frac{(x-2)(x-4)(x-6)}{16} + \frac{(x-2)(x-4)(x-6)(x-\ldots)}{64}$$

$$y_n = \frac{x^2-2x-4x+8}{8} - \frac{[x^2-2x-4x+8][x-6]}{16} + \frac{[x^2-2x-4x+8][x^2-6x-8x+48]}{64}$$

$$y_n = \frac{x^2-6x+8}{8} - \frac{[x^3-2x^2-4x^2+8x-6x^2+12x+24x-48]}{16}$$

$$+ \frac{x^4-6x^3-8x^3+48x^2-2x^3+12x+16x^2-96x-4x^3+24x^2+32x^2+192x+8x^2-48x-64x+404}{64}$$

$$y_n = \frac{x^2-6x+8}{8} - \frac{[x^3-12x^2+44x-48]}{16} + \frac{x^4-20x^3+44x+404}{64}$$

$$y_n = \frac{x^3-6x+8}{8} - \frac{x^3+12x^2-44x+48}{16} + \frac{x^4-20x^3+44x+404}{64}$$

$$y_n = \frac{8x^3-48x+64-4x^3+48x^2-176x+192+x^4-20x^3+44x+404}{64}$$

$$y_n = x^4-16x^3+48x^2-180x+660$$

6
48
8
404

192
36
156
48
+108
64
44

404
192
64
160 44
176
176
48
-224
44
188

176
48
4
192

Date 10/7/18

i) find the no. of students from the following data who secured marks not more than 45

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	35 +	48 +	70 +	40 +	22

Difference table

Marks (x) (below)	No. of Students (y)	1st	2nd	3rd	4th
40	35				
		48			
50	83		22		
		70		-52	
60	153		-30		64
		40		12	
70	193		-18		
		22			
80	215				

From Newton's forward interpolation formula.

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

y_n . $n = \frac{x - x_0}{h}$; $x = 45$, $x_0 = 40$, $h = 10$

why we took 40 instead of 30

$$n = \frac{45-40}{10} = \frac{5}{10} = \frac{1}{2} = 0.5$$

$$y(45) = 35 + (0.5)(48) + \frac{(0.5)(0.5-1)}{2!} 22 + \frac{(0.5)(0.5-1)(0.5-2)}{6} \times -52 + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24} \times 64$$

$$y(45) = 35 + 24 + 2.75 + \frac{(0.5)(-0.5)(-1.5)(-26)}{3} + \frac{(0.5)(-0.5)(-1.5)(-2.5)}{3} \times 8$$

$$y(45) = 35 + 24 - 2.75 - 3.25 - 2.5$$

$\therefore y(45) = 50.5$

$\therefore$ No. of students who secured below 45 marks $= 50.5$

$= 51$ (approximate)

ii) No.of students in between 40 and 45 =
No.of students secured 45 marks - No.of students secured 40 marks

$= 51 - 35$

$= 16$

above 45

$= 215 - 51$

$= 164$

3) find the no.of men getting the wages between Rs.10 and Rs.15 from the following table

wages	0-10	10-20	20-30	30-40
Frequency	9 +	30 +	35 +	42

Difference Table

x (below)	y	1st	2nd	3rd
10	9			
		30		
20	39		5	
		35		2
30	74		7	
		42		
40	116			

From Newtons Forward interpolation formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \cdots$$

$n = \frac{x - x_0}{h}$ $x = 15$; $x_0 = 10$; $h = 10$

$n = \frac{15-10}{10} = \frac{5}{10} = \frac{1}{2} = 0.5$

$$y(15) = 9 + 30(0.5) + \frac{(0.5)(0.5-1)}{2} 5 + \frac{(0.5)(0.5-1)(0.5-2)}{\cancel{6}3} \cancel{2}$$

$$= 9 + 15.0 + \frac{(0.5)(-0.5)}{2} . 5 + \frac{(0.5)(-0.5)(-1.5)}{3}$$

$$= 9 + 15 - 0.625 + 0.125$$

$y(15) = 23.5$

$\therefore$ No.of men got the wages below Rs.15 = 23.5

= 24 (approximately)

the wages in between Rs.10 and Rs.15

No.of men who got below Rs.15 - below Rs.10

$= 24 - 9 = 15$

$= 9+42+210+105 \quad = 366$

40 Using Newtons Backward interpolation formula, find $e^{-1.9}$ from the following table

x :	1	1.25	1.5	1.75	2
$y=e^{-x}$:	0.3679	0.2865	0.2231	0.1738	0.1353

Solu) Difference table

x	$y=e^{-x}$	1st	2nd	3rd	4th
1	0.3679				
		~~-0.0321~~ -0.0814			
1.25	0.2865		0.018 ~~0.5687~~		
		-0.0634		-0.0039	
1.5	0.2231		0.0141 ~~0.1127~~		0.0006
		-0.0493		-0.0033	
1.75	0.1738		0.0108		
		-0.0385			
2	0.1353				

From Newton's Backward Interpolation formula

$$y_n = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}$$

$[y_n = 0.1353 +]$ $\quad n = \frac{x - x_0}{h}$ $\quad h = 0.25; \ x = 1.9; \ x_0 = 2$

$$n = \frac{1.9-2}{0.25} = \frac{-0.1}{0.25} = -0.4$$

$$y_{1.9} = 0.1353 + (-0.4)(-0.0385) + \frac{(-0.4)(-0.4+1)(0.0108)}{1.2} + \frac{(-0.4)(-0.4+1)(-0.4+2)}{1.2.3}(-0.0033) + \frac{(-0.4)(-0.4+1)(-0.4+2)(-0.4+3)}{1.2.3.4} \times 0.0006$$

$[y_n = 0.1353 + 0.0154 + 6.264\times10^{-3} - 8.448\times10^{-4}$

$= 3.6756]$

$y_{7.9} = 0.1353 + 0.0154 - 0.00\frac{934}{1296} + 0.0002112 + 0.00002496$

$y_{7.9} = 0.13797614 = 0.138$

41. Find the $\cos(25^\circ)$ and $\cos(75^\circ)$ from the following data.

H.W

x	10	20	30	40	50	60	70	80
$y = \cos x$	0.9848	0.9397	0.866	0.766	0.6428	0.5	0.3420	0.1727

42. Using Newton's backward formulae find the value of y and $x = 36$ from the following data.

xus

x :	21	25	29	33	37
y :	18.4	17.8	17.1	16.3	15.5

Solu) 41.

x	y	1st	2nd	3rd	4th	5th	6th	7th
10	0.9848							
		-0.0451						
20	0.9397		-0.0286					
		-0.0737		0.0023				
30	0.866		-0.0263		0.0008			
		-0.1		0.0031		-0.003		
40	0.766		-0.0232		0.0005		[illegible]	
		-0.1232		0.0036		0.0003		-0.0016
50	0.6428		-0.0196		0.0008		[illegible]	
		-0.1028		0.0044		-0.0013		
60	0.5		-0.0152		-0.0005			
		-0.158		0.0039				
70	0.3420		-0.0113					
		-0.1693						
80	0.1727							

Newton's Forward Interpolation Formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + \frac{n(n-1)(n-2)}{3!}\Delta^3 y_0 + \frac{n(n-1)(n-2)(n-3)}{4!}\Delta^4 y_0$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)}{5!}\Delta^5 y_0 + \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)}{6!}\Delta^6 y_0$$

$$+ \frac{n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)}{7!}\Delta^7 y_0$$

$n = \frac{x - x_0}{h}$; $x = 25$; $x_0 = 10$; $h = 10$ $\quad n = \frac{25-10}{10} = \frac{15}{10} = 1.5$

$$y_n = 0.9848 + 1.5(-0.0451) + \frac{1.5(1.5-1)}{2}(-0.0286) + \frac{(1.5)(1.5-1)(1.5-2)}{6}\,0.002$$

$$+ \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)}{24}\,0.0008 + \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)}{120}(-0.0003)$$

$$+ \frac{(1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)}{720}(0.0006)$$

$$+ (1.5)(1.5-1)(1.5-2)(1.5-3)(1.5-4)(1.5-5)(1.5-6)/5040\,(-0.0016)$$

$$y_n = 0.9848 - 0.06765 - \frac{0.02145}{2} - 0.0625 \times 0.0023$$
$$+ 0.0234375 \times 0.0008 + 7.8125 \times 10^{-4} \times 0.0003 +$$
$$+ 6.510416667 \times 10^{-5} \times 0.0006 + 4.650297619 \times 10^{-6} \times 0.0016$$

$$y_n = 0.9848 - 0.06765 - 0.010725 - 0.00014375 + 0.00001875$$
$$+ 0.00000023475 + 0.0000000390625 + 0.0000000074404761 9$$

$y(\cos(25)) = 0.9063002809$

Newtons Backward Interpolation formulae.

$$y_n = y_n + n\nabla y_n + \frac{n(n+1)}{2!}\nabla^2 y_n + \frac{n(n+1)(n+2)}{3!}\nabla^3 y_n + \frac{n(n+1)(n+2)(n+3)}{4!}\nabla^4 y_n$$
$$+ \frac{n(n+1)(n+2)(n+3)(n+4)}{5!}\nabla^5 y_n + \frac{n(n+1)(n+2)(n+3)(n+4)(n+5)}{6!}\nabla^6 y_n$$
$$+ \frac{n(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}{7!}\nabla^7 y_n$$

$n = \frac{x - x_0}{h}$; $x = 75$, $x_0 = 80$; $h = 10$ $\quad n = \frac{75-80}{10} = \frac{-5}{10} = -0.5$

$$y_n = \cancel{-0.5}$$
$$0.9848 + (-0.5)(-0.0451) + \frac{(-0.5)(-0.5+1)}{2!}(-0.0286)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)}{6} \times 0.0023 + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{24} 0.000$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{120}(-0.0003)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)}{720} \times 0.0006$$

wrong within the brackets

$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)(-0.5+6)}{5040}(-0.0016)$$

$$y_n = 0.9848 + 0.02255 + \frac{0.00715}{2} + \frac{0.0008625}{6} - \frac{0.00075}{24} + \frac{0.00098437}{120}$$
$$- \frac{0.008859375}{720} + \frac{0.1299375}{5040}$$

$$\cos(75) = 0.9848 + 0.02255 + 0.003575 + 0.00014375 - 0.00003125$$
$$+ 0.000008203125 - 0.0000123046875 + 0.00002578125$$

$\cos(75) = 1.01105918$

$$= 0.1727 + (-0.5)(-0.1693) + \frac{(-0.5)(-0.5+1)}{2}(-0.0113)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(0.0039) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(0.0005)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)}{5!}(-0.0013)$$
$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)(-0.5+4)(-0.5+5)}{6!}(-0.001)$$

Date 13/7/18

Legranges Interpolation Formula

Consider $y=f(x)$ be the given function. x takes the values $x_0, x_1, x_2, x_3, x_4, \ldots$ the corresponding y values are $y_0, y_1, y_2, y_3, y_4, \ldots$ respectively. Then

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

Date 14/7/18

1. Using Legranges formula to find $f(6)$ from the following table.

	x_0	x_1	x_2	x_3	x_4
x	2	5	7	10	12
$f(x)$	18	180	448	1210	2028
	y_0	y_1	y_2	y_3	y_4

By Legranges interpolation formula

solu)
$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4$$

$$y(6) = \frac{(6-5)(6-7)(6-10)(6-12)}{(2-5)(2-7)(2-10)(2-12)} \cdot 18 + \frac{(6-2)(6-7)(6-10)(6-12)}{(5-2)(5-7)(5-10)(5-12)} \times 180$$

$$+ \frac{(6-2)(6-5)(6-10)(6-12)}{(7-2)(7-5)(7-10)(7-12)} \times 448 + \frac{(6-2)(6-5)(6-7)(6-12)}{(10-2)(10-5)(10-7)(10-12)} \times 1210$$

$$+ \frac{(6-2)(6-5)(6-7)(6-10)}{(12-2)(12-5)(12-7)(12-10)} \times 2028$$

$$y(6) = \frac{1(-1)(-4)(-6)}{(-3)(-5)(-8)(-10)} \times 18 + \frac{4(-1)(-4)(-6)}{3(-2)(-5)(-7)} \times 180$$

$$+ \frac{4(1)(-4)(-6)}{5(2)(-3)(-5)} \times 448 + \frac{(4)(1)(-1)(-6)}{8\cdot 5\cdot 3\cdot 2} \times 1210 + \frac{4\cdot 1\cdot(-1)(-4)}{10\cdot 7\cdot 5\cdot 2}$$

$$y(6) = \frac{-\cancel{24}^{9}}{\cancel{1200}\,\cancel{100}\,\cancel{50}\,25} \times \cancel{189} - \frac{\cancel{72}^{36}}{-\cancel{210}\,15} 180 + \frac{72}{150} \times 448 - \frac{\cancel{24}^{1}}{\cancel{200}\,10} \times \cancel{1210}^{121} + \frac{16}{700} \times 2028$$

$$y(6) = \frac{-9}{25} + \frac{6480}{15} + \frac{72}{150} \times 448 - 121 + \frac{16 \times 2028}{700}$$

$$y(6) = -0.36 + 432 + 215 - 121 + 46.354$$

$$y(6) = 572.714$$

$$y(6) = \frac{-9}{25} + \frac{576}{7} + \frac{7168}{25} - 121 + \frac{8112}{175}$$

$$y(6) = -121 + \frac{-9 + 7168}{25} + \frac{576}{7} + \frac{8112}{175}$$

$$y(6) = -121 + \frac{7159}{25} + \frac{576}{7} + \frac{8112}{175}$$

$$y(6) = -121 + 286.36 + 82.2857 + 46.3542$$

$$\therefore y(6) = 293.9999 = 294$$

2. Using the Legranges interpolation formulae find the value of $y(10)$ from the following table

x:	5^{x_0}	6^{x_1}	9^{x_2}	11^{x_3}
y:	12_{y_0}	13_{y_1}	14_{y_2}	16_{y_3}

Solu) By Legranges interpolation formulae

$$y(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x_1-x_2)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$y(10) = \frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} 12 + \frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} 13$$
$$+ \frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} 14 + \frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)} \cdot 16$$

$$y(10) = \frac{\cancel{4} \cdot 1 \cdot (-1)}{(-1)(-\cancel{4})(-6)} \cdot \cancel{12}^{2} + \frac{5 \cdot 1 \cdot (-1)}{1(-3)(-5)} 13 + \frac{5(\cancel{4})(-1)}{\cancel{4}(3)(-2)} \cancel{14}^{7}$$
$$+ \frac{5(\cancel{4})(1)}{\cancel{6}\,3 \cdot 5 \cdot \cancel{2}} \cdot \cancel{16}^{8}$$

$$y(10) = +\frac{\cancel{2}^{1}}{\cancel{2}} - \frac{13}{3} + \frac{35}{3} + \frac{16}{3} = \frac{6 - 13 + 35 + 16}{3}$$

180

1210

2028

$y(10) = \frac{44}{3}$

$\therefore y(10) = 14.667$

3. find the Cubic Legranges Interpolating polynomial from the following data.

x :	0 x_0	1 x_1	2 x_2	5 x_3
$f(x)$:	2 y_0	3 y_1	12 y_2	147 y_3

solu) The Legranges Interpolation formula

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$= \frac{(x-1)(x-2)(x-5)}{(0-1)(0-2)(0-5)} 2 + \frac{(x-0)(x-2)(x-5)}{(1-0)(1-2)(1-5)} 3$$

$$+ \frac{(x-0)(x-2)(x-5)}{(2-0)(2-1)(2-5)} 12 + \frac{(x-0)(x-1)(x-2)}{(5-0)(5-1)(5-2)} 147$$

$$= \frac{(x-1)(x-2)(x-5)}{(-1)(-\cancel{2})(-5)} \cancel{2} + \frac{x(x-2)(x-5)}{1(-1)(-4)} 3$$

$$+ \frac{x(x-1)(x-5)}{2 \cdot 1 \cdot (-\cancel{3})} \cancel{12}^{2} + \frac{x(x-1)(x-2)}{5(4)(3)} 147$$

$$= -\frac{(x-1)(x-2)(x-5)}{5} + \frac{x(x-2)(x-5)}{4} \times 3$$

$$- x(x-1)(x-5) \times 2 + \frac{x(x-1)(x-2)}{60} \times 147$$

$$= -\frac{(x^2 - x - 2x + 2)(x-5)}{5} + \frac{(x^2 - 2x)(x-5)}{4} \times 3$$

$$- (x^2 - x)(x-5) 2 + \frac{(x^2 - x)(x-2)}{60} \times 147$$

$$= -[x^3 - x^2 - 2x^2 + 2x - 5x^2 + 5x + 10x - 10]/5$$

$$+ \frac{[x^3 - 2x^2 - 5x^2 + 10x] 3}{4} - [x^3 - x^2 - 5x^2 + 5x] 2$$

$$+ \frac{[x^3 - x^2 - 2x^2 + 2x] \times 147}{60}$$

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$= -[x^3-7x^2+10x]$
$= -\frac{[x^3-8x^2+17x-10]}{5} + \frac{[x^3-7x^2+10x]3}{4} - \frac{[x^3-6x^2+5x]}{2}$

$+ \frac{[x^3-3x^2+2x]\,49}{20}$

$= \frac{-x^3+8x^2-17x+10}{5} + \frac{3x^3-21x^2+30x}{4} - 2x^3+12x^2-10x$

$+x^3-3x^2+2x + \frac{49x^3-147x^2+98x}{20}$

$= \frac{-4x^3+32x^2-68x+40+15x^3-105x^2+150x-40x^3+240x^2-200x+49x^3-147x^2+98x}{20}$

$= \frac{20x^3+20x^2-20x+40}{20}$

$= \frac{20(x^3+x^2-x+20)}{20}$

$\therefore f(x) = x^3+x^2-x+20$

4. find the Legranges interpolating polynomial for the given data.

x :	1 (x_0)	2 (x_1)	3 (x_2)	4 (x_3)
$f(x)$:	1 (y_0)	8 (y_1)	27 (y_2)	64 (y_3)

Solu) The Legranges interpolation formulae.

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1$$
$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$f(x) = \frac{(x-2)(x-3)(x-4)}{(1-2)(1-3)(1-4)}1 + \frac{(x-1)(x-3)(x-4)}{(2-1)(2-3)(2-4)}8$$
$$+ \frac{(x-1)(x-2)(x-4)}{(3-1)(3-2)(3-4)}27 + \frac{(x-1)(x-2)(x-3)}{(4-1)(4-2)(4-3)}64$$

$$f(x) = \frac{(x-2)(x-3)(x-4)}{(-1)(-2)(-3)} + \frac{(x-1)(x-3)(x-4)}{1(-1)(-2)}8\,4.$$
$$+ \frac{(x-1)(x-2)(x-4)}{2(1)(-1)}27 + \frac{(x-1)(x-2)(x-3)}{3(2)(1)}64\,32$$

$$= \frac{(x^2-2x-3x+6)(x-4)}{-6} + (x^2-x-3x+3)(x-4)\,4$$
$$+ \frac{(x^2-x-2x+2)(x-4)}{-2}27 + \frac{[x^2-x-2x+2](x-3)}{3}32$$

$$= \frac{-(x^2-5x+6)(x-4)}{6} + (x^2-4x+3)(x-3)\, 32$$

$$- \frac{(x^2-3x+2)(x-4)}{2}\, 27 + \frac{[x^2-3x+2][x-3]}{3}\, 32$$

$$[= \frac{-[x^3-5x^2+6x-4x^2+20x-24]}{6} + [x^3-4x^2+3x-3x^2+12x+9]\,4$$

$$- \frac{[x^3-3x^2+2x-4x^2+12x-8]}{2}\, 27 + \frac{[x^3-3x^2+2x-3x^2+9x-6]}{3}\, 32$$

$$= \frac{-x^3+5x^2-6x+4x^2-20x+24}{6} + 4x^3-16x^2+12x-12x^2+48x+36$$

$$- \frac{27x^3+81x^2-54x+108x^2-324x+216}{2} + \frac{32x^3-96x^2+64x-96x^2+288x-192}{3}$$

wrong

$$= \frac{-x^3+5x^2-6x+4x^2-20x+24+24x^3-96x^2+72x-72x^2+288x+216-81x^3+243x^2-162x+324x^2-972x+648+64x^3-192x^2+128x-192x^2+576x-384}{6}$$

$$= 6x^3]$$

$$= \frac{-[x^3-7x^2+12x-2x^2+14x-24]+24(x^3-7x^2+12x-x^2+7x-12)-27\times3(x^3-6x^2+8x-x^2+6x-8]+64(x^3-5x^2+6x-x^2+5x-6)}{6}$$

$$= \frac{-[x^3-9x^2+26x-24]+24[x^3-8x^2+19x-12]-27[x^3-7x^2+14x-8]+64[x^3-6x^2+11x-6]}{6}$$

$$= \frac{+[x^3+9x^2-26x+24]+24x^3-192x^2+456x-288-81x^3+567x^2-1134x+648+64x^3-384x^2+704x-384}{6}$$

$$= \frac{1}{6}[6x^3+0+0+0] = x^3$$

5. Using Legranges Interpolation formula to fit a polynomial to the following data

	x_0	x_1	x_2	x_3
x :	-1	0	2	3
	u_0	u_1	u_2	u_3
u_x :	-8	3	1	12

And also find the value u_1

solu) By Lagranges interpolation formulae

$$u_x = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} u_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} u_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} u_3$$

$$u_x = \frac{(x-0)(x-2)(x-3)}{(-1-0)(-1-2)(-1-3)}(-8) + \frac{(x+1)(x-2)(x-3)}{(0+1)(0-2)(0-3)} 3$$

$$+ \frac{(x+1)(x-0)(x-3)}{(2+1)(2-0)(2-3)} 1 + \frac{(x+1)(x-0)(x-2)}{(3+1)(3-0)(3-2)} 12$$

$$\left[u_x = \frac{(x-0)(x-2)(x-3)}{-1\times -3\times +4}(+8) + \frac{(x+1)(x-2)(x-3)}{-24\times 1} 12$$

$$+ \frac{(x+1)x(x-3)}{3\cdot 2(-1)} + \frac{(x+1)(x)(x-2)}{4\cdot 3\cdot 1} 12$$

$$u_x = \frac{2x[x^2-2x-3x+6]}{3} + \frac{x[x^2+x-2x-2]}{4}$$

wrong

$$+ \frac{[x^2+x-3x-3]x}{-6} + \frac{[x^2+x-2x-2]x}{1}$$

$$u_x = \frac{2x^3-4x^2-6x^2+12x}{3} + \frac{x^3+x^2-2x^2-2x}{4}$$

$$\frac{x^3+x^2-3x^2-3x}{6} + \frac{x^3+x^2-2x^2-2x}{}$$

$$u_x = \frac{2x^3-10x^2+12x}{3} + x^3-5x^2\Big]$$

$$u_x = 2x^3 \cdot \frac{x(x^2-5x+6)}{-1\times 3\times -4} \times -8^2 + \frac{(x+1)(x^2-5x+6)}{1\times -2\times -3} \times 3$$

$$+ \frac{(x+1)(x^2-3x)}{3\times 2\times -1} + \frac{(x+1)(x^2-2x)}{4\times 2\times 1} \times 12^3$$

$$u_x = \frac{2x^3-10x^2+12x}{3} + \frac{x^3-5x^2+6x+x^2-5x+6}{2} - \frac{(x^3-3x^2+x^2-3x)}{6}$$

$$+ (x^3-2x^2+x^2-2x)$$

$$u_x = \frac{2(2x^3-10x^2+12x)+3(x^3-4x^2+x+6)-(x^3-3x^2+x^2-3x) + 6(x^3-2x^2-2x)}{6}$$

$$u_x = \frac{1}{6}\{4x^3-20x^2+24x+3x^3-12x^2+3x+18-x^3+2x^2+3x+6x^3-6x^2-12x\}$$

$$= \frac{12x^3-36x^2+18x+18}{6} = 2x^3-6x^2+3x+3 \quad \text{If } x=1$$

$$u_1 = 2(1)^3-6(1)^2+3(1)+3 = 2$$

Date 17/7/18

Central Differences

Gauss - Forward Interpolating Formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2}$$

$$+\frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-2} + \cdots$$

Gauss - Backward Interpolating Formulae

$$y_n = y_0 + n\Delta y_{-1} + \frac{(n+1)n}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2} + \frac{(n+2)(n+1)n(n-1)}{4!}$$

$$\Delta^4 y_{-2} + \frac{(n+2)(n+1)n(n-1)(n-2)}{5!}\Delta^5 y_{-3} + \cdots$$

1 find $f(2.5)$ using the following table.

x: 1 2 3 4

$f(x)$: 1 8 27 64 Difference table

why we used only Gauss forward why should's Newton's forward

Solu)

x	f_y	1st	2nd	3rd
1 x_{-1}	1 y_{-1}			
		7 y_{-1}		
			12 y_{-1}	
2 x_0	8 y_{0}			6 y_{-1}
		19 y_0		
3 x_1	27 y_1		18 y_0	
		37 y_1		
4 x_2	64 y_2			

Amma
Nanna
Chinni

Gauss forward interpolating formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1}$$

$n = \frac{x - x_0}{h}$; $x_0 = 2.0$ $x = 2.5$; $h = 1$

$n = \frac{2.5 - 2}{1} = 0.5$ why we took $x_0 = 2$

$$y_n = 8 + (0.5)(19) + \frac{(0.5)(0.5-1)}{2!}12 + \frac{(0.5+1)(0.5)(0.5-1)}{3!}6$$

$$= 8 + 9.5 + \frac{(0.5)(-0.5)}{\cancel{2}}\cancel{12}^{6} + \frac{(1.5)(0.5)(-0.5)}{\cancel{2}\,\cancel{3}}\times\cancel{6}$$

$$= 8 + 9.5 - \cancel{5.75}\,1.5 - 0.375$$

$$y(2.5) = 15.625$$

2. From the following table find y when $x = 38$

x: 30 35 40 45 50

y: 15.9 14.9 14.1 13.3 12.5

Difference table

x	y	1st	2nd	3rd	4th
30 x_{-1}	15.9 y_{-1}				
		-1 y_{-1}			
35 x_0	14.9 y_0		0.2 y_{-1}	-0.2 y_{-1}	
		-0.8 y_0			0.2 y_{-1}
40	14.1 y_1		0 y_0		
		-0.8 y_1		0 y_0	
45	13.3 y_2		0 y_1		
		-0.8 y_2			
50	12.5 y_3				

By applying Gauss forword interpolating formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} \left[+\frac{(n+1)n(n-1)(n-2)}{4!}\right]\Delta^4 y$$

$n = \frac{x - x_0}{h}$; $x = 38$ $x_0 = 35$; $h = 5$

$n = \frac{38-35}{5} = \frac{3}{5} = 0.6$

$$y_n = 14.9 + (0.6)(-0.8) + \frac{(0.6)(0.6-1)}{2!}(0.2) + \frac{(0.6+1)(0.6)(0.6-1)}{3!}(-0.2)$$

$$\left[+\frac{(0.6+1)(0.6)(0.6-1)(0.6-2)}{4!}(0.2)\right]$$

$y_{(38)} = 14.9 - 0.48 - 0.024 + 0.0128 + [0.00]$

$y_{(38)} = 14.4088$

3. Using Gauss forword interpolating formulae we find $f(3.3)$ from the following data

Difference table

why we used Gauss forword & Gauss backword

x: 1 2 3 4 5

$f(x)$: 15.3 15.1 15 14.5 14

x	y	1st	2nd	3rd	4th
1 x_{-2}	15.3 y_{-2}	-0.2 y_{-2}	-0.1 y_{-2}	-0.5 y_{-2}	-2.5
2 x_{-1}	15.1 y_{-1}				0.9 y_{-2}
		-0.1 y_{-1}			
3 x_0	15 y_0		-0.6 y_{-1}	$+0.6$ y_{-1}	
		$+0.5$ y_0			
4	14.5		-0.0 y_0		
		-0.5			
5	14				

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By applying Gauss forward interpolating formulae

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+1)n(n-1)(n-2)}{4!}\Delta^4 y_{-2}$$

$n = \frac{x-x_0}{h}$ $x = 3.3$; $x_0 = 3$; $h = 1$; $n = \frac{3.3-3}{1} = 0.3$

$$y(3.3) = 15 + 0.3(-0.5) + \frac{(0.3)(0.3-1)}{2!}(-0.4) + \frac{(0.3+1)(0.3)(0.3-1)}{3!}(0.4)$$

$$+ \frac{(0.3+1)(0.3)(0.3-1)(0.3-2)}{4!} \times 0.9$$

$y(3.3) = 15 - 0.15 + 0.042 \,\cancel{-0.0286} + \cancel{0.02016]} - 0.0182 + 0.0580125$

$y(3.3) = \cancel{14.88656}$ $\cancel{14.9318125}$

$= 15 - 0.15 + 0.042 - 0.182 + 0.01740375 = 14.89120375$

4. find the polynomial which quit the data in the following table using ~~Gauss~~ Newtons forward formula

x	3	5	7	9	11
y	6	24	58	108	174

solu) Difference table

x	y	1st	2nd	3rd	4th
3	6				
		18			
5	24		16		
		34		0	
7	58		16		0
		50		0	
9	108		16		
		66			
11	174				

By applying Newtons forward formula

$$y_n = y_0 + n\Delta y_0 + \frac{n(n-1)}{2!}\Delta^2 y_0 + .$$

$n = \frac{x-x_0}{h}$ $x_0 = 3$; $x_n = x$ $h = 2$ $n = \frac{3x}{2}$.

$n = \frac{x-3}{2}$

$$y_n = 6 + \left(\frac{x-3}{\cancel{2}}\right)\cancel{18}\,9 + \frac{\left(\frac{x-3}{2}\right)\left(\frac{x-3}{2}-1\right)}{\cancel{2}}\,\cancel{16}\,8$$

$$= 6 + 9x - 27 + \left(\frac{x-3}{\cancel{2}}\right)\left(\frac{x-5}{\cancel{2}}\right) \times \cancel{16}\,\cancel{8}\,\cancel{4}\,2 \frac{1}{\cancel{2}}$$

$$= 6 + 9x - 27 + [x-3][x-5]\,2$$

$$= 6 + 9x - 27 + [x^2 - 3x - 5x + 15]\,2$$

$$= -21 + 9x + 2x^2 - 6x - 10x + 30$$

$y_n = 2x^2 - 7x + 9$

Date 7/8/18

5 By using Gauss Backward interpolating formulae find the value of y and $x = 3.3$ from the following data

x	1	2	3	4	5
y	15.3	15.1	15	14.5	14

Solu) Difference table

x	y	1st	2nd	3rd	4th
1 x_{-2}	15.3 y_{-2}				
		-0.2 y_{-2}			
2 x_{-1}	15.1 y_{-1}		-0.2 y_{-2}		
		-0.1 y_{-1}		-0.5 y_{-2}	
3 x_0	15 y_0		-0.4 y_{-1}		0.9 y_{-2}
		0.5 y_0		$+0.4$ y_{-1}	
4	14.5		0 y_0		
		-0.5			
5	14				

By using ~~Newton's~~ Gauss Backword formula

$$y_n = y_0 + n\Delta y_{-1} + \frac{n(n+1)}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-1} + \frac{(n+2)(n+1)n(n-1)}{4!}\Delta^4 y$$

$n = \frac{x - x_0}{h}$; $x = 3.3$; $x_0 = 3$; $h = 1$; $n = \frac{3.3 - 3}{1} = 0.3$

$$y(3.3) = 15 + (0.3)(-0.1) + \frac{(0.3+1)(0.3)}{2!}(-0.4) + \frac{(0.3+1)(0.3)(0.3-1)}{3!}(-0.5)$$

$$+ \frac{(0.3+2)(0.3+1)(0.3)(0.3-1)}{4!}(+0.9)$$

$[y_n = 15 + 4.53 - 0.0195 + 0.0182]$

$y(3.3) = 15 - 0.03 - 0.078 + 0.02275 - 0.02354625$

$y(3.3) = 14.89120375$

$y(3.3) = 14.8912$

6 From the following table find the value of y when $x = 1.35$

x:	1	1.2	1.4	1.6	1.8	2
y:	0.0	-0.112	-0.016	0.336	0.992	2

why we used Gauss backward

x	y	1st	2nd	3rd	4th	
1	0.0	$\underline{-0.112}\ y_{-1}$				
$\underline{1.2}\ x_0$	$\underline{-0.112}\ y_0$	0.096	$\underline{0.208}\ y_{-1}$	$\underline{0.048}\ y_{-1}$	0	0
1.4	-0.016	0.352	0.256	0.048	0	
1.6	0.336	0.656	0.304	0.048		
1.8	0.992	-0.992	0.352			
2	2					

By applying Gauss backward interpolating formula

$$y_n = y_0 + n\Delta y_{-1} + \frac{(n+1)n}{2!}\Delta^2 y_{-1} + \frac{(n+1)n(n-1)}{3!}\Delta^3 y_{-2}$$

$n = \frac{x-x_0}{h}$ $\quad x = 1.35 \quad x_0 = 1.2$; $h = 0.2$; $n = \frac{1.35-1.2}{0.2} = 0.7$

$$y(1.35) = (-0.112) + (0.75)(-0.112) + \frac{(0.75)(0.75+1)}{2!}(0.208)$$

$$= -0.112 - 0.084 + 0.1365$$

$$\therefore y(1.35) = -0.0595$$

Unit-5

IV: Numerical Integration & The Solutions of Ordinary Differential Equation.

There are three rules

1. Trapizoidal Rule
2. Simpson $\frac{1}{3}$ rule
3. Simpson $\frac{3}{8}$ Rule

In Numerical integration, we solve the given problem by using the above rules.

Trapizoidal Rule:

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+y_3+\cdots+y_{n-1})\right]$$

Simpson $\frac{1}{3}$ Rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_n)+4(y_1+y_3+y_5+\cdots)+2(y_2+y_4+y_6+\cdots)\right]$$

Simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_n)+3(y_1+y_2+y_4+y_5+y_7+y_8+\cdots)+2(y_3+y_6+y_9+\cdots)\right]$$

where $h=\frac{b-a}{n}$ and $y=f(x)$ is the given curve

1. Evaluate $\int_0^1 \frac{1}{1+x^2}\,dx$ where $n=4$

$b=1$, $n=4$; $h=\frac{b-a}{n}=\frac{1-0}{4}$

Solu: $f(x)=\frac{1}{1+x^2}=y$; $a=0$;

$h=\frac{1}{4}$

$x_0=a=0$

$\therefore y_0=f(x_0)=f(0)=\frac{1}{1+0^2}=1$

$x_1=x_0+h = 0+\frac{1}{4} = \frac{1}{4}$

$y_1=f(x_1) = \frac{1}{1+x_1^2}$; $x_1=\frac{1}{4}$

$=\frac{1}{1+(\frac{1}{4})^2}=\frac{16}{17}$

$x_2 = x_1 + h$ $\quad y_2 = f(x_2)$ $\quad x_3 = x_2 + h$

$= \frac{1}{4} + \frac{1}{4}$ $\quad = \frac{1}{1+x_2^2}$, $x_2 = \frac{1}{2}$ $\quad = \frac{1}{2} + \frac{1}{4}$

$= \frac{1}{2}$ $\quad = \frac{3}{4}$

$= \frac{1}{1+(\frac{1}{2})^2} = \frac{1}{1+\frac{1}{4}} = \frac{4}{5}$

$y_3 = f(x_3)$. $\quad x_4 = x_3 + h$ $\quad y_4 = f(x_4)$

$= \frac{1}{1+x_3^2}$, $x_3 = 3/4$ $\quad = \frac{3}{4} + \frac{1}{4}$ $\quad = \frac{1}{1+x_4^2}$, $x_4 = 1$

$= 1$ $\quad = \frac{1}{1+1^2} = \frac{1}{1+1}$

$= \frac{1}{1+(\frac{3}{4})^2} = \frac{1}{1+\frac{9}{16}} = \frac{16}{25}$ $\quad = \frac{1}{2}$

$\therefore y_0 = 1$; $y_1 = 0.9412$; $y_2 = 0.8$; $y_3 = 0.64$; $y_4 = 0.5$

① Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3+\cdots)\right]$$

$$= \frac{\frac{1}{4}}{2}\left[(1+0.5) + 2(0.9412+0.8+0.64)\right]$$

$$= \frac{1}{8}\left[1.5 + 2(2.3812)\right]$$

$$= \frac{1}{8}\left[1.5 + 4.7624\right]$$

$$= 0.7828$$

② Sympson $\frac{1}{3}$ Rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2 \cancel{+y_4})\right]$$

$$= \frac{\frac{1}{4}}{3}\left[(1+0.5) + 4(0.9412+0.64) + 2(0.8)\right]$$

$$= \frac{1}{12}\left[1.5 + 4(1.5812) + 1.6\right]$$

$$= \frac{1}{12}\left[1.5 + 6.3248 + 1.6\right]$$

$$= \frac{1}{12}\left[9.4248\right]$$

$$= 0.7854$$

0.64
0.9412
1.5812

③ Sympson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_4)+3(y_1+y_2)+2(y_3)\right]$$

$$= \frac{3\frac{1}{4}}{8}\left[(1+0.5)+3(0.9412+0.8)+2(0.64)\right]$$

$$= \frac{3}{32}\left[1.5+3(1.7412)+1.28\right]$$

$$= \frac{3}{32}\left[1.5+5.2236+1.28\right]$$

$$= \frac{3}{32}\left[8.0036\right]$$

$$= \frac{24.0108}{32} = 0.7503375 = 0.7503$$

2. Evoluate $\int_1^2 \frac{1}{x}\,dx$ where $n=4$

solu) $f(x) = \frac{1}{x}$, $n=4$, $a=1$, $b=2$ $h = \frac{b-a}{n} = \frac{2-1}{4} = \frac{1}{4}$

$x_0 = a = 1$; $y_0 = f(x_0) = \frac{1}{x_0} = \frac{1}{1} = 1$

$x_1 = x_0 + h$ $y_1 = f(x_1)$

$= 1 + \frac{1}{4}$ $= \frac{1}{x_1}$

$= \frac{5}{4}$ $= \frac{1}{\frac{5}{4}} = \frac{4}{5} =$

$x_2 = x_1 + h$ $y_2 = f(x_2)$

$= \frac{5}{4} + \frac{1}{4}$ $= \frac{1}{x_2}$

$= \frac{6}{4} = \frac{3}{2}$ $= \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$y_3 = f(x_3)$

$x_3 = x_2 + h$ $= \frac{1}{x_3}$

$= \frac{3}{2} + \frac{1}{4}$ $= \frac{1}{\frac{7}{4}}$

$= \frac{6+1}{4}$

$= \frac{7}{4}$ $= \frac{4}{7}$

$y_4 = f(x_4)$

$x_4 = x_3 + h$ $= \frac{1}{x_4}$

$= \frac{7}{4} + \frac{1}{4}$ $= \frac{1}{2}$

$= \frac{8}{4} = 2$

$y_0 = 1$, $y_1 = 0.8$; $y_2 = 0.6667$; $y_3 = 0.5714$; $y_4 = 0.5$

① By Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$= \frac{\frac{1}{4}}{2}\left[(1+0.5) + 2(0.8+0.6667+0.5714)\right]$$

$$= \frac{1}{8}\left[1.5 + 2(2.0384)\right]$$

$$= \frac{1}{8}\left[1.5 + 4.0768\right]$$

$$= \frac{5.5768}{8} = 0.4647333 = \cancel{0.4648} = 0.6971$$

2. Syimpson $\frac{1}{3}$ Rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2y_2\right]$$

$$= \frac{\frac{1}{4}}{3}\left[(1+0.5) + 4(0.8+0.5714) + 2(0.6667)\right]$$

$$= \frac{1}{12}\left[1.5 + 4(1.3714) + 1.3334\right]$$

$$= \frac{1}{12}\left[1.5 + 5.4856 + 1.3334\right]$$

$$= \frac{8.319}{12} = 0.69325$$

3. Simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_4) + 3(y_1+y_2) + 2(y_3)\right]$$

$$= \frac{3\left(\frac{1}{4}\right)}{8}\left[(1+0.5) + 3(0.8+0.6667) + 2(0.5714)\right]$$

$$= \frac{3}{32}\left[1.5 + 3(1.4667) + 2(0.5714)\right]$$

$$= \frac{3}{32}\left[1.5 + 4.4001 + 1.1428\right]$$

$$= \frac{3}{32}\left[7.0429\right]$$

$$= \frac{21.1287}{32}$$

$$= 0.66027187\overset{}{5} = 0.6602$$

3. Evaluate $\int_0^1 \frac{1}{x+1}\,dx$, $n=5$

$y=f(x)=\frac{1}{x+1}$, $n=5$ $a=0$, $b=1$ $h=\frac{b-a}{n}=\frac{1-0}{5}=\frac{1}{5}$

$h=\frac{1}{5}$

$x_0=a=0$ $y_0=f(x_0)$ $y_1=f(x_1)$

$=\frac{1}{0+1}$ $=\frac{1}{\frac{1}{5}+1}$

$x_1=x_0+h$

$=0+\frac{1}{5}$ $=\frac{1}{1}$ $y_1=\frac{5}{6}$

$=\frac{1}{5}$ $=1$

$x_2=x_1+h$ $y_2=f(x_2)$ $x_5=x_4+h$

$=\frac{1}{5}+\frac{1}{5}$ $=\frac{1}{\frac{2}{5}+1}$ $=\frac{4}{5}+\frac{1}{5}$

$=1$

$=\frac{2}{5}$ $=\frac{5}{7}$ $y_5=f(x_4)$

$x_3=x_2+h$ $y_3=f(x_3)$ $=\frac{1}{1+1}$

$=\frac{2}{5}+\frac{1}{5}$ $=\frac{1}{\frac{3}{5}+1}$ $=\frac{1}{2}$

$=0.5$

$=\frac{3}{5}$ $=\frac{5}{8}$

$x_4=x_3+h$ $y_4=f(x_4)$

$=\frac{3}{5}+\frac{1}{5}$ $=\frac{1}{\frac{4}{5}+1}$

$=\frac{4}{5}$ $=\frac{5}{9}$

$x_0=0$, $y_0=1$; $y_1=0.8333$; $y_2=0.71438$

$y_3=0.625$; $y_4=0.5556$; $y_5=0.5$; $h=0.2$

① By Trapizoidal Rule.

$\int_a^b y\,dx=\frac{h}{2}[(y_0+y_5)+\cancel{2(y_1+y_2)}+2(y_1+y_2+y_3+y_4)]$

$=\frac{0.2}{2}[(1+0.5)+2(0.8333+0.71438\ 0.625+0.5556)]$

$=0.1[1.5+2[2.7282]]$

$=0.1[1.5+5.4564]$

$=(0.1)(6.9564)$

$=0.69564$

② By simpson $\frac{1}{3}$ rule

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_5)+4(y_1+y_3)+2(y_2+y_4)\right]$$

$$= \frac{0.2}{3}\left[(1+0.5)+4(0.8333+0.625)+2(0.7143+0.5556)\right]$$

$$= \frac{0.2}{3}\left[1.5+4(1.4583)+2(1.2699)\right]$$

$$= \frac{0.2}{3}\left[1.5+5.8332+2.5398\right]$$

$$= \frac{0.2}{3}\left[9.873\right]$$

$$= 0.2\left[3.291\right]$$

$$= 0.6582$$

③ By simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_5)+3(y_1+y_2+y_4)+2(y_3)\right]$$

$$= \frac{3(0.2)}{8}\left[(1+0.5)+3(0.8333+0.7143+0.5556)+2(0.625)\right]$$

$$= \frac{0.6}{8}\left[1.5+3(2.1032)+2(0.625)\right]$$

$$= \frac{0.6}{8}\left[1.5+6.3096+1.25\right]$$

$$= 0.6\,\frac{[9.0596]}{8}$$

$$= (0.6)(1.1324)$$

$$= 0.6795$$

4 Evaluate $\int_0^6 \frac{dx}{1+x^2}$ $n=6$

solu $y=f(x)=\frac{1}{1+x}$ $a=0$, $b=6$; $n=6$ $h=\frac{b-a}{n}=\frac{6-0}{6}=1$

$x_0=a=0$, $y_0=f(x_0)=\frac{1}{1+0}=1$

$x_1=x_0+h$ $\quad y_1=f(x_1)$

$=1$ $\quad =\frac{1}{1+1}=\frac{1}{2}=0.5$

$x_2=x_1+h$ $\quad y_2=f(x_2)$

$=1+1$ $\quad =\frac{1}{1+2}=\frac{1}{3}=0.3334$

$=2$

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$x_3 = x_2 + h$
$= 2+1$
$= 3$

$y_3 = f(x_3)$
$= \frac{1}{1+3}$
$= \frac{1}{4} = 0.25$

$x_6 = x_5 + h$
$= 5+1$
$= 6$

$y_6 = f(x_6)$
$= \frac{1}{1+6} = \frac{1}{7}$
$= 0.1428$

$x_4 = x_3 + h$
$= 3+1$
$= 4$

$y_4 = f(x_4)$
$= \frac{1}{1+4} = \frac{1}{5} = 0.2$

$x_5 = x_4 + h$
$= 4+1$
$= 5$

$y_5 = f(x_5)$
$= \frac{1}{1+5} = \frac{1}{6} = 0.16667$

$x_0 = 0,\ y_0 = 1,\ y_1 = 0.5,\ y_2 = 0.3334,\ y_3 = 0.25;\ y_4 = 0.2$
$y_5 = 0.1667$
$y_6 = 0.1428$

① Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)]$$
$$= \frac{1}{2}[(1+0.1428) + 2(0.5+0.3334+0.25+0.2+0.1667)]$$
$$= 0.5[(1.1428) + 2(1.4501)]$$
$$= 0.5[1.1428 + 2.9002]$$
$$= 0.5[4.043]$$
$$= 2.0215$$

② simpson $\frac{1}{3}$ Rule.

$$\int_a^b y\,dx = \frac{3h}{8}[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)]$$
$$= \frac{1}{3}[(1+0.1428) + 4(0.5+0.25+0.1667) + 2(0.3334+0.2)]$$
$$= 0.3334[1.1428 + 4(0.9167) + 2(0.5334)]$$
$$= 0.3334[1.1428 + 3.6668 + 1.0668]$$
$$= 0.3334[5.8764]$$
$$= 1.95919176$$

③ simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$= \frac{3(1)}{8}\left[(1+0.1428)+3(0.5+0.3334+0.2+0.1667)+2(0.25)\right]$$

$$= 0.375\left[1.1428 + 3(1.2001) + 0.5\right]$$

$$= 0.375\left[1.1428 + 3.6003 + 0.5\right]$$

$$= 0.375\left[5.2431\right]$$

$$= 1.9661625$$

5. Evaluate $\int_0^4 e^x\,dx$ given, $e = 2.72$ $e^2 = 7.39$ $e^3 = 20.09$

$e^4 = 54.6$; $n = 4$

Solu) $y = f(x) = e^x$, $n = 4$, $a = 0$, $b = 4$ $h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

$x_0 = a = 0$ $\quad y_0 = f(x_0) = e^0 = 1$

$x_1 = x_0 + h$ $\quad y_1 = f(x_1) = e^{x_1} = e^1 = 2.72$

$= 1$

$x_2 = x_1 + h$ $\quad y_2 = f(x_2)$

$= 1+1$ $\quad = e^{x_2} = e^2 = 7.39$

$= 2$

$x_3 = x_2 + h$ $\quad y_3 = f(x_3)$

$= 2+1$ $\quad = e^{x_3} = e^3 = 20.09$

$= 3$

$x_4 = x_3 + h$ $\quad y_4 = f(x_4)$

$= 3+1$ $\quad = e^{x_4} = e^4 = 54.6$

$= 4$

$x_0 = 0$; $y_0 = 1$; $y_1 = 2.72$; $y_2 = 7.39$, $y_3 = 20.09$; $y_4 = 54.6$

1) Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_4) + 2(y_1+y_2+y_3)\right]$$

$$= \frac{1}{2}\left[(1+54.6)+2(2.72+7.39+20.09)\right]$$

$$= 0.5\left[55.6 + 2(30.2)\right]$$

$$= 0.5\left[55.6 + 60.4\right]$$

$$= 0.5\left[116\right]$$

$$= 58$$

2)10(0.5
54.6
1.
20.09
7.

2) Simpson $\frac{1}{3}$ rule:

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2(y_2)\right]$$

$$= \frac{1}{3}\left[(1+54.6)+4(2.72+20.09)+2(7.39)\right]$$

$$= \frac{1}{3}\left[55.6+4(22.81)+14.78\right]$$

$$= \frac{1}{3}\left[55.6+91.24+14.78\right]$$

$$= \frac{161.62}{3}$$

$$= 53.8733$$

3) Simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_4)+3(y_1+y_2)+2(y_3)\right]$$

$$= \frac{3(1)}{8}\left[(1+54.6)+3(2.72+7.39)+2(20.09)\right]$$

$$= \frac{3}{8}\left[55.6+3(10.11)+2(20.09)\right]$$

$$= 0.375\left[55.6+30.33+40.18\right]$$

$$= 0.375\left[126.11\right]$$

$$= 47.29125$$

6. The velocity of a car running on a straight line at intravals of 2 minutes are given below.

Date 23/1/18

Time :	0	2	4	6	8	10	12
velocity :	0	22	30	27	18	7	0

find the distance covered by the car.

Solu) Since we know that rate of change of displacement is called velocity. i.e., [Rate of change of velocity is called accelcration]

$$\frac{ds}{dt} = v$$

$$ds = v\,dt$$

$$s = \int_0^{12} v\,dt$$

1) Trapizoidal Rule

$$s = \int_0^{12} v\,dt = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{2}{2}\left[(0+0) + 2(22+30+27+18+7)\right]$$

$$= 2[104]$$

$$= 208$$

$\frac{104}{2}$

2) Simpson $\frac{1}{3}^{rd}$ Rule

$$S = \int_0^{12} v\,dt = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{2}{3}\left[(0+0) + 4(22+27+7) + 2(30+18)\right]$$

$$= \frac{2}{3}\left[4(56) + 2(48)\right]$$

$$= \frac{2}{3}\left[224+96\right]$$

$$= \frac{2}{3}\left[320\right]$$

$$= 213.333$$

$3)\overline{2016}$ 18

224 196 320

56 4 224

3) Simpson $\frac{3}{8}$ Rule

$$S = \int_0^{12} v\,dt = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$= \frac{3(\cancel{2})}{\cancel{8}_4}\left[(0+0) + 3(22+30+18+7) + 2(27)\right]$$

$$= \frac{3}{4}\left[3(77) + 54\right]$$

$$= \frac{3}{4}\left[231+54\right]$$

$$= \frac{3}{4} \times 285$$

$$= 213.75.$$

77 3 231

7 The velocity v of a particle at a distance s from a point on its path is given by the table below.

S	0	10	20	30	40	50	60
velocity	47	58	64	65	61	52	38

Estimate the time taken to travel 60 m by using the Rules.

Solu] Since we know that the rate of change of displacement is called the velocity.

$$v = \frac{ds}{dt} \Rightarrow dt = \frac{1}{v} ds$$

$$t = \int_0^{60} \frac{1}{v} ds.$$

$\frac{1}{v} = \frac{1}{47} = 0.0212 (y_0)$; $\frac{1}{58} = 0.0172 (y_1)$; $\frac{1}{64} = 0.0156 (y_2)$

$\frac{1}{65} = 0.0153 (y_3)$; $\frac{1}{61} = 0.0164 (y_4)$; $\frac{1}{52} = 0.0192 (y_5)$

$\frac{1}{38} = 0.0263 (y_6)$

1) Trapizoidal Rule

$$t = \int_0^{60} \frac{1}{v} ds = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{10}{2}\left[(0.0212 + 0.0263) + 2(0.0172 + 0.0156 + 0.0154 + 0.0164 + 0.0192)\right]$$

$$= 5\left[(0.0475) + 2(\right.$$

$$= 5\left[(0.0475) + 0.1676\right]$$

$$= 1.0755 \text{ sec.}$$

2) Simpson $\frac{1}{3}$ Rule

$$t = \int_0^{60} \frac{1}{v} ds = \frac{3h}{8}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{10}{3}\left[(0.0212 + 0.0263) + 4(0.0172 + 0.0154 + 0.0192) + 2(0.0156 + 0.0164)\right]$$

0.0475

$$= \frac{10}{3}\left[(0.0475) + 0.2072 + 0.064\right]$$

$$= \frac{10}{3}\left[0.3187\right]$$

$$= 1.06233$$

$= \frac{h}{2}[253 \ldots]$

$= 1151.01$

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10 A river is 80 meters wide. The depth y of the river at a distance 'x' from one bank is given by the following table

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

find the approximate area of the cross section of the given river.

solu) Since we know that the cross section area of the given river is

$$A = \int_0^{80} y\,dx$$

1) By trapizoidal Rule.

$$\int_0^{80} y\,dx = \frac{h}{2}\left[(y_0+y_8) + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$= \frac{10}{2}\left[(0+80)+2(10+20+30+40+50+60+70)\right]$ $= \frac{10}{2}\left[(0+3)+2(4+7+9+12+15+14+8)\right]$

$= 5(80+2(280))$ $= 5[3+2(69)]$

$= 5[80+560]$ $= 5[3+138]$

$= 5 \times 640$ $= 5[141] = 705$ sq. units

$= 3200]$

$141 \times 5 = 705$

2) Simpson $\frac{1}{3}$rd Rule

$$\int_0^{80} y\,dx = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$= \frac{10}{3}\left[(0+3)+4(4+9+15+14)+2(7+12+14)\right]$

$= \frac{10}{3}\left[3+4(42)+2(33)\right]$

$= \frac{10}{3}\left[3+168+66\right]$

$= \frac{10}{3} \times 233 = 710$ sq. units

3) Simpson $\frac{3}{8}$ Rule

$$\int_0^{} y\,dx = \frac{3h}{8}\left[(y_0+y_8)+3(y_1+y_2+y_4+y_5+y_7)+2(y_3+y_6)\right]$$

$$= \frac{3(10)}{8}\left[(0+3)+3(4+7+12+15+8)+2(9+14)\right]$$

$$= \frac{30}{8}\left[3+3(46)+2(23)\right]$$

$$= \frac{\cancel{30}^{15}}{\cancel{8}_{4}}\left[3+138+46\right]$$

$$= \frac{15}{4}\times 167$$

$$= 701.25$$

11. A train is moving at the speed of 30m/s. suddenly breaks are applied. The speed of the train per second after t seconds is given by

time	0	5	10	15	20	25	30	35	40	45
speed	30_{y_0}	24_{y_1}	19_{y_2}	16_{y_3}	13_{y_4}	11_{y_5}	10_{y_6}	8_{y_7}	7_{y_8}	5_{y_9}

find the distance moved by the train in 45 seconds.

Solu) Since we know that

$$v = \frac{ds}{dt}$$

$$\Rightarrow ds = v\,dt$$

$$s = \int_0^{45} v\,dt \qquad h=5$$

1) Trapizoidal Rule

$$\int_0^{45} v\,dt = \frac{h}{2}\left[(y_0+y_9)+2(y_1+y_2+y_3+y_4+y_5+y_6+y_7+y_8)\right]$$

$$= \frac{5}{2}\left[(30+5)+2(24+19+16+13+11+10+8+7)\right]$$

$$= \frac{5}{2}\left[35+2(108)\right]$$

$$= \frac{5}{2}\left[35+216\right]$$

$$= \frac{5}{2}\left[251\right] = \frac{1255}{2}$$

$$= 627.5$$

2) Simpson $\frac{1}{3}$ Rule

$$\int_0^{45} v\,dt = \frac{h}{3}\left[(y_0+y_9)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{5}{3}\left[(30+5)+4(\cancel{30}+24+16+11+8)+2(19\cancel{8}+13+10+7)\right]$$

$$= \frac{5}{3}\left[(35)+4(\cancel{103}\ 59)+2(49)\right]$$

$$= \frac{5}{3}\left[35+\cancel{412}\ 236+98\right]$$

$$\left[= \frac{5}{3}[\cancel{53}1]\right] = \frac{5}{3} \quad \frac{1845}{3}$$

$$\left[= \cancel{885}\right]\ 615.$$

412
84
35
531

3) Simpson $\frac{3}{8}$ Rule

$$\int_0^{45} v\,dt = \frac{3h}{8}\left[(y_0+y_9)+3(y_1+y_2+y_4+y_5+y_7+y_8)+2(y_3+y_6)\right]$$

$$= \frac{3\times5}{8}\left[(30+5)+3(24+19\cancel{8}+13+11+8+7)+2(16+10)\right]$$

$$= \frac{15}{8}\left[35+3(\cancel{161}\ 82)+2(26)\right] = \frac{15}{8}\left[35+483+52\right]$$

$$= \frac{15}{8}\left[\cancel{870}\ 333\right] = \cancel{1068.75} = \frac{4995}{8} = 624.375$$

12. In an experiment, a quantity, G was measured as follows. Solu) G(2~~6~~) = 101.24

G(20) = 95.9

G(21) = 96.85

G(22) = 97.77

G(23) = 98.68

G(24) = 99.56

G(25) = 100.41

G(26) =

compute $\int_{20}^{26} G(x)\,dx$

Solu)

x	20	21	22	23	24	25	26
y	95.9	96.85	97.77	98.68	99.56	100.41	101.24

1) Given G(26) = 101.24

1) Trapizoidal Rule

$$\int_{20}^{26} G(x)\,dx = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$

$$= \frac{1}{2}\left[(95.9+101.24) + 2(96.85+97.77+98.68+99.56+100.41)\right]$$

$$= 0.5\left[197.14 + 2(493.27)\right]$$

$$= 0.5\left[197.14 + 986.54\right]$$

$$= 0.5\left[1183.68\right]$$

$$= 591.84$$

2) simpson $\frac{1}{3}$ rd Rule

$$\int_{20}^{26} G(x)\,dx = \frac{h}{3}\left[(y_0+y_6) + 4(y_1+y_3+y_5) + 2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(95.9+101.24) + 4(96.85+98.68+100.41) + 2(97.77+99.56)\right]$$

$$= 0.3333\left[197.14 + 4(295.94) + 2(197.33)\right]$$

$$= 0.3333\left[197.14 + 1183.76 + 394.66\right]$$

$$= 0.3333\left[1775.56\right]$$

$$= 591.794148$$

3) simpson $\frac{3}{8}$ Rule

$$\int_{20}^{26} G(x)\,dx = \frac{3h}{8}\left[(y_0+y_6) + 3(y_1+y_2+y_4+y_5) + 2(y_3)\right]$$

$$= \frac{3}{8}\left[(95.9+101.24) + 3(96.85+97.77+99.56+100.41) + 2(98.68)\right]$$

$$= 0.375\left[197.14 + 3(394.59) + 2(98.68)\right]$$

$$= 0.375\left[197.14 + \cancel{1882.84}\ 1183.77 + 197.36\right]$$

$$= 0.375\left[1578.27\right]$$

$$= 591.85125$$

13 The speed of a train at varies times after leaving one station until it stops at another station are given in the following table

	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
speed:	0	13	33	39.5	40	40	36	15	0
time:	0	0.5	1	1.5	2	2.5	3	3.5	4

Find the distance between the two stations.

Solu) Since we know that the rate of change of displacement is called velocity

$$\frac{ds}{dt} = v$$

$$ds = v\,dt$$

$$s = \int_0^4 v\,dt$$

1) Trapizoidal Rule

$$\int_0^4 v\,dt = \frac{h}{2}\left[(y_0+y_8) + 2(y_1+y_2+y_3+y_4+y_5+y_6+y_7)\right]$$

$$= \frac{0.5}{2}\left[(0+0) + 2(13+33+39.5+40+40+36+15)\right]$$

$$= \frac{0.5}{2}\left[2(216.5)\right]$$

$$= 0.25[433]$$

$$= 108.25$$

2) Simpson $\frac{1}{3}$rd Rule

$$\int_0^4 v\,dt = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{0.5}{3}\left[(0+0) + 4(13+39.5+40+15) + 2(33+40+36)\right]$$

$$= \frac{\cancel{0.1667}\,0.5}{3}\left[4(107.5)+2(109)\right]$$

$$= \frac{0.5}{3}\cancel{0.05556}\left[430+218\right]$$

$$= \cancel{0.05556}\,\frac{0.5}{3}\left[648\right]$$

$$= \cancel{36}.\ \frac{324}{3} = 108$$

3) Simpson $\frac{3}{8}$ Rule

$$\int_0^4 v\,dt = \frac{3h}{8}\left[(y_0+y_8) + 3(y_1+y_2+y_4+y_5+y_7) + 2(y_3+y_6)\right]$$

$$= \frac{3h}{8}\left[(y_0+y_8) + 3(13+33+40+40+15) + 2(39.5+36)\right]$$

$$= \frac{3(0.5)}{8}\left[(0+0) + 3(13+33+40+40+15)\right.$$

$$= 0.1875\left[(141)3 + 2(75.5)\right]$$
$$= 0.1875\,[423 + 151]$$
$$= 0.1875\,(574)$$
$$= 107.625$$

14. Evaluate $\int_0^6 \frac{dx}{1+x^4}$ $n = 6$

Solu) Given

$$\int_0^6 \frac{dx}{1+x^4} \; ; \; n=6 \quad ; h = \frac{b-a}{n} = \frac{6-0}{6} = 1$$

$x_0 = a = 0$ $y_0 = f(x_0) = \frac{1}{1+(0)^4} = \frac{1}{1} = 1 = 1$

$x_1 = x_0 + h = 0+1 = 1$ $y_2 = f(x_2) = \frac{1}{1+2^4} = \frac{1}{1+16} = \frac{1}{17} = 0.05882$

$x_2 = x_1 + h = 1+1 = 2$ $y_3 = f(x_3) = \frac{1}{1+3^4} = \frac{1}{1+81} = \frac{1}{82} = 0.012195$

$y_4 = f(x_4) = \frac{1}{1+4^4} = \frac{1}{1+256} = \frac{1}{257} = 0.003891$

$x_3 = x_2 + h = 2+1 = 3$ $y_5 = f(x_5) = \frac{1}{1+5^4} = \frac{1}{1+625} = \frac{1}{626} = 0.001597$

$y_1 = f(x_1) = \frac{1}{1+1} = \frac{1}{2} = 0.5$

$x_4 = x_3 + h = 3+1 = 4$ $x_5 = x_4 + h = 4+1 = 5$ $x_6 = x_5 + h = 5+1 = 6$

$y_6 = f(x_6) = \frac{1}{1+6^4} = \frac{1}{1+1296} = \frac{1}{1297} = 0.00077101$

1) Trapizoidal Rule

$$\int_a^b y\,dx = \frac{h}{2}\left[(y_0+y_6) + 2(y_1+y_2+y_3+y_4+y_5)\right]$$
$$= \frac{1}{2}\left[(1+0.00071) + 2(0.5 + 0.05882 + 0.012195 + 0.003891 + 0.001597)\right]$$
$$= \frac{1}{2}\left[1.00071 + 2(0.576503)\right]$$
$$= 0.5\,[1.00071 + 1.153006]$$
$$= 0.5\,[2.153716]$$
$$= 1.076858$$

2) Simpson $\frac{1}{3}$rd Rule.

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0+y_6)+4(y_1+y_3+y_5)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(1+0.00071)+4(0.5+0.012195+0.001597) + 2(0.05882+0.003891)\right]$$

$$= \frac{1}{3}\left[1.00071+4(0.513792)+2(0.062711)\right]$$

$$= \frac{1}{3}\left[1.00071+2.055168+0.125422\right]$$

$$= \frac{1}{3}\left[3.1813\right]$$

$$= 1.06043333$$

3) Simpson $\frac{3}{8}$ Rule

$$\int_a^b y\,dx = \frac{3h}{8}\left[(y_0+y_6)+3(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$= \frac{3}{8}\left[(1+0.00071)+3(0.5+0.05882+0.003891+0.001597) + 2(0.012195)\right]$$

$$= 0.375\left[1.00071+3(0.564308)+2(0.012195)\right]$$

$$= 0.375\left[1.00071+1.692924+0.02439\right]$$

$$= 0.375\left[2.718024\right]$$

$$= 1.019259$$

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5. Find the value of $\log_e 2$ from $\int_0^1 \frac{x^2}{1+x^3}dx$ by using Simpson $\frac{1}{3}$rd Rule by dividing the range of integration into 4 equal parts

Solu Given $\int_0^1 \frac{x^2}{1+x^3}dx$

$a=0,\ b=1,\ n=4,\ h=\frac{b-a}{n}=\frac{1}{4}=0.25$

$$y=\frac{x^2}{1+x^3}$$

$$x_0=0 \Rightarrow y_0=\frac{x_0^2}{1+x_0^3}=\frac{0}{1+0}=0$$

$x_1=x_0+h$ $\quad y_1=\frac{x_1^2}{1+x_1^3}=\frac{(0.25)^2}{1+(0.25)^3}=\frac{0.0625}{1+0.015625}=\frac{0.0625}{1.015625}$

$=0+0.25$

$=0.25$ $\quad =0.061538$

$=0.0615$

$x_2 = x_1 + h$ $\quad y_2 = \frac{x_2^2}{1+x_2^3} = \frac{(0.5)^2}{1+(0.5)^3} = \frac{0.25}{1+0.125} = \frac{0.25}{1.125}$

$= 0.25 + 0.25$

$= 0.5$ $\quad = 0.2222$

$x_3 = x_2 + h$ $\quad y_3 = \frac{x_3^2}{1+x_3^3} = \frac{(0.75)^2}{1+(0.75)^3} = \frac{0.5625}{1+0.421875} = \frac{0.5625}{1.421875}$

$= 0.5 + 0.25$

$= 0.75$ $\quad = 0.3956$

$x_4 = x_3 + h$ $\quad y_4 = \frac{x_4^2}{1+x_4^3} = \frac{1^2}{1+1^3} = \frac{1}{2} = 0.5$

$= 0.75 + 0.25$

$= 1$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^1 y\,dy = \frac{h}{3}\left[(y_0+y_4)+4(y_1+y_3)+2y_2\right]$$

$$= \frac{0.25}{3}\left[(0+0.5)+4[(0.0615)+0.3956]+2(0.2222)\right]$$

$$= \frac{0.25}{3}\left[0.5+4(0.4571)+2(0.2222)\right]$$

$$= \frac{0.25}{3}\left[0.5+1.8284+0.4444\right]$$

$$= \frac{0.25}{3}\left[2.7724\right]$$

$$= 0.231066$$

$$= 0.2311$$

$$\begin{array}{r} 0.0615 \\ 0.3956 \\ \hline 0.4571 \end{array} \qquad \begin{array}{r} 0.4444 \\ 1.8284 \\ 0.5 \\ \hline 2.7724 \end{array}$$

16. Find an approximate value of $\log_e 5$ by calculating to four decimal places by Simpson $\frac{1}{3}$rd Rule. $\int_0^5 \frac{1}{4x+5}\,dx$ dividing the range into 10 equal parts

Solu) Given $\int_0^5 \frac{1}{4x+5}$ $\quad a = 0,\ b = 5,\ n = 10,\ h = \frac{b-a}{n} = \frac{5-0}{10} = \frac{1}{2} = 0.5$

$y = x_0 = 0$ $\quad y_0 = \frac{1}{4x_0+5}$

$x_1 = x_0 + h$

$= 0 + 0.5$ $\quad y_0 = \frac{1}{4(0)+5}$

$= 0.5$ $\quad y_0 = \frac{1}{5} = 0.2$

$x_2 = x_1 + h$ $\quad y_1 = \frac{1}{4x_1+5}$

$= 0.5 + 0.5$

$= 1.0$ $\quad y_1 = \frac{1}{4(0.5)+5} = \frac{1}{2+5} = \frac{1}{7} = 0.1428$

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$x_3 = x_2 + h$
$= 1.0 + 0.5$
$= 1.5$

$y_2 = \frac{1}{4x_2+5} = \frac{1}{4(1)+5} = \frac{1}{4+5} = \frac{1}{9} = 0.1111$

$x_4 = x_3 + h$
$= 1.5 + 0.5$
$= 2$

$y_3 = \frac{1}{4x_3+5} = \frac{1}{4(1.5)+5} = \frac{1}{6+5} = \frac{1}{11} = 0.0909$

$y_4 = \frac{1}{4x_4+5} = \frac{1}{4(2)+5} = \frac{1}{13} = 0.0769$

$x_5 = x_4 + h$
$= 2 + 0.5$
$= 2.5$

$y_5 = \frac{1}{4x_5+5} = \frac{1}{4(2.5)+5} = \frac{1}{10+5} = \frac{1}{15} = 0.0667$

$x_6 = x_5 + h$
$= 2.5 + 0.5$
$= 3.0$

$y_6 = \frac{1}{4x_6+5} = \frac{1}{4(3.0)+5} = \frac{1}{17} = 0.0588$

$x_7 = x_6 + h$
$= 3 + 0.5$
$= 3.5$

$y_7 = \frac{1}{4x_7+5} = \frac{1}{4(3.5)+5} = \frac{1}{14+5} = \frac{1}{19} = 0.05263$

$x_8 = x_7 + h$
$= 3.5 + 0.5$
$= 4.0$

$y_8 = \frac{1}{4x_8+5} = \frac{1}{4(4)+5} = \frac{1}{21} = 0.0476$

$x_9 = x_8 + h$
$= 4.0 + 0.5$
$= 4.5$

$y_9 = \frac{1}{4x_9+5} = \frac{1}{4(4.5)+5} = \frac{1}{18+5} = \frac{1}{23} = 0.0435$

$x_{10} = x_9 + h$
$= 4.5 + 0.5$
$= 5$

$y_{10} = \frac{1}{4x_{10}+5} = \frac{1}{4(5)+5} = \frac{1}{20+5} = \frac{1}{25} = 0.04$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^5 y\,dx = \frac{h}{3}\left[(y_0+y_{10}) + 4(y_1+y_3+y_5+y_7+y_9) + 2(y_2+y_4+y_6+y_8)\right]$$

$$= \frac{0.5}{3}\left[(0.2+0.04) + 4(0.1429+0.0909+0.0667+0.05263+0.0435) + 2(0.1111+0.0769+0.0588+0.0476)\right]$$

$$= \frac{0.5}{3}\left[0.24 + 4(0.39663) + 2(0.2943)\right]$$

$$= \frac{0.5}{3}\left[0.24 + 1.58652 + 0.5892\right]$$

$$= \frac{0.5}{3}\left[2.41572\right] = 0.40253 = 0.4025$$

17. Evaluate $\int_0^2 \frac{dx}{x^2+x+1}$ to three decimal dividing the range into eight equal parts.

solu) Given $a=0$, $b=2$, $n=8$; $h=\frac{b-a}{n}=\frac{2-0}{8}=\frac{2}{8}=0.25$

$y=f(x)=\frac{1}{x^2+x+1}$

$x_0=a=0 \implies y_0=\frac{1}{x_0^2+x_0+1}=\frac{1}{0^2+0+1}=\frac{1}{1}=1$

$x_1=x_0+h$ $\quad y_1=\frac{1}{x_1^2+x_1+1}$ $\quad y_1=\frac{1}{(0.25)^2+(0.25)+1}=\cancel{1}\quad 0.7619$

$=0+0.25$

$=0.25$

$x_2=x_1+h$ $\quad y_2=\frac{1}{x_2^2+x_2+1}=\frac{1}{(0.5)^2+0.5+1}=\frac{1}{1.75}=0.5714$

$=0.25+0.25$

$=0.5$

$x_3=x_2+h$ $\quad y_3=\frac{1}{x_3^2+x_3+1}$

$=0.5+0.25$ $\quad =\frac{1}{(0.75)^2+0.75+1}=0.4324$

$=0.75$

$x_4=x_3+h$ $\quad y_4=\frac{1}{x_4^2+x_4+1}=\frac{1}{1+1+1}=\frac{1}{3}=0.3333$

$=0.75+0.25$

$=1$

$x_5=x_4+h$ $\quad y_5=\frac{1}{x_5^2+x_5+1}=\frac{1}{(1.25)^2+1.25+1}=0.2622$

$=1+0.25$

$=1.25$

$x_6=x_5+h$ $\quad y_6=\frac{1}{x_6^2+x_6+1}=\frac{1}{(1.5)^2+1.5+1}=0.2105$

$=1.25+0.25$

$=1.5$

$x_7=x_6+h$ $\quad y_7=\frac{1}{x_7^2+x_7+1}=\frac{1}{(1.75)^2+1.75+1}=0.17204$

$=1.5+0.25$

$=1.75$

$x_8=x_7+h$ $\quad y_8=\frac{1}{x_8^2+x_8+1}=\frac{1}{2^2+2+1}=\frac{1}{7}=0.14285$

$=1.75+0.25$

$=2$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^2 y\,dx = \frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\right]$$

$$= \frac{0.25}{3}\left[(1+0.14285)+4(0.7619+0.4324+0.2622+0.17204)+2(0.5714+0.3333+0.2105)\right]$$

$$= \frac{0.25}{3}\left[1.1428+2(1.1149)+4(1.62854)\right]$$

$$= \frac{0.25}{3}\left[1.1428+2.2298+6.51416\right]$$

$$= \frac{0.25}{3}\left[9.88676\right] = \frac{2.47184}{3}$$

$$= 0.824$$

18. Evaluate $\int_0^6 \frac{x}{1+x^5}$ by using simpson $\frac{3}{8}$ Rule where $n=6$

H.W

Solu) Given that $a=0,\ b=6,\ n=6$; $h=\frac{b-a}{n}=\frac{6-0}{6}=1$

$y=f(x)=\frac{x}{1+x^5}$

$x_0=a=0,\quad y_0=\frac{x_0}{1+x_0^5}=\frac{0}{1+0}=0$

$x_1=x_0+h = 0+1 = 1 \quad y_1=\frac{x_1}{1+x_1^5}=\frac{1}{1+1}=\frac{1}{2}=0.5$

$2x^2=4$
$2x^2=4$
$2=2$
32

$x_2=x_1+h = 1+1 = 2 \quad y_2=\frac{2}{1+2^5}=\frac{2}{1+32}=\frac{2}{33}=0.0303 \quad 0.0606$

$x_3=x_2+h = 2+1 = 3 \quad y_3=\frac{3}{1+3^5}=\frac{3}{1+243}=\frac{3}{244}=0.0040983 \quad 0.0122$

$x_4=x_3+h = 3+1 = 4 \quad y_4=\frac{x_4}{1+x_4^5}=\frac{4}{1+4^5}=\frac{4}{1+1024}=\frac{1}{1025}=\cancel{0.0009756} \quad 0.003902$

$x_5=x_4+h = 4+1 = 5 \quad y_5=\frac{x_5}{1+x_5^5}=\frac{5}{1+5^5}=\frac{5}{1+3125}=\frac{5}{3126}=0.00031989 \quad 0.001599$

$x_6=x_5+h = 5+1 = 6 \quad y_6=\frac{x_6}{1+x_6^6}=\frac{6}{1+6^6}=\frac{6}{1+7776}=\frac{6}{7777}=0.00012858 \quad 0.007722$

Simpson $\frac{3}{8}$ Rule

$$\int_0^6 y\,dx = \frac{3h}{8}\left[(y_0+y_6)+4(y_1+y_2+y_4+y_5)+2(y_3)\right]$$

$$\left[\left[= \frac{3(1)}{8}\left[(0+0.000128)+4(0.5+0.0040+0.000319)+2(0.0303+0.000975)\right)\right.\right.$$

wrong

$$= \frac{3}{8}\left[0.000128+4(0.504319)+2(0.031275)\right]$$

$$= 0.375[0.000128+2.017276+0.06255]$$

$$= 0.375[2.079954]$$

$$= 0.77998]]$$

$$= \frac{3}{8}[0+0.0077]+3[0.5+0.0606+0.0039+0.0015+0.0077]$$

$$+2[0.0122]$$

$$= \frac{3}{8}[0.0077+3(0.566)+0.0244]$$

$$= \frac{3}{8}[0.0077+1.698+0.0244]$$

$$= \frac{3}{8}[1.7301]$$

$$= \frac{5.1903}{8} = 0.6487875$$

19. Evaluate $\int_0^1 \frac{dx}{1+x}$ by using Simpson $\frac{1}{3}$rd Rule where $h=0.2$

$\left[x \quad \underset{x_0}{0} \quad \underset{x_1}{0.2} \quad \underset{x_2}{0.4} \quad \underset{x_3}{0.6} \quad \underset{x_4}{0.8} \quad \underset{x_5}{1}\right]$ and also find $\log_e 2$

solu) Given that

$$\int_0^1 \frac{dx}{1+x}$$

put $1+x=t$

$dx = dt$

$x=0,\ t=1+x=1+0=1$

$x=1,\ t=1+x=1+1=2$

$$\int_0^1 \frac{dx}{1+x} = \int_1^2 \frac{dt}{t}$$

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$$= [\log t]_1^2$$
$$= \log_e^2 - \log_e^1$$
$$= \log_e^2 - 0$$
$$\int_0^2 \frac{dt}{1+x} = \log_e^2 \rightarrow (1)$$

By simpson $\frac{1}{3}$rd Rule

$$\int_0^1 \frac{1}{1+x}\,dx = \frac{h}{3}\left[(y_0+y_5) + 4(y_1+y_3) + 2(y_2+y_4)\right]$$

$h = 0.2$

$$y = f(x) = \frac{1}{1+x}$$

$x_0 = a = 0$

$x_1 = x_0 + h = 0 + 0.2 = 0.2$

$x_2 = x_1 + h = 0.2 + 0.2 = 0.4$

$x_3 = x_2 + h = 0.4 + 0.2 = 0.6$

$x_4 = x_3 + h = 0.6 + 0.2 = 0.8$

$x_5 = x_4 + h = 0.8 + 0.2 = 1.0$

$$y_0 = \frac{1}{1+x_0} = \frac{1}{1+0} = \frac{1}{1} = 1$$
$$y_1 = \frac{1}{1+x_1} = \frac{1}{1+0.2} = \frac{1}{1.2} = 0.8333$$
$$y_2 = \frac{1}{1+x_2} = \frac{1}{1+0.4} = \frac{1}{1.4} = 0.71428$$
$$y_3 = \frac{1}{1+x_3} = \frac{1}{1+0.6} = \frac{1}{1.6} = 0.625$$
$$y_4 = \frac{1}{1+x_4} = \frac{1}{1+0.8} = \frac{1}{1.8} = 0.5555$$
$$y_5 = \frac{1}{1+x_5} = \frac{1}{1+1} = \frac{1}{2} = 0.5$$

$$= \frac{0.2}{3}\left[(1+0.5) + 4(0.8333+0.625) + 2(0.71428+0.5555)\right]$$
$$= 0.0666\left[1.5 + 4(1.4583) + 2(1.26978)\right]$$
$$= 0.0666\left[1.5 + 5.8332 + 2.53958\right]$$
$$= 0.0666\left[9.87276\right]$$
$$= 0.658118$$

20. Evaluate $\int_0^5 \frac{dx}{4x+5}$ by using simpson $\frac{1}{3}$rd Rule where $h=1$ and also find $\log_e^5$

Solu) $\int_0^5 \frac{dx}{4x+5}$.

put $4x+5=t$

$4\,dx = dt$

$dx = \frac{dt}{4}$

Put $x=0$ $t=4(0)+5=5$

$x=5$ $t=4(5)+5=25$

$$\int_0^5 \frac{dx}{4x+5} = \int_5^{25} \frac{dt}{4t} = \frac{1}{4}\int_5^{25}\frac{dt}{t}$$

$$= \frac{1}{4}\left[\log t\right]_5^{25}$$

$$= \frac{1}{4}\left[\log_e^{25} - \log_e^5\right]$$

$$= \frac{1}{4}\left[2\log_e^5 - \log_e^5\right)$$

$$= \frac{1}{4}\left[2-1\right]\log_e^5$$

$$\therefore \int_0^5 \frac{dx}{4x+5} = \frac{1}{4}\log_e^5 \rightarrow (1) \text{ (or) } \log_e^5 = 4\int_0^5 \frac{dx}{4x+5}$$

NOW

$F(x) = \frac{1}{4x+5}$ $\quad a=0=x_0$

$x_1 = x_0+h$ $\quad x_2 = x_1+h$ $\quad x_3 = x_2+h$ $\quad x_4 = x_3+h$ $\quad x_5 = x_4+h$

$=0+1$ $\quad =1+1$ $\quad =2+1$ $\quad =3+1$ $\quad =4+1$

$=1$ $\quad =2$ $\quad =3$ $\quad =4$ $\quad =5$

$y_0 = \frac{1}{4x_0+5} = \frac{1}{4(0)+5} = \frac{1}{5} = 0.2$

$y_1 = \frac{1}{4x_1+5} = \frac{1}{4(1)+5} = \frac{1}{9} = 0.1111$

$y_2 = \frac{1}{4x_2+5} = \frac{1}{4(2)+5} = \frac{1}{13} = 0.07692$

$y_3 = \frac{1}{4x_3+5} = \frac{1}{4(3)+5} = \frac{1}{17} = 0.05882$

$y_4 = \frac{1}{4x_4+5} = \frac{1}{4(4)+5} = \frac{1}{21} = 0.04761$

$y_5 = \frac{1}{4x_5+5} = \frac{1}{4(5)+5} = \frac{1}{25} = 0.04$

Simpson $\frac{1}{3}$rd Rule

$$\int_0^5 \frac{dx}{4x+5} = \frac{h}{3}\left[(y_0+y_5)+4(y_1+y_3)+2(y_2+y_4)\right]$$

$$= \frac{1}{3}\left[(0.2+0.04)+4(0.1111+0.0588)+2(0.0769+0.0476)\right]$$

$$= \frac{1}{3}\left[0.24+4(0.1699)+2(0.12451)\right]$$

$$= \frac{1}{3}\left[0.24+0.6796+0.24902\right]$$

$$= 0.3333\left[1.16862\right]$$

$$= 0.389501 \rightarrow (2)$$

$$\frac{1}{4}\log_e 5 = 0.389501 \quad [\text{from } (1) \,\&\, (2)]$$

$$\log_e 5 = 4\times 0.38950$$

$$= 1.558004$$

Date 30/7/18

Numerical Solution for the Ordinary differential Equations.

Picards Method:

Consider $\frac{dy}{dx} = F(x,y)$ then

$y_1^{(1)} = y_0 + \int_{x_0}^{x} F(x, y_0)\,dx$ is called first approximation

$y^{(2)} = y_0 + \int_{x_0}^{x} F(x, y^{(1)})\,dx$ is called second approximation

$y^{(3)} = y_0 + \int_{x_0}^{x} F(x, y^{(2)})\,dx$ is called third approximation

$y^{(4)} = y_0 + \int_{x_0}^{x} F(x, y^{(3)})\,dx$ is called fourth approximation

1) Using picards method to find the value of y and $x=0.1$, $x=0.2$ given $\frac{dy}{dx}=x-y$ if intial condition $y=1$ when $x=0$

Solu) Given $\frac{dy}{dx} = x-y$ if initial condition $y=1$ when $x=0$

solu $f(x,y) = x-y \longrightarrow (1)$

Given

$y=1$, when $x=0 \Rightarrow x_0=0,\ y_0=1$

By picards method

$$y^{(1)} = y_0+\int_{x_0}^{x} f(x_0,y_0)\,dx$$

$$f(x,y_0) = x-y_0$$
$$= x-1,\ x_0=0$$

$$y^{(1)} = 1+\int_0^x (x-1)\,dx$$
$$= 1+\left[\frac{x^2}{2}-x\right]_0^x$$
$$= 1+\frac{x^2}{2}-x$$
$$y^{(1)} = 1-x+\frac{x^2}{2} \rightarrow (2)$$

$$y^{(2)} = y_0+\int_{x_0}^{x} f(x,y^{(1)})\,dx$$

$$f(x,y^{(1)}) = x-y^{(1)}$$
$$= x-\left(1-x+\frac{x^2}{2}\right)$$
$$= x-1+x-\frac{x^2}{2}$$
$$= 2x-1-\frac{x^2}{2}$$

$$y^{(2)} = 1+\int_0^x \left(-1+2x-\frac{x^2}{2}\right)dx$$
$$= 1+\left[-x+\cancel{2}\frac{x^2}{\cancel{2}}-\frac{1}{2}\frac{x^3}{3}\right]_0^x$$

$$y^{(2)} = 1-x+x^2-\frac{x^3}{6} \rightarrow (3)$$

$$y^{(3)} = y_0+\int_{x_0}^{x} f(x,y^{(2)})\,dx$$

$$f(x,y^{(2)}) = x-y^{(2)}$$
$$= x-\left(1-x+x^2-\frac{x^3}{6}\right)$$
$$= x-1+x-x^2+\frac{x^3}{6}$$

$$= -1 + 2x - x^2 + \frac{x^3}{6}$$

$$y^{(3)} = -1 + 2x - x^2 + \frac{x^3}{6} \rightarrow (4)$$

$$y^{(3)} = 1 + \int_0^x \left(-1 + 2x - x^2 + \frac{x^3}{6}\right) dx$$

$$= 1 + \left[-x + \frac{2x^2}{2} - \frac{x^3}{3} + \frac{1}{6}\frac{x^4}{4}\right]_0^x$$

$$y^{(3)} = 1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{24}$$

$$y^{(4)} = y_0 + \int_{x_0}^x f(x, y^{(3)})\, dx$$

$$f(x, y^{(3)}) = x - \left(1 - x + x^2 - \frac{x^3}{3} + \frac{x^4}{24}\right)_0^x$$

$$= x - 1 + x - x^2 + \frac{x^3}{3} - \frac{x^4}{24}$$

$$y = 2x - x^2 - 1 + \frac{x^3}{3} - \frac{x^4}{24}$$

$$y^{(4)} = 1 + \int_0^x \left(2x - x^2 - 1 + \frac{x^3}{3} - \frac{x^4}{24}\right) dx$$

$$= 1 + \left[\frac{2x^2}{2} - \frac{x^3}{3} + x + \frac{x^4}{4 \cdot 3} - \frac{x^5}{5 \times 24}\right]_0^x$$

$$y^{(4)} = 1 + x^2 - \frac{x^3}{3} + x + \frac{x^4}{12} - \frac{x^5}{120}$$

$$y = 1 - x + x^2 - \frac{x^3}{3} \cancel{-x} + \frac{x^4}{12} - \frac{x^5}{120}$$

at $x = 0.1$

$$y = 1 - 0.1 + (0.1)^2 - \frac{(0.1)^3}{3} + \frac{(0.1)^4}{12} - \frac{(0.1)^5}{120}$$

$$= +0.9 + 0.01 - \frac{0.001}{3} + \frac{0.0001}{12} - \frac{0.00001}{120}$$

$$= 0.91 \cancel{-0.9096} + -0.000333 + 0.00000083 - 0.0000000833$$

$$= 0.90967522$$

$$y(0.1) = 0.9097$$

at $x = 0.2$

$$y = 1 - 0.2 + (0.2)^2 - \frac{(0.2)^3}{3} + \frac{(0.2)^4}{12} - \frac{(0.2)^5}{120}$$

$$= 0.8 + 0.04 - \frac{0.008}{3} + \frac{0.00016}{12} - \frac{0.00032}{120}$$

$$= 0.84 - 0.0026667 + 0.0001333 - 0.0000026667$$

$$y(0.2) = 0.8375$$

0.1
0.1
01
00
0.01
0.1
0.01
000

0.8
0.04
0.84

2
4
8
12

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2. If $\frac{dy}{dx} = \frac{y-x}{y+x}$, find the value of y and $x=0.1$ using picards method given that $y(0)=1$

solu] Given that

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

$$f(x,y) = \frac{y-x}{y+x} \rightarrow ①$$

Given $y(0)=1 \Rightarrow x_0=0,\ y_0=1$

By picards method

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x,y_0)\,dx$$

$$f(x,y_0) = \frac{y_0-x}{y_0+x} = \frac{1-x}{1+x}$$

$$y^{(1)} = 1 + \int_0^x \left[\frac{1-x}{1+x}\right] dx$$

1-x=t
-1=dt
-dx=dt

$$= 1 + \left[\frac{(1+x)(-1)-(1-x)(1)}{(1+x)^2}\right]_0^x = \cancel{1+}$$

put $1+x=t$

$1\,dx = dt$

$$= 1 + \left[\frac{-1-\cancel{x}-1+\cancel{x}}{(1+x)^2}\right]_0^x$$

lower limit $x=0$, $t=1+x$

$t=1$

$$= 1+\left[-2\right]$$

$x=x$, $t=1+x$

$$= 1 + \int_1^{1+x} \frac{1-(t-1)}{t}\,dt$$

$$= 1 + \int_1^{1+x} \frac{\cancel{1}-t+\cancel{1}}{t}\,dt$$

$$= 1 + \int_1^{1+x} \frac{2-t}{t}\,dt$$

$$= 1 + \int_1^{1+x} \left[\frac{2}{t}-1\right] dt$$

$$= 1 + \left[2\log t - t\right]_1^{1+x}$$

$$= 1 + \left[2\log(1+x) - (1+x)\right] - \left[2\log(1) - 1\right]$$

$$= 1 + \left[2\log(1+x) - (1+x)\right] + 1$$

$$= 1 + 2\log(1+x) - \cancel{x} - x + \cancel{x}$$

$$y^{(1)} = 1 - x + 2\log(1+x)$$

$$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})\, dx$$

$$f(x, y^{(1)}) = \frac{y^{(1)} - x}{y^{(1)} + x}$$

$$= \frac{1 - x + 2\log(1+x) - x}{1 - \cancel{x} + 2\log(1+x) + \cancel{x}}$$

$$= \frac{1 - 2x + 2\log(1+x)}{1 + 2\log(1+x)}$$

$$y^{(2)} = 1 + \int_0^x \frac{1 - 2x + 2\log(1+x)}{1 + 2\log(1+x)}\, dx$$

It is not defined

The solution of the given differential Equation is

$$y = 1 - x + 2\log(1+x)$$

put $x = 0.1$

$$y = 1 - (0.1) + 2\log(1 + 0.1)$$

$$y = 0.9 + 2\log(1.1)$$

$$y = 0.9 + 2(0.0953)$$

$$y = 0.9 + 0.19062 = 0.9906$$

$$y = 1.0906$$

Date 17/18 3. find the solution of $\frac{dy}{dx} = 1 + xy$, $y(0) = 1$ in the intraval $(0, 0.05)$ correct to three decimal places taking $h = 0.1$

Solu Given $\frac{dy}{dx} = 1 + xy \Rightarrow f(x, y) = 1 + xy \rightarrow$ ①

$y(0) = 1$, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

By picards method

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$f(x, y^0) = 1 + x \cdot y_0, \quad y_0 = 1$$

$$= 1 + x$$

$$y^{(1)} = 1 + \int_0^x (1+x)\, dx$$

$$= 1+\left[x+\frac{x^2}{2}\right]_0^1$$

$$= 1+\left[x+\frac{x^2}{2}\right]$$

$$y^{(1)} = 1+x+\frac{x^2}{2} \rightarrow (2)$$

$$y^{(2)} = y_0+\int_{x_0}^{x} f(x, y^{(1)})\,dx$$

$$f(x, y^{(1)}) = 1+xy^{(1)}$$

$$= 1+x\left[1+x+\frac{x^2}{2}\right]$$

$$= 1+x+x^2+\frac{x^3}{2}$$

$$y^{(2)} = 1+\int_0^x \left[1+x+x^2+\frac{x^3}{2}\right]dx$$

$$= 1+\left[x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{8}\right]_0^x$$

$$y^{(2)} = 1+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{8} \rightarrow (3)$$

$$y^{(3)} = y_0+\int_{x_0}^{x} f(x, y^{(2)})\,dx$$

$$f(x, y^{(2)}) = 1+xy^{(2)} = 1+x\left[1+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{8}\right]$$

$$= 1+x+x^2+\frac{x^3}{2}+\frac{x^4}{3}+\frac{x^5}{8}$$

$$y^{(3)} = 1+\int_0^x \left[1+x+x^2+\frac{x^3}{2}+\frac{x^4}{3}+\frac{x^5}{8}\right]dx$$

$$= 1+\left[x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{2\times 4}+\frac{x^5}{3\times 5}+\frac{x^6}{6\times 8}\right]$$

$$y^{(3)} = 1+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{8}+\frac{x^5}{15}+\frac{x^6}{48}$$

$$y^{(4)} = y_0+\int_{x_0}^{x} f(x, y^{(3)})\,dx$$

Not defined

$$\therefore\ y = 1+x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{8}+\frac{x^5}{15}+\frac{x^6}{48}$$

$x_1 = x_0+h$
$= 0+0.1$
$= 0.1$

$$y(0.1) = 1+0.1+\frac{(0.1)^2}{2}+\frac{(0.1)^3}{3}+\frac{(0.1)^4}{8}+\frac{(0.1)^5}{15}+\frac{(0.1)^6}{48}$$

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$$y(0.1) = 1+0.1+\frac{0.01}{2}+\frac{0.001}{3}+\frac{0.0001}{8}+\frac{0.00001}{15}+\frac{0.000001}{48}$$

$$= 1.1+0.005+0.00033+0.000025+0.00000067$$

$$+0.000000021$$

$$y(0.1) = 1.105343191$$

$$y(0.1) = 1.105$$

$x_2 = x_1+h$
$= 0.1+0.1$
$= 0.2$

$$y(0.2) = 1+0.2+\frac{(0.2)^2}{2}+\frac{(0.2)^3}{3}+\frac{(0.2)^4}{8}+\frac{(0.2)^5}{15}+\frac{(0.2)^6}{48}$$

$$y(0.2) = 1.2+\frac{0.04}{2}+\frac{0.008}{3}+\frac{0.0016}{8}+\frac{0.00032}{15}+\frac{0.0000064}{48}$$

$$= 1.2+0.02+0.00267+0.0002+0.0000213+0.00000133$$

$$= 1.22289263$$

$$= 1.223$$

$x_3 = x_2+h$
$= 0.2+0.1$
$= 0.3$

$$y(0.3) = 1+0.3+\frac{(0.3)^2}{2}+\frac{(0.3)^3}{3}+\frac{(0.3)^4}{8}+\frac{(0.3)^5}{15}+\frac{(0.3)^6}{48}$$

$$y(0.3) = 1.3+\frac{0.03}{2}+\frac{0.027}{3}+\frac{0.0081}{8}+\frac{0.00243}{15}+\frac{0.000729}{48}$$

$$= 1.3+0.045+0.009+0.00101+0.000162+0.0000151$$

$$= 1.3551871$$

$$y(0.3) = 1.355$$

$x_4 = x_3+h$
$= 0.3+0.1$
$= 0.4$

$$y(0.4) = 1+0.4+\frac{(0.4)^2}{2}+\frac{(0.4)^3}{3}+\frac{(0.4)^4}{8}+\frac{(0.4)^5}{15}+\frac{(0.3)^6}{48}$$

$$y(0.4) = 1+0.4+\frac{0.96}{2}+\frac{0.064}{3}+\frac{0.0256}{8}+\frac{0.01024}{15}+\frac{0.004096}{48}$$

$$= 1.4+0.08+0.02133+0.0032+0.0006826+0.00008$$

$$= 1.505265$$

$$y(0.4) = 1.505$$

$x_5 = x_4 + h$ $\quad y(0.5) = 1 + 0.5 + \frac{(0.5)^2}{2} + \frac{(0.5)^3}{3} + \frac{(0.5)^4}{8} + \frac{(0.5)^5}{15} + \frac{(0.5)^6}{48}$
$= 0.4 + 1$
$= 0.5$

$y(0.5) = 1.5 + \frac{0.25}{2} + \frac{0.125}{3} + \frac{0.0625}{8} + \frac{0.03125}{15} + \frac{0.015625}{48}$

$= 1.5 + 0.125 + 0.04167 + 0.0078125 + 0.00208333 + 0.00032$

$= 1.6798155$

$y(0.5) = 1.68$

4. For the differential Equation $\frac{dy}{dx} = x - y^2$, $y(0) = 0$ Calculate $y(0.2)$ by using picards method to third approximation and round of the value into four decimal places

Solu) Given $\frac{dy}{dx} = x - y^2$

$f(x, y) = x - y^2 \rightarrow ①$

$y(0) = 0, \quad x_0 = 0, \quad y_0 = 0$

By picards method

$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$

$f(x, y_0) = x - y_0^2$
$= x - 0$
$= x$

$y^{(1)} = 0 + \int_0^x x\, dx$

$y(1) = \left[\frac{x^2}{2}\right]_0^x = \frac{x^2}{2} \rightarrow ②$

$y^{(2)} = y_0 + \int_{x_0}^{x} f(x, y^{(1)})\, dx$

$f(x, y^{(1)}) = x - y_1^2$
$= x - \left(\frac{x^2}{2}\right)^2$
$= x - \frac{x^4}{4}$

$y^{(2)} = 0 + \int_0^x \left[x - \frac{x^4}{4}\right] dx$

$y(2) = \frac{x^2}{2} - \frac{x^5}{20} \rightarrow ③$

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$$y^{(3)} = y_0 + \int_{x_0}^{x} f(x, y^{(2)})\, dx$$

$$f(x, y^{(2)}) = x - (y_{(2)})^2$$

$$= x - \left[\frac{x^2}{2} - \frac{x^5}{20}\right]^2$$

$$= x - \left[\frac{x^4}{4} + \frac{x^{10}}{400} - \frac{2x^7}{40}\right]$$

$$= x - \frac{x^4}{4} - \frac{x^{10}}{400} + \frac{x^7}{20}$$

$$y^{(3)} = 0 + \int_0^x \left[x - \frac{x^4}{4} - \frac{x^{10}}{400} + \frac{x^7}{20}\right]$$

$$y^{(3)} = \left[\frac{x^2}{2} - \frac{x^5}{20} - \frac{x^{11}}{4400} + \frac{x^8}{160}\right] \to (4)$$

$$y = \frac{x^2}{2} - \frac{x^5}{20} - \frac{x^{11}}{4400} + \frac{x^8}{160}$$

$$y(0.2) = \frac{(0.2)^2}{2} - \frac{(0.2)^5}{20} - \frac{(0.2)^{11}}{4400} + \frac{(0.2)^8}{160}$$

$$= \frac{0.04}{2} - \frac{0.00032}{20} - \frac{0.0000000204}{4400} + \frac{0.00000256}{160}$$

$$= 0.02 - 0.000016 - 0.00000000000004654$$
$$+ 0.000000016$$

$$= 0.0199984016$$

$$y(0.2) = 0.02$$

5. find an approximate value of y when $x = 0.1$, if $\frac{dy}{dx} = x - y^2$ and $y = 1$, at $x = 0$ using picards method upto ~~three~~ two approximations

t.w

solu) Given that

$$\frac{dy}{dx} = x - y^2$$

$$f(x, y) = x - y^2 \to (1)$$

$$y = 1, \quad x = 0.1, \quad x_0 = 0, \quad y_0 = 1$$

By picards method

$$y^{(1)} = y_0 + \int_{x_0}^{x} f(x, y_0)\, dx$$

$$f(x, y_0) = x - y_0^2$$

$$= x - 1^2$$

$= x-1$

$y^{(1)} = 9 + \int_0^x (x-1)\,dx$

$y(1) = 1 + \left[\frac{x^2}{2} - x\right] \rightarrow ②$

$y(2) = y_0 + \int_{x_0}^{x} f(x, y^{(1)})\,dx$

$f(x, y^{(1)}) = x - y_1^2$

$= x - \left[1 + \frac{x^2}{2} - x\right]^2$

$= x - 1 - \frac{x^2}{2} + x$

$= 2x - 1 - \frac{x^2}{2}$

$y(2) = 1 + \int_0^x \left[2x - 1 - \frac{x^2}{2}\right]$

$= 1 + \frac{2x^2}{2} - x - \frac{x^3}{6}$

$y(2) = 1 + x^2 - x - \frac{x^3}{6}$

$y = 1 - x + x^2 - \frac{x^3}{6}$

$y(0.1) = 1 - 0.1 + (0.1)^2 - \frac{(0.1)^3}{6}$

$= 1 - 0.1 + 0.01 - \frac{0.001}{6}$

$= 1 - 0.1 + 0.01 - 1.00016$

$= -0.09016$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$= x - \left[1 + \frac{x^4}{4} - x^2 + \frac{2x^2}{2} - \frac{2x^2}{2}x - 2x^2\right]$

$= x - \left[1 + \frac{x^4}{4} + \cancel{x^2} + \cancel{x^2} - x^3 - 2x\right]$

$= x - \left[1 + 2x^2 + \frac{x^4}{4} - x^3 - 2x\right]$

$= x - \left[1 + 2x^2 - 2x + \frac{x^4}{4} - x^3\right]$

$y(2) = 1 + \int_0^x \left[x - 1 - 2x^2 + 2x - \frac{x^4}{4} + x^3\right]$

$y(2) = 1 + \frac{x^2}{2} - x - \frac{2x^3}{3} + \frac{x^2}{2} - \frac{x^5}{4 \times 5} + \frac{x^4}{4}$

$y(2) = 1 + \frac{x^2}{2} - x - \frac{2}{3}x^3 + x^2 - \frac{x^5}{20} + \frac{x^4}{4}$

c)

$= 0.2 + 0.02 + 0.00013$

$= 0.22013$

$y(0.4) = 0.4 + \frac{(0.4)^2}{2} + \frac{(0.4)^4}{12}$

$= 0.4 + \frac{0.16}{2} + \frac{0.0256}{12}$

$= 0.4 + 0.08 + 0.00213$

$= 0.48213$

$y(0.1) = 1 + \frac{(0.1)^2}{2} - 0.1 - \frac{2}{3}(0.1)^3 + (0.1)^2 - \frac{(0.1)^5}{20} + \frac{(0.1)^4}{4}$

$= 1 + \frac{0.01}{2} - 0.1 - \frac{2}{3}(0.001) + 0.01 - \frac{0.00001}{20} + 0.0001$

$= 0.9101 + 0.0005$

$= 0.9099328 - (0.6667)(0.001)$

1/8/18

7. R-K Method of 4th order

Consider $\frac{dy}{dx} = f(x, y)$

$$y_1 = y_0 + k$$

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = h \cdot f(x_0, y_0)$$

$$k_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_0 + h, y_0 + k_3)$$

Similarly

$$y_2 = y_1 + k$$

$$k = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$$

$$k_1 = h \cdot f(x_1, y_1)$$

$$k_2 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$k_3 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$k_4 = h \cdot f(x_1 + h, y_1 + k_3)$$

1. Use R-K method of 4th order. Find the value of y at $x=0.1$ given $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$,

Solu) Given

$$\frac{dy}{dx} = \frac{y-x}{y+x}$$

$$\Rightarrow f(x,y) = \frac{y-x}{y+x} \rightarrow ①$$

$y(0)=1 \Rightarrow x_0=0, \; y_0=1 \; ; \; h=0.1$

$$k_1 = h.f(x_0, y_0)$$

$$= h.f(0,1)$$

$$= 0.1 \; \frac{1-0}{1+0}$$

$$= 0.1 \times 1 = 0.1$$

$$\boxed{k_1 = 0.1}$$

$$k_2 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}\right)$$

$$= h.f\left(0+\frac{0.1}{2}, 1+\frac{0.1}{2}\right)$$

$$= hf\left(\frac{0.1}{2}, \frac{2.1}{2}\right)$$

$$= 0.1 f(0.05, 1.05)$$

$$= 0.1\left[\frac{1.05-0.05}{1.05+0.05}\right]$$

$$= 0.1 \times \frac{1}{1.1}$$

$$\boxed{k_2 = 0.0909}$$

$$k_3 = h.f\left(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}\right)$$

$$= h.f\left(0+\frac{0.1}{2}, 1+\frac{0.0909}{2}\right)$$

$$= h.f(0.05, 1+0.04545)$$

$$= h.f(0.05, 1.04545)$$

$$= 0.1\left[\frac{1.04545-0.05}{1.04545+0.05}\right]$$

$$= 0.1 \times \frac{0.99545}{1.09545}$$

$= 0.09087133$

$k_3 = 0.0909$

$K_4 = h.f(x_0+h, y_0+K_3)$

$= h.f(0+0.1, 1+0.0909)$

$= h.f(0.1, 1.0909)$

$= 0.1\left[\frac{1.0909-0.1}{1.0909+0.1}\right]$

$= 0.1 \times \frac{0.9909}{1.1909}$

$\boxed{K_4 = 0.0832}$

$K = \frac{1}{6}\left[K_1+2K_2+2K_3+K_4\right]$

$= \frac{1}{6}\left[0.1+2(0.0909)+2(0.0909)+0.0832\right]$

$= \frac{1}{6}\left[0.1+0.1818+0.1818+0.0832\right]$

$= \frac{0.5468}{6}$

$K = 0.09113$

$K = 0.0911$

2. $y_1 = y_0+K$

$= 1+0.0911$

$y_1 = 1.0911, \quad x_1 = 0.1$

2. Use R.K. method of 4^{th} order to find the value of y ah $x=0.1$, given $y' = xy+1$, $y(0) = 1$

solu) Given $\frac{dy}{dx} = xy+1 = y'$

$f(x, y) = xy+1 \longrightarrow ①$

$y(0) = 1, \quad x_0 = 0, \quad y_0 = 1, \quad h = 0.1$

$K_1 = h.f(x_0, y_0)$

$= hf(0,1)$

$= h\left[0(1)+1\right]$

$= 0.1 \times 1$

$\boxed{k_1 = 0.1}$

$k_2 = h \cdot f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$= hf(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2})$

$= hf(0.05, 1 + 0.05)$

$= hf(0.05, 1.05)$

$= 0.1 [(0.05)(1.05) + 1]$

$= 0.1 [1.0525]$

$k_2 = 0.10525$

$\boxed{k_2 = 0.1053}$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$= hf(0 + \frac{0.1}{2}, 1 + \frac{0.10523}{2})$

$= hf(0.05, 1 + 0.05265)$

$= hf(0.05, 1.05265)$

$= 0.1 [(0.05)(1.05265) + 1]$

$= 0.1 [1.0526325]$

$k_3 = 0.1052 6325 = 0.1053$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= hf(0 + 0.1, 1 + 0.1053)$

$= hf(0.1, 1.1053)$

$= h[(0.1)(1.1053) + 1]$

$= 0.1 [1.11053]$

$= 0.111053$

$k_4 = 0.1111$

$k = \frac{1}{6} [k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6} [0.1 + 2(0.10525) + 2(0.1053) + 0.1111]$

$= \frac{1}{6}[0.1 + 0.2106 + 0.2106 + 0.1111]$

$= \frac{0.6323}{6}$

$= 0.10538$

$K = 0.1054$

$y_1 = y_0 + K$

$= 1 + 0.1054$

$y_1 = 1.1054$; $x_1 = 0.1$

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3. Using R.K method of 4^{th} order find y when $x = 0.1$ and 0.2, Given that $x = 0$, when $y = 1$ and $\frac{dy}{dx} = x+y$

Solu| Given

$\frac{dy}{dx} = x+y$

$f(x,y) = x+y \rightarrow ①$

$x=0$, $y=1$, $x_0=0$, $y_0=1$, $h=0.1$

Case(i)

$K_1 = h \cdot f(x_0, y_0)$

$x_1 = x_0 + h$

$= 0.1 + 0.1$

$= 0.2$

$K_1 = h \cdot f(0,1)$

$= h \cdot f[0+1]$

$K_1 = 0.1$

$K_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= h \cdot f\left(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}\right)$

$= h \cdot f(0.05, 1 + 0.05)$

$= h \cdot f(0.05, 1.05)$

$= 0.1[0.05 + 1.05]$

$= 0.1[1.1]$

$K_2 = 0.11$

$k_3 = h \cdot f\left(x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right)$

$= h \cdot f\left(0 + \frac{0.1}{2},\ 1 + \frac{0.11}{2}\right)$

$= h \cdot f(0.05,\ 1 + 0.055)$

$= h \cdot f(0.05,\ 1.055)$

$= 0.1\,[0.05 + 1.055]$

$= 0.1\,[1.105]$

$k_3 = 0.1105$

$k_4 = h \cdot f(x_0 + h,\ y_0 + k_3)$

$= h \cdot f(0 + 0.1,\ 1 + 0.1105)$

$= h \cdot f(0.1,\ 1.1105)$

$= 0.1\,[0.1 + 1.1105]$

$= 0.1\,[1.2105]$

$k_4 = 0.12105 = 0.1211$

$k = \frac{1}{6}\,[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}\,[0.1 + 2(0.11) + 2(0.1105) + 0.12105]$

$= \frac{1}{6}\,[0.1 + 0.22 + 0.221 + 0.1219]$

$= \frac{1}{6}\,[0.6621]$

$= 0.0736833 \quad 0.11035$

$k = 0.1104$

$y_1 = y_0 + k$

$= 1 + 0.1104$

$= 1.1104 \quad x_1 = 0.1$

Case (ii)

$x_2 = x_1 + h$

$= 0.1 + 0.1$

$= 0.2$

$k_1 = h \cdot f(x_q, y_q)$

$= h \cdot f(0.1, 1.1104)$

$= h\,[0.1 + 1.1104]$

$= h\,[1.2104]$

$= 0.1 \times 1.2104$

$k_1 = 0.12104 = 0.121$

$k_2 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$

$= h \cdot f\left(0.1 + \frac{0.1}{2}, 1.1104 + \frac{0.121}{2}\right)$

$= h \cdot f(0.1 + 0.05, 1.1104 + 0.0605)$

$= h \cdot f(0.15, 1.1709)$

$= 0.1\,[0.15 + 1.1709]$

$= 0.1\,[1.3209]$

$= 0.13209$

$k_2 = 0.1321$

$k_3 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$

$= h \cdot f\left(0.1 + \frac{0.1}{2}, 1.1104 + \frac{0.1321}{2}\right)$

$= h \cdot f(0.1 + 0.05, 1.1104 + 0.06605)$

$= h \cdot f(0.15, 1.17645)$

$= 0.1\,[0.15 + 1.17645]$

$= 0.1\,[1.32645]$

$k_3 = 0.132645 = 0.1327$

$k_4 = h \cdot f(x_1 + h, y_1 + k_3)$

$= h \cdot f(0.1 + 0.1, 1.1104 + 0.1327)$

$= h \cdot f(0.2, 1.2431)$

$= 0.1\,[0.2 + 1.2431]$

$= 0.1\,[1.4431]$

$= 0.14431$

$K_4 = 0.1443$

$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.121 + 2(0.1321) + 2(0.1327) + 0.1443]$

$= \frac{1}{6}[0.121 + 0.2642 + 0.2654 + 0.1443]$

$= \frac{1}{6}[0.7949]$

$= 0.132483$

$K = 0.13249$

$y_2 = y_1 + K$

$= 1.1104 + 0.13249$

$= 1.24289$

$y_2 = 1.2429$

4. Use R-K method of 4th order to find y when $x = 1.2$ given $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.5$

solu) Given that

$$\frac{dy}{dx} = x^2 + y^2$$

$$f(x_0, y_0) = x^2 + y^2 \rightarrow (1)$$

$y(1) = 1.5$, $x_0 = 1$, $y_0 = 1.5$, $h = 0.1$

Case (i) $x_1 = x_0 + h$

$= 1 + 0.1$

$= 1.1$

$K_1 = h.f(x_0, y_0)$

$= h.f(1, 1.5)$

$= h.[1 + (1.5)^2]$

$= 0.1[1 + 2.25]$

$= 0.1[3.25]$

$K_1 = 0.325$

$$K_2 = h\cdot f\left(x_0+\frac{h}{2},\ y_0+\frac{k_1}{2}\right)$$

$$= h\cdot f\left(1+\frac{0.1}{2},\ 1.5+\frac{0.325}{2}\right)$$

$$= h\cdot f\left(1+0.05,\ 1.5+0.1625\right)$$

$$= h\cdot f(1.05,\ 1.6625)$$

$$= 0.1\left[(1.05)^2+(1.6625)^2\right]$$

$$= 0.1\left[1.1025+2.76390\right]$$

$$= 0.1\left[3.8664\right]$$

$$= 0.38664$$

$$K_2 = 0.3866$$

$$K_3 = h\cdot f\left(x_0+\frac{h}{2},\ y_0+\frac{K_2}{2}\right)$$

$$= h\cdot f\left(1+\frac{0.1}{2},\ 1.5+\frac{0.3866}{2}\right)$$

$$= h\cdot f\left(1+0.05,\ 1.5+0.1933\right)$$

$$= h\cdot f(1.05,\ 1.6933)$$

$$= 0.1\left[(1.05)^2+(1.6933)^2\right]$$

$$= 0.1\left[1.1025+2.8673\right]$$

$$= 0.1\left[3.9698\right]$$

$$= 0.39698$$

$$K_3 = 0.397$$

$$K_4 = h\cdot f(x_0+h,\ y_0+K_3)$$

$$= h\cdot f(1+0.1,\ 1.5+0.397)$$

$$= h\cdot f(1.1,\ 1.897)$$

$$= 0.1\left[(1.1)^2+(1.897)^2\right]$$

$$= 0.1\left[1.21+3.599\right]$$

$$= 0.1\left[4.809\right]$$

$$K_4 = 0.4809$$

$$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$= \frac{1}{6}[0.325 + 2(0.3866) + 2(0.397) + 0.4809]$$

$$= \frac{1}{6}[0.325 + 0.7732 + 0.794 + 0.4809]$$

$$= \frac{1}{6}[2.3731]$$

$$= 0.395516$$

$$K = 0.3956$$

$$y_1 = y_0 + K$$

$$= 1.5 + 0.3956$$

$$y_1 = 1.8956, \quad x_1 = 1.1$$ pending

case (ii)

$$x_2 = x_1 + h$$

$$= 1.1 + 0.1$$

$$= 1.2$$

$$K_1 = h.f(x_q, y_q)$$

$$= h.f(\cancel{1.1, 1.5})$$

$$= h.f(1.1, 1.8956)$$

$$= h.f[(1.1)^2 + (1.8956)^2]$$

$$= 0.1[1.21 + 3.5933]$$

$$= 0.1[4.8033]$$

$$K_1 = 0.48033$$

$$K_2 = h.f\left(x_q + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$$

$$= h.f\left(1.1 + \frac{0.1}{2}, 1.8956 + \frac{0.48033}{2}\right)$$

$$= h.f(1.1 + 0.05, 1.8956 + 0.240165)$$

$$= h.f(1.15, 2.13577)$$

$$= 0.1[(1.15)^2 + (2.13577)^2]$$

$$= 0.1[1.3225 + 4.5615]$$

$= 0.1[5.884]$

$k_2 = 0.5884$

$K_3 = h.f\left(x_1 + \frac{h}{2},\ y_1 + \frac{K_2}{2}\right)$

$= h.f\left(1.1 + \frac{0.1}{2},\ 1.8956 + \frac{0.5884}{2}\right)$

$= h.f(1.1 + 0.05,\ 1.8956 + 0.2942)$

$= h.f(1.15,\ 2.1898)$

$= 0.1[(1.15)^2 + (2.1898)^2]$

$= 0.1[1.3225 + 4.7953]$

$= 0.1[6.1178]$

$K_3 = 0.61178 = 0.6118$

$K_4 = h.f(x_1 + h,\ y_1 + K_3)$

$= h.f(1.1 + 0.1,\ 1.8956 + 0.6118)$

$= h.f(1.2,\ 2.5074)$

$= 0.1[(1.2)^2 + (2.5074)]$

$= 0.1[1.44 + 6.28705]$

$= 0.1[7.72705]$

$K_4 = 0.7727$

$K = \frac{1}{6}[k_1 + 2k_2 + 2K_3 + K_4]$

$= \frac{1}{6}[0.48033 + 2(0.5884) + 2(0.6118) + 0.7727]$

$= \frac{1}{6}[0.48033 + 1.1768 + 1.2236 + 0.7727]$

$= \frac{1}{6}[3.65343]$

$= 0.608905$

$K = 0.6089$

$y_2 = y_1 + K$

$= 1.8956 + 0.6089$

Date 3/8/18 $y_2 = 2.5045$, $x_2 = 1.2$.

4. Given the initial value problem $y' = 1+y^2$, $y(0)=0$, find $y(0.6)$ by R.K method of 4th order taking $h=0.2$

Solu) Given

$y' = 1+y^2$

$f(x_0, y_0) = 1+y^2 \rightarrow (1)$

$y(0)=0$, $x_0=0$, $y_0=0$

case (i)

$x_1 = x_0+h$

$= 0+0.2$

$= 0.2$

$K_1 = hf(x_0, y_0)$

$= h.f(0, 0)$

$= 0.2[1+0^2]$

$K_1 = 0.2$

$K_2 = h.f(x_0+\frac{h}{2}, y_0+\frac{K_1}{2})$

$= h.f(0+\frac{0.2}{2}, 0+\frac{0.2}{2})$

$= h.f(0.1, 0.1)$

$= 0.2[1+(0.1)^2]$

$= 0.2[1+0.01]$

$= 0.2[1.01]$

$K_2 = 0.202$

$K_3 = h.f(x_0+\frac{h}{2}, y_0+\frac{K_2}{2})$

$= h.f(0+\frac{0.2}{2}, 0+\frac{0.202}{2})$

$= h.f(0.1, 0.101)$

$= 0.2[1+(0.101)^2]$

$= 0.2[1+0.010201]$

$= 0.2[1.010201]$

$\begin{array}{r} 0.1 \\ 0.1 \\ \hline 01 \\ 00 \end{array}$

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$k_3 = 0.2020402$

$k_3 = 0.202$

$k_4 = h.f(x_0+h,\ y_0+k_3)$

$= h.f(0+0.2,\ 0+0.2020)$

$= h.f(0.2,\ 0.2020)$

$= 0.2[1+(0.202)^2]$

$= 0.2[1+0.0408004]$

$= 0.2[1.0408004]$

$= 0.2081608$

$k_4 = 0.2082$

$K = \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$= \frac{1}{6}[0.2+2(0.202)+2(0.202)+0.2082]$

$= \frac{1}{6}[0.2+0.404+0.404+0.2082]$

$= \frac{1}{6}[1.2162]$

$K = 0.2027$

$y_1 = y_0+K$

$= 0+0.2027$

$y_1 = 0.2027 \quad x_1 = 0.2$

case(ii)

$x_2 = x_1+h$

$= 0.2+0.2$

$x_2 = 0.4$

$k_1 = h.f(x_1,\ y_1)$

$= h.f(0.2,\ 0.2027)$

$= h.[1+(0.2027)^2]$

$= 0.2[1+0.04108729]$

$= 0.2[1.04108729]$

$k_1 = 0.2082$

$K_2 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right)$

$= h \cdot f\left(0.2 + \frac{0.2}{2}, 0.2027 + \frac{0.2082}{2}\right)$

$= h \cdot f(0.2 + 0.1, 0.2027 + 0.1041)$

$= h \cdot f(0.3, 0.3068)$

$= 0.2[1 + (0.3068)^2]$

$= 0.2[1 + 0.09412624]$

$= 0.2[1.09412624]$

$= 0.218825248$

$K_2 = 0.2188$

$K_3 = h \cdot f\left(x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right)$

$= h \cdot f\left(0.2 + \frac{0.2}{2}, 0.2027 + \frac{0.2188}{2}\right)$

$= h \cdot f(0.2 + 0.1, 0.2027 + 0.10948)$

$= h \cdot f(0.3, 0.31218)$

$= 0.2[1 + (0.31218)^2]$

$= 0.2[1 + 0.097422015]$

$= 0.2[1.097422015]$

$= 0.219484403$

$K_3 = 0.2194$

$K_4 = h \cdot f(x_1 + h, y_1 + K_3)$

$= h \cdot f(0.2 + 0.2, 0.2027 + 0.2195)$

$= h \cdot f(0.4, 0.4222)$

$= 0.2[1 + (0.4222)^2]$

$= 0.2[1 + 0.17825284]$

$= 0.2[1.17825284]$

$= 0.235650568$

$K_4 = 0.2357$

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$$K = \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$$

$$= \frac{1}{6}[0.2082 + 2(0.2188) + 2(0.2195) + 0.2357]$$

$$= \frac{1}{6}[0.2082 + 0.4376 + 0.439 + 0.2357]$$

$$= \frac{1}{6}[1.3205]$$

$$= 0.2200083333$$

$$K = 0.2201$$

$$y_2 = y_1 + K$$

$$= 0.2027 + 0.2201$$

$$y_2 = 0.4228 \quad x_2, 0.4$$

case(iii)

$$x_3 = x_2 + h$$

$$= 0.4 + 0.2$$

$$x_3 = 0.6$$

$$K_1 = h \cdot f(x_2, y_2)$$

$$= h \cdot f(0.4, 0.4228)$$

$$= 0.2[1 + (0.4228)^2]$$

$$= 0.2[1 + 0.17875984]$$

$$= 0.2[1.17875984]$$

$$= 0.235751968$$

$$K_1 = 0.2358$$

$$K_2 = h \cdot f\left(x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2}\right)$$

$$= h \cdot f\left(0.4 + \frac{0.2}{2}, 0.4228 + \frac{0.2358}{2}\right)$$

$$= h \cdot f(0.5, 0.4228 + 0.1179)$$

$$= h \cdot f(0.5, 0.5407)$$

$$= 0.2[1 + (0.5407)^2]$$

$$= 0.2[1 + 0.29235649]$$

$$= 0.2[1.29235649]$$

$= 0.258471298$

$K_2 = 0.2585$

$K_3 = h.f\left(x_2+\frac{h}{2}, y_2+\frac{K_2}{2}\right)$

$= h.f\left(0.4+\frac{0.2}{2}, 0.4228+\frac{0.2585}{2}\right)$

$= h.f(0.5, 0.4228+0.12925)$

$= h.f(0.5, 0.55205)$

$= 0.2[1+(0.55205)^2]$

$= 0.2[1+0.3047592202]$

$= 0.2[1.304759203]$

$= 0.26095184$

$K_3 = 0.2601$

$K_4 = h.f(x_2+h, y_2+K_3)$

$= h.f(0.4+0.2, 0.4228+0.261)$

$= h.f(0.6, 0.6838)$

$= 0.2[1+(0.6838)^2]$

$= 0.2[1+0.46758244]$

$= 0.2[1.46758244]$

$= 0.293516488$

$K_4 = 0.2935$

$K = \frac{1}{6}[K_1+2K_2+2K_3+K_4]$

$= \frac{1}{6}[0.2358+2(0.2585)+2(0.261)+0.2935)$

$= \frac{1}{6}[0.2358+0.517+0.522+0.2935)$

$= \frac{1}{6}[1.5683]$

$= 0.26138333$

$K = 0.2614$

$y_3 = y_2+K$

$= 0.4228+0.2614 = 0.6842$

5. find the value of $y(1.1)$ using R-K method of 4th order

Hw Given that $\frac{dy}{dx} = 3x+y^2$, $y(1)=1$

6 find the value of $y(1.1)$ using R-K method of 4th order given $\frac{dy}{dx} = y^2+xy$; $y(1) = 1$

solu/ Given that

$$\frac{dy}{dx} = 3x+y^2$$

$$f(x_0, y_0) = 3x+y^2 \rightarrow (1)$$

$$y(1)=1, \quad x_0 = 1, \quad y_0 = 1$$

$K_1 = h.f(x_0, y_0)$

$= h.f(1,1)$

$= 0.1[3(1)+1]$

$= 0.1[3+1]$

$K_1 = 0.1[4] = 0.4$

$K_2 = h.f(x_0+\frac{h}{2}, y_0+\frac{K_1}{2})$

$K_2 = h.f(1+\frac{0.1}{2}, 1+\frac{0.4}{2})$

$= h.f(1+0.05, 1+0.2)$

$= h.f(1.05, 1.2)$

$= 0.1[3(1.05)+(1.2)^2]$

$= 0.1[3.15+1.44]$

$= 0.1[4.59]$

$K_2 = 0.459$

$K_3 = h.f(x_0+\frac{h}{2}, y_0+\frac{K_2}{2})$

$= h.f(1+\frac{0.1}{2}, 1+\frac{0.459}{2})$

$= h.f(1.05, 1+0.2295)$

$= h.f(1.05, 1.2295)$

$= 0.1[3(1.05)+(1.2295)^2]$

$= 0.1[3.15+1.51167025]$

$= 0.1[4.66167025]$

$= 0.466167025$

$= 0.4662$

$K_4 = h.f(x_0+h, y_0+K_3)$

$= h.f(1+0.1, 1+0.4662)$

$= h.f(1.1, 1.4662)$

$= 0.1[3(1.1)+(1.4662)^2]$

$= 0.1[3.3+2.14974244]$

$= 0.1[5.44974244]$

$= 0.544974244$

$K_4 = 0.5441$

$K = \frac{1}{6}[K_1+2K_2+2K_3+K_4]$

$= \frac{1}{6}[0.4+2(0.459)+2(0.4662)+0.5441]$

$= \frac{1}{6}[0.4+0.918+0.9324+0.5441]$

$= \frac{1}{6}[2.7945]$

$= 0.46575$

$K = 0.4658$

$y_1 = y_0 + K$

$= 1 + 0.4658$

$= 1.4658$

6. Given that

$\frac{dy}{dx} = y^2+xy$

$f(x_0, y_0) = y^2+xy \rightarrow ①$

$y(1)=1, \quad x_0=1, \quad y_0=1$

$K_1 = h.f(x_0, y_0)$

$= h.f(1,1)$

$= 0.1[1+1(1)]$

$= 0.1 \times 2$

$K_1 = 0.2$

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$K_2 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right)$

$= h \cdot f\left(1 + \frac{0.1}{2}, 1 + \frac{0.2}{2}\right)$

$= h \cdot f(1 + 0.05, 1 + 0.1)$

$= h \cdot f(1.05, 1.1)$

$= 0.1 \left[(1.1)^2 + (1.05)(1.1)\right]$

$= 0.1 [1.21 + 1.155]$

$= 0.1 [2.365]$

$K_2 = 0.2365$

$K_3 = h \cdot f\left(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right)$

$= h \cdot f\left(1 + \frac{0.1}{2}, 1 + \frac{0.2365}{2}\right)$

$= h \cdot f(1 + 0.05, 1 + 0.11825)$

$= h \cdot f(1.05, 1.11825)$

$= 0.1 \left[(1.11825)^2 + (1.05)(1.11825)\right]$

$= 0.1 [1.250483063 + 1.1741625]$

$= 0.1 [2.424645563]$

$= 0.2424645563$

$K_3 = 0.2425$

$K_4 = h \cdot f(x_0 + h, y_0 + K_3)$

$= h \cdot f(1 + 0.1, 1 + 0.2425)$

$= h \cdot f(1.1, 1.2425)$

$= 0.1 \left[(1.2425)^2 + (1.1)(1.2425)\right]$

$= 0.1 [1.54380625 + 1.36675]$

$= 0.1 [2.91055625]$

$= 0.291055625$

$K_4 = 0.2919$

$K = \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$

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$= \frac{1}{6}[0.2 + 2(0.2365) + 2(0.2425) + 0.2919]$

$= \frac{1}{6}[0.2 + 0.473 + 0.485 + 0.2911]$

$= \frac{1}{6}[1.4491]$

$= 0.2415166667$

$K = 0.2415$

$y_1 = y_0 + K$

$= 1 + 0.2415$

$y_1 = 1.2415$

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Code No: R201201 **R20** SET - 1

I B. Tech II Semester Regular Examinations, September-2021

MATHEMATICS-II

(Com. to All Branches)

Time: 3 hours Max. Marks: 70

Answer any five Questions one Question from Each Unit
All Questions Carry Equal Marks

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UNIT-I

1. a) Reduce the matrix $A = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}$ to its normal form and hence find the rank. (7M)

   b) Show that the only real value of λ for which the following equations have non-trivial solution is 6 and solve them, when $\lambda=6$. $x+2y+3z=\lambda x$; $3x+y+z=\lambda y$; $2x+3y+z=\lambda z$. (7M)

Or

2. a) Prove that the product of the Eigen values is equal to determinant of the matrix. (7M)

   b) Test the consistency of the system $x+ y+ z = 6$, $x-y+2z=5$, $3x+y+z=-8$, hence solve. (7M)

UNIT-II

3. a) Verify Cayley -Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ also find A^{-1} (7M)

   b) Find the nature, rank, index and signature of the quadratic from by reduce in to canonical form $x^2 + y^2 + 2z^2 + 2xy - 4xz + 4yz$ (7M)

Or

4. a) Find the orthogonal matrix P such that A is diagonalize where $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}$ (7M)

   b) Find the nature, rank, index and signature of the quadratic from by reduce in canonical form $2x^2 + y^2 - 3z^2 + 12xy - 4xz - 8yz$. (7M)

UNIT-III

5. a) Find the real root of the equation $x = \sin x$ using bisection method. (7M)

   b) Find the real root of the equation $x^3-x-1 =0$ using iteration method. (7M)

Or

6. a) Find the real root of the equation $\tan x = x$ using Newton Raphson method . (7M)

   b) Solve the following system of equations using Gauss-Jacobi method (7M)

$$8x - 3y + 2z = 20,\ \ 4x + 11y - z = 33, 6x + 3y + 12z = 35$$
~~~~~~

UNIT-IV

7. a) Fit a polynomial for the following data (7M)

$$y_0 = -5,\ y_1 = -1,\ y_2 = 9,\ y_3 = 25,\ y_4 = 55,\ y_5 = 105$$

b) Find the y(4) for the following data (7M)

x	0	2	3	6
y	707	819	866	966

Or

8. a) Prove that $1 + \mu^2\delta^2 = \left(1 + \frac{1}{2}\delta^2\right)^2$ (7M)

b) Interpolate by means of Gauss's backward formula, the population of a town for the year 1974, given that (7M)

year	1939	1949	1959	1969	1979	1989
population	12	15	20	27	39	52

UNIT-V

9. a) Evaluate $\int_0^1 x^3\,dx$ using Simpson's 1/3rd and Simpson's 3/8th Rules. (7M)

b) Find y(0.1) using Picard's If $\frac{dy}{dx} = 2e^x + y$,y(0) = 1 (7M)

Or

10 a) Find y(0.1), y(0.2) using Taylor's series method If $\frac{dy}{dx} = e^x - 2y$,y(0) = 1 (7M)

b) Find the solution of $\frac{dy}{dx} = x - y$, y (0) =1at x=0.1, 0.2, 0.3, 0.4& 0.5 using Euler's method. (7M)